CONSIDERATIONS OF FOCUSING A SUPERCONDUCTING HELIX - PROTON - LINEAR ACCELERATOR BY ALTERNATING THE SYNCHRONOUS PHASE

H.P. Junior, J. Klabunde, H. Deitinghoff, H. Klein
Institut für angewandte Physik der Universität
Frankfurt/Main, Germany

ABSTRACT

The high acceleration rate achievable in a superconducting helix linac as discussed recently suggests a reappraisal of the focusing method, where adjacent helix sections are tuned such, that the synchronous phase changes periodically. This method proves rather weak in a conventional linac with 1 MeV/m. Nevertheless at acceleration rates as high as 3 - 4 MeV/m axial and radial acceptances of particles turn out to be comparable to those of a machine focused strongly by quadrupole-doublets.

Computations at proper helix parameters are discussed for a proton structure similiarto the 0.75 - 20 MeV machine, which is part of the prototype of a superconducting linac designed at the Kernforschungszentrum Karlsruhe.

I. Introduction

The properties of the helix as an accelerating structure have been studied at our institute for some years now[1,2,3]. In cooperation with the KFZ Karlsruhe this structure is designed for the energy range from 0.75 to 20 MeV as a part of the prototype of a superconducting linear accelerator for protons.

The high acceleration rates of 3 to 4 MeV/m suggest to reconsider[4] the focusing method[5,6,7] of alternating the synchronous phase of particles by tuning of the RF phase in adjacent helix sections. This can be obtained quite easily, when adjacent helices have proper distances or shieldings.

This paper contributes a short survey of the principle, the foundations of our calculations and some primary results of computed radial and axial acceptances.

II. Foundations

Beam dynamics have been treated extensively in [2,8,9], the system of equations describing the axial and radial particle motion can be written quite generally:

$$m\,\ddot{z} = I_o(\gamma r)\; e\, E_o \cos\left(\omega t - \int_o^z k(z')\,dz' - \rho_s - \Delta\rho\right)$$
$$m\,\ddot{r} = -\frac{k}{\gamma}\, I_1(\gamma r)\; e\, E_o \sin\left(\omega t - \int_o^z k(z')\,dz' - \rho_s - \Delta\rho\right) \qquad (*)$$

I_o and I_1 stand for the modified Besselfunctions of zero and first order, ω, E_o, ρ_s represent frequency, amplitude and synchronous phase of the axial symmetric RF-field, γ and k stand for the radial and axial wave numbers respectively. As there does not exist any analytic solution of the system (*), linear approximation is used in order to gain stability with respect to helix parameters. The linear equations can be derived by means of expanding the Besselfunctions into power series and taking the first terms only:

$$m\,\ddot{z} = e\,E_o \cos(\omega t - \int_o^z k(z')\,dz' - \rho_s - \Delta\rho)$$

$$m\,\ddot{r} = \frac{1}{2}\,k\,r\,e\,E_o \sin(\omega t - \int_o^z k(z')\,dz' - \rho_s - \Delta\rho)$$

This restricts us to small radial deviations. If we restrict ourselves as well to small axial deviations $\Delta z = z - z_s$, where z_s stands for the synchronous axial co-ordinate of the particle, the system degenerates into the two linear equations:

$$\frac{d^2}{dt^2}(r) - \frac{e\,\omega\,E_o \sin\rho_s}{2\cdot m\cdot v}\,r = 0$$

$$\frac{d^2}{dt^2}(\Delta z) + \frac{e\,\omega\,E_o \sin\rho_s}{m\cdot v}\,\Delta z = 0 \qquad (**)$$

As can be seen easily from these expressions, focusing and defocusing alternate with respect to both components of motion, if the synchronous phase alternates between - say $+30^o$ and -30^o - in adjacent helix sections. Then the axial motion corresponds to a FD channel, the radial motion does to a DF channel. For stability of motion both phase shifts μ (defined by one half the trace of the corresponding transfer matrices of two adjacent helix sections) must satisfy the condition $0 < \mu < \pi$. Iso-μ-curves within this range are shown in fig. 1. These Iso-μ-curves are drawn at different excitations

$$Q_r = \sqrt{\frac{e\,\omega\,E_o|\sin\rho_s|}{2m\cdot v_s}}\;\tau$$

$$Q_{ax} = \sqrt{2}\cdot Q_r$$

of helix sections with an average synchronous phase velocity v_s and a particle transit time τ through a helix section. For cooling reasons[10] it is advantagous to construct superconducting helix sections modulo $\beta\frac{\lambda}{2}$ and τ may then be written:

$$\tau = \frac{n}{2}\,\frac{\lambda}{c} \qquad n = 1,2,3\ldots$$

where c stands for the velocity of light and λ for the wave length of the travelling wave on the axis.

By way of synchrotron theory which we successfully used in our designs[3,8,11,12], helix parameters were varied and maximum acceptances computed. With the parameters gained within this linear pattern, the non linear equations (*) were then numerically computed, the principle of these computations being described in[9]

III. Results

Some primary results are presented in the following figures and tables. Figures 2, 3 and 4 together with table I represent axial acceptances in phase space Δρ (phase deviation) -ΔT (energy deviation). In all examples the accelerator corresponds to a DF channel with respect to radial motion and a FD channel, when axial motion is regarded. It turns out, that comparatively large acceptances are achieved with parameters as chosen in example 3. The high maximum energy deviation, accepted by the linac, suggested the limitation of the output energy deviation. Corresponding radial acceptances of representative particles within these areas are shown in figures 5,6,7. Initial axial conditions are given in table II.

Again the acceptance areas for example 3 turn out larger and are comparable to acceptance areas of a conventionally AG-focused superconducting helix linac, where magnetic quadrupoles are used.

A more detailed comparision with a conventionally focused machine will be published elsewhere, as soon as we have more computational results.

IV. Conclusion

The computations suggest in the first instance acceleration rates as large as possible at relatively low frequencies. Comparing results derived when using the linear approximations (**) with those gained from solving the non linear equations (*) distinct differences are apparent. It is therefore absolutely necessary to compute in the non linear pattern.

Acknowledgement

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Table I

Example	$f=\omega/2\pi$ [MHz]	E_0 [MV/m]	Drift-length [cm]	τ_1	τ_2	Output energy resolution	Phase acc. envelop in figs. 2-4
1	90	3.464	0			not ltd.	————
			10	$\frac{3\lambda}{2c}$	$\frac{4\lambda}{2c}$	not ltd.	—·—·—
			10			1%	— — —
2	50	3.464	0			not ltd.	————
			10	$\frac{2\lambda}{2c}$	$\frac{4\lambda}{2c}$	not ltd.	—·—·—
			10			1 %	— — —
3	50	4.620	0			not ltd.	————
			10			not ltd.	—·—·—
			10			1%	— — —

Table II

Example	$f=\omega/2\pi$ [MHz]	E_0 [MV/m]	Initial axial conditions $\Delta\varphi$ [°]	ΔT [keV]	Normalized acceptance area [cm·mrad]	Radial acc. envelop in figs. 5-7
1	90	3.464	0	0	2.5	————
			0	-10		— — —
			-10	-15		…………
			20	100		—·—·—
			10	100		—x—x—
2	50	3.464	0	0	5.1	————
			0	-20		— — —
			10	0		…………
			-10	0		—·—·—
			0	200		—x—x—
3	50	4.620	0	0	7.2	————
			0	-20		— — —
			10	0		…………
			-10	0		—·—·—
			0	200		—x—x—

Examples of table II correspond to those of table I. Radial acceptance calculations did not include drift-lengths between the helix sections.

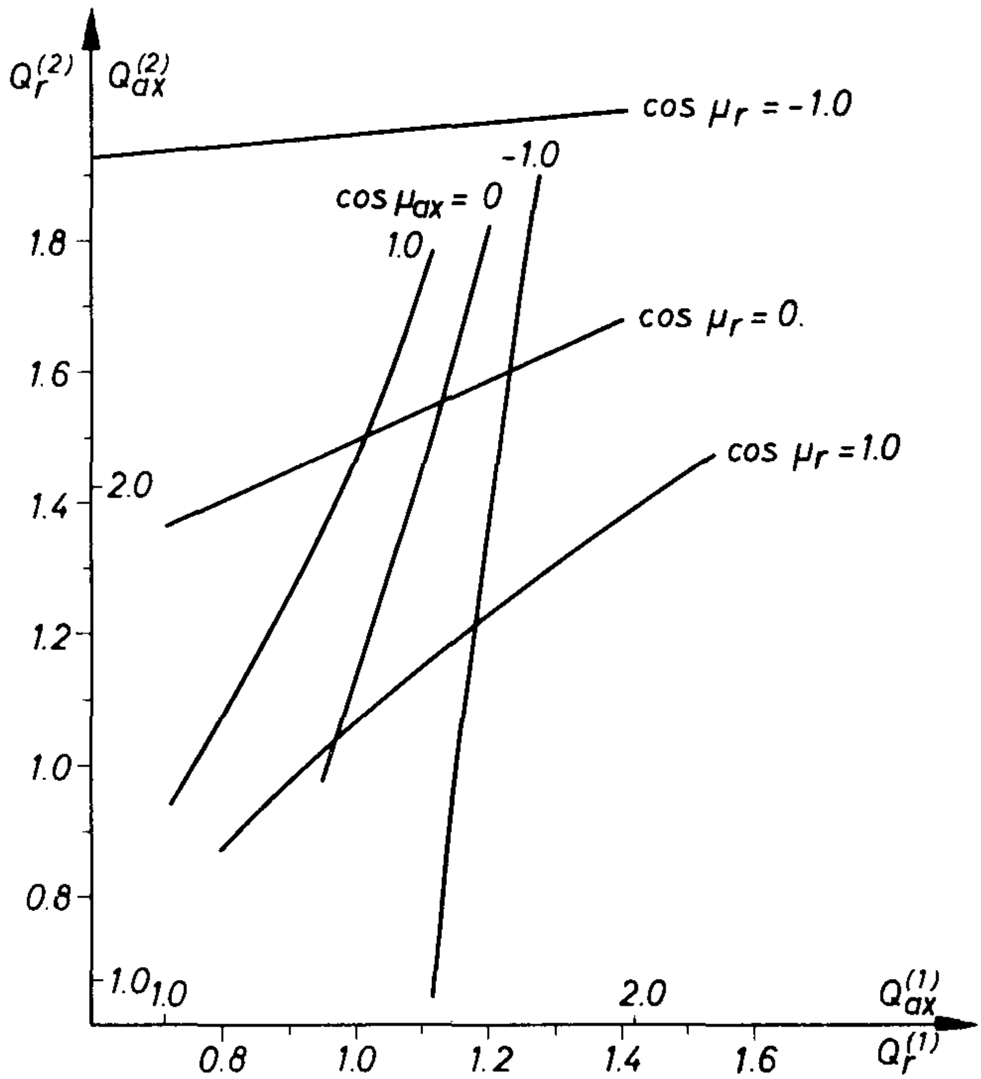


Fig. 1a

Iso - μ - curves at different excitations of two adjacent helix sections

1.section	axial focusing radial defocusing
2.section	radial focusing axial defocusing

$\tau_1 : \tau_2 = 3:4$

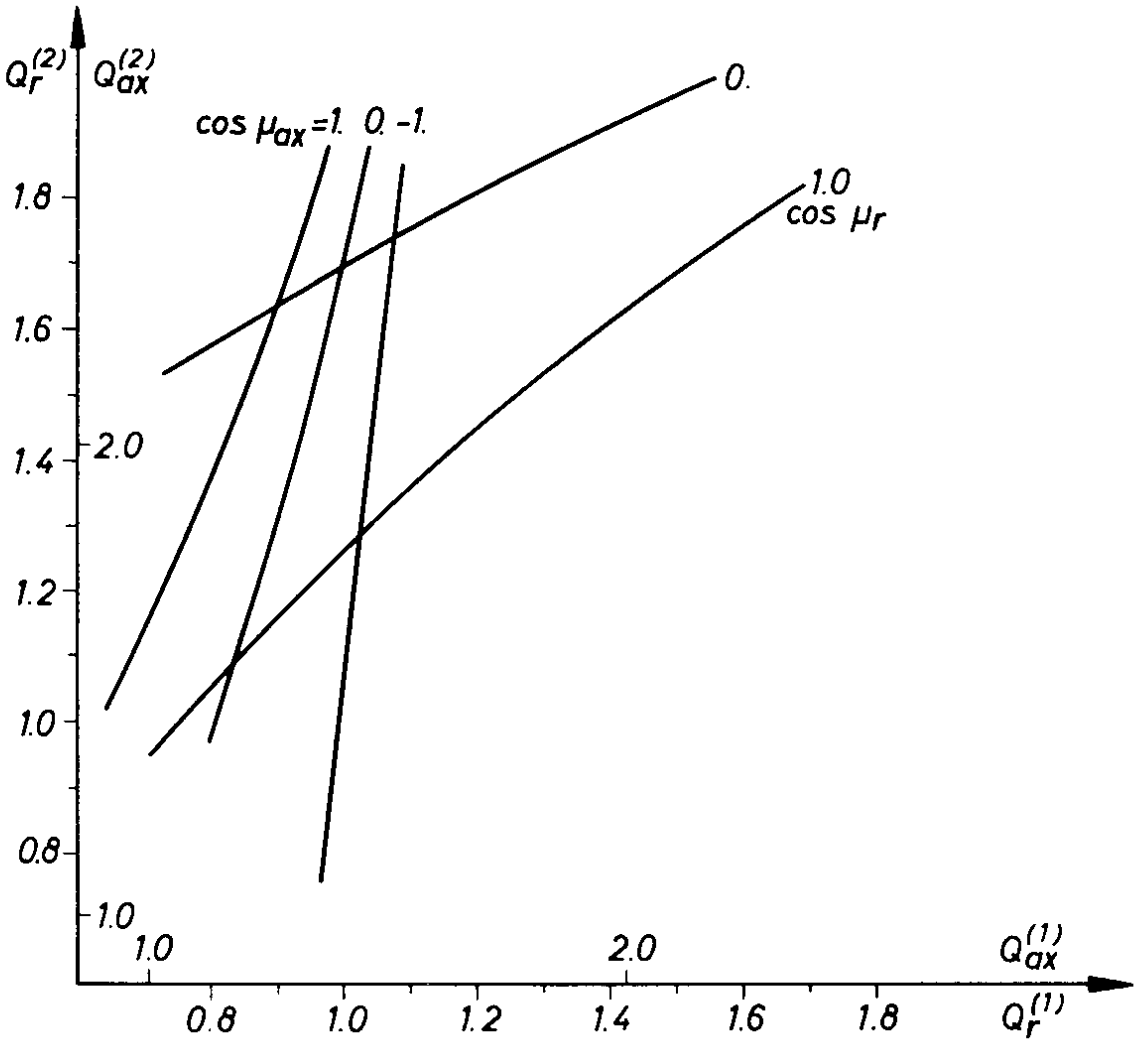


Fig. 1b
Iso - μ - curves at different excitations of two adjacent helix sections
1. section axial focusing
radial defocusing
2. section radial focusing
axial defocusing
$\tau_1 : \tau_2 = 1:2$

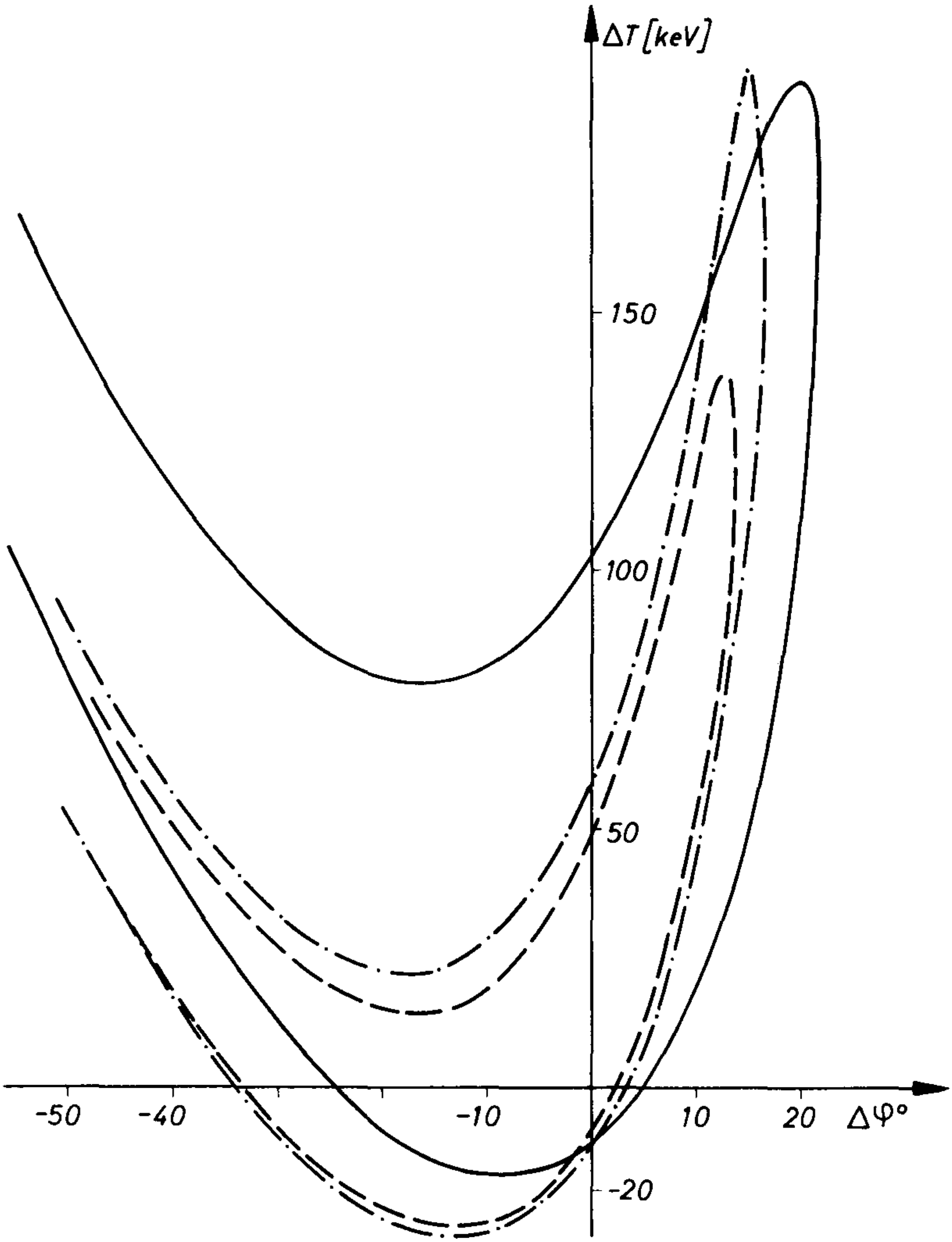


Fig. 2 Phase acceptances with $\varphi_s = \pm 30°$ corresponding to example 1 table I.

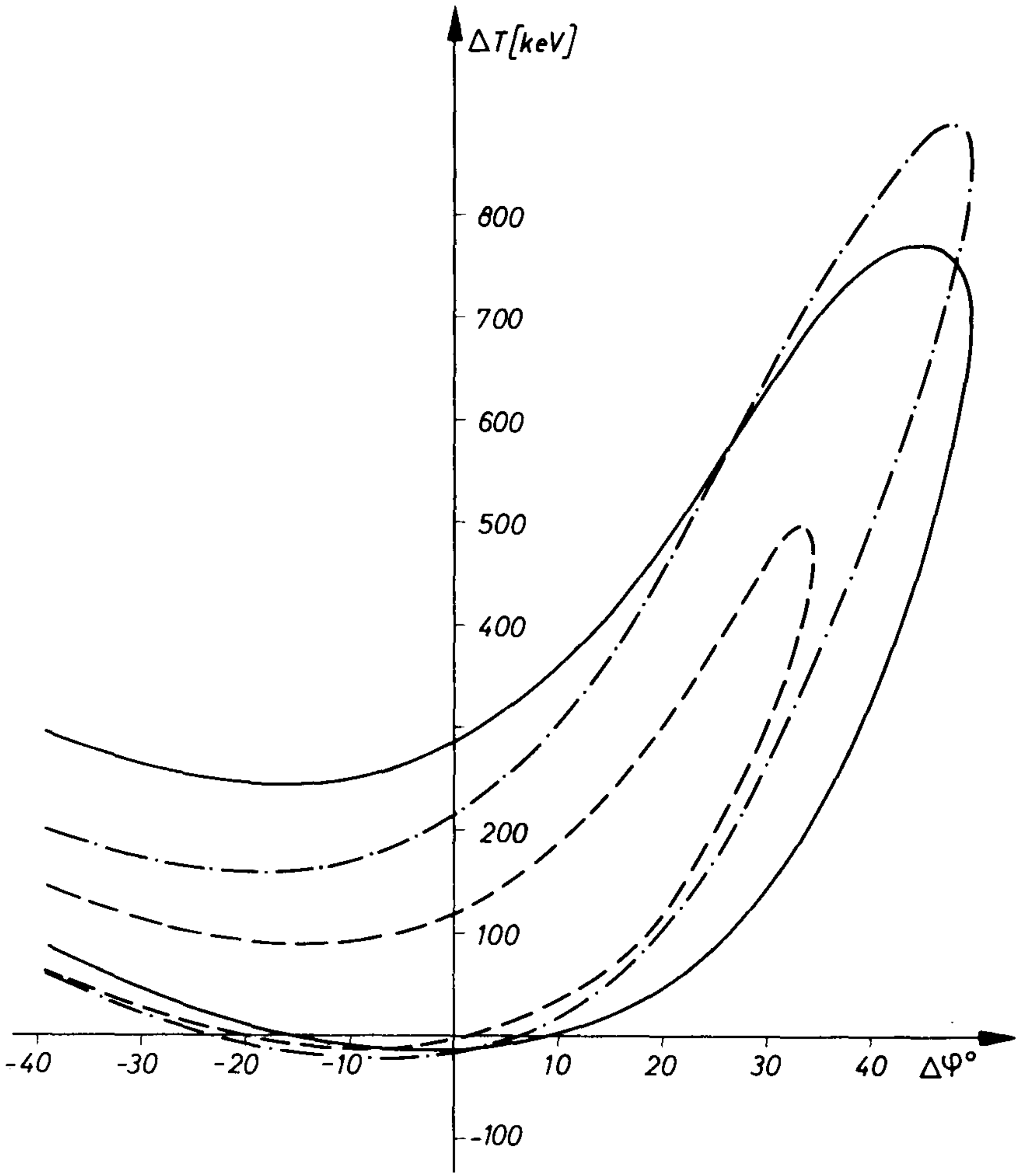


Fig. 3 Phase acceptances with $\varphi_s = +30°$ corresponding to example 2 table 1.

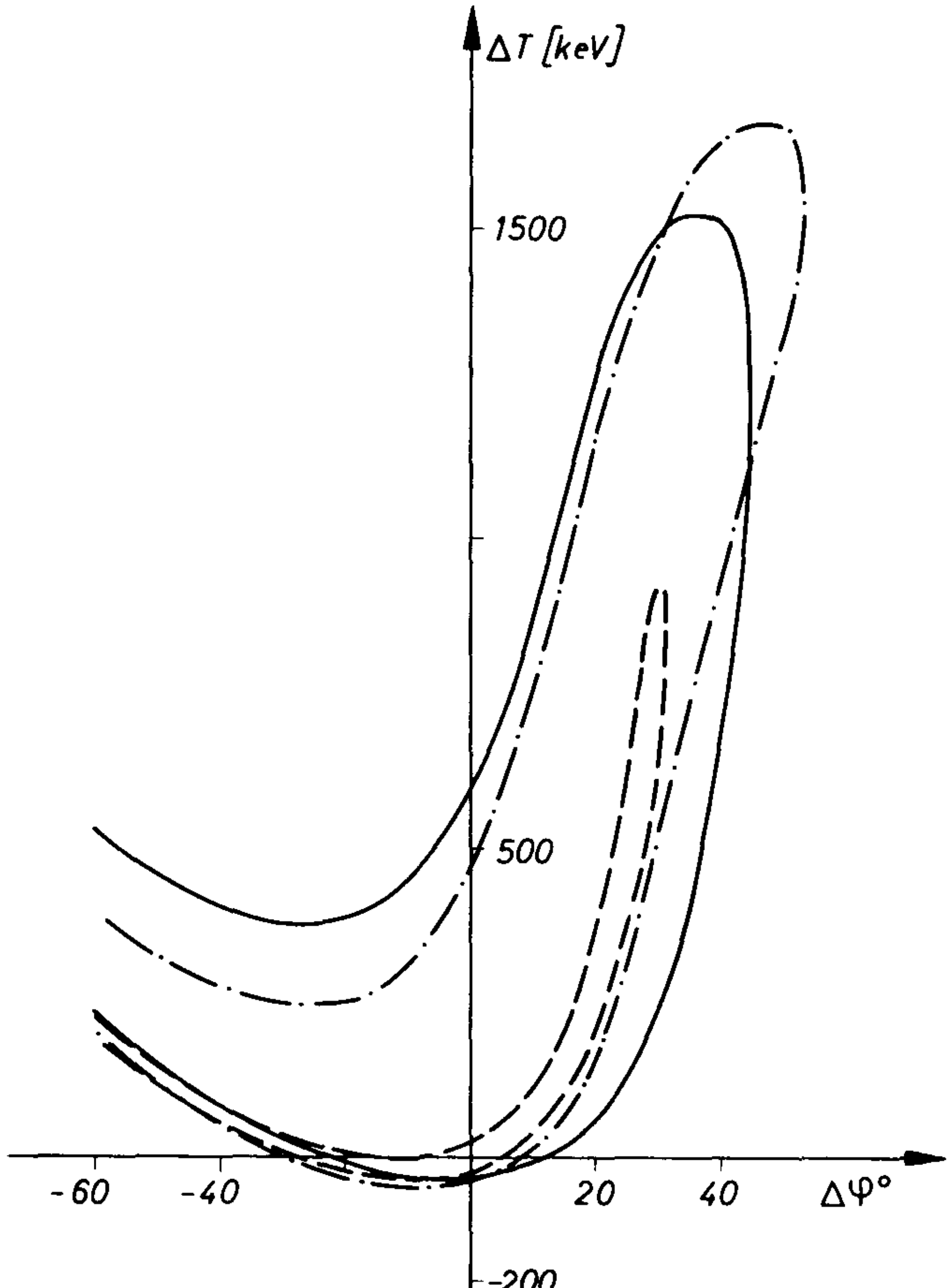


Fig. 4 Phase acceptances with $\varphi_s = \pm 30°$ corresponding to example 3 table 1.

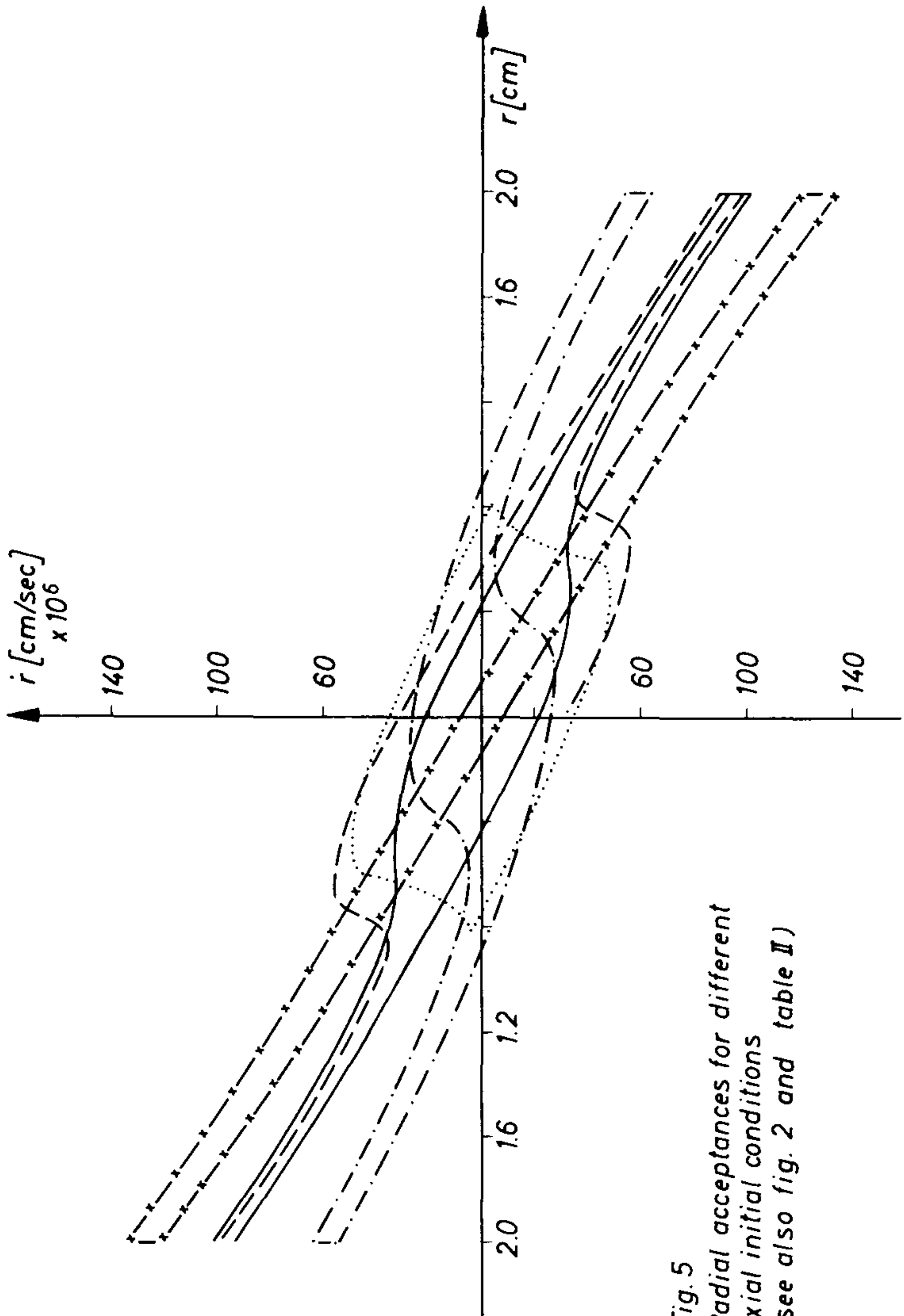
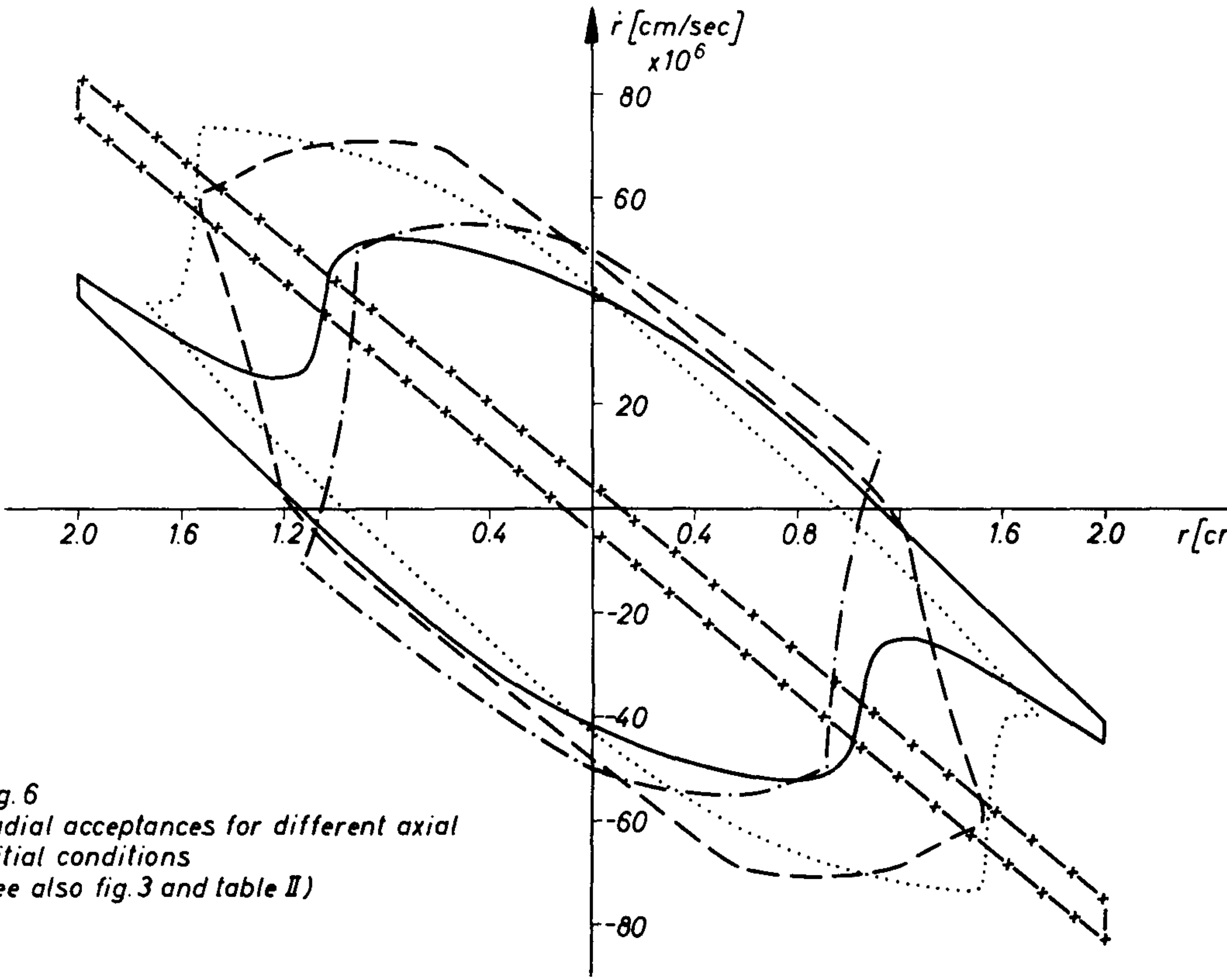


Fig. 5
Radial acceptances for different axial initial conditions (see also fig. 2 and table II)



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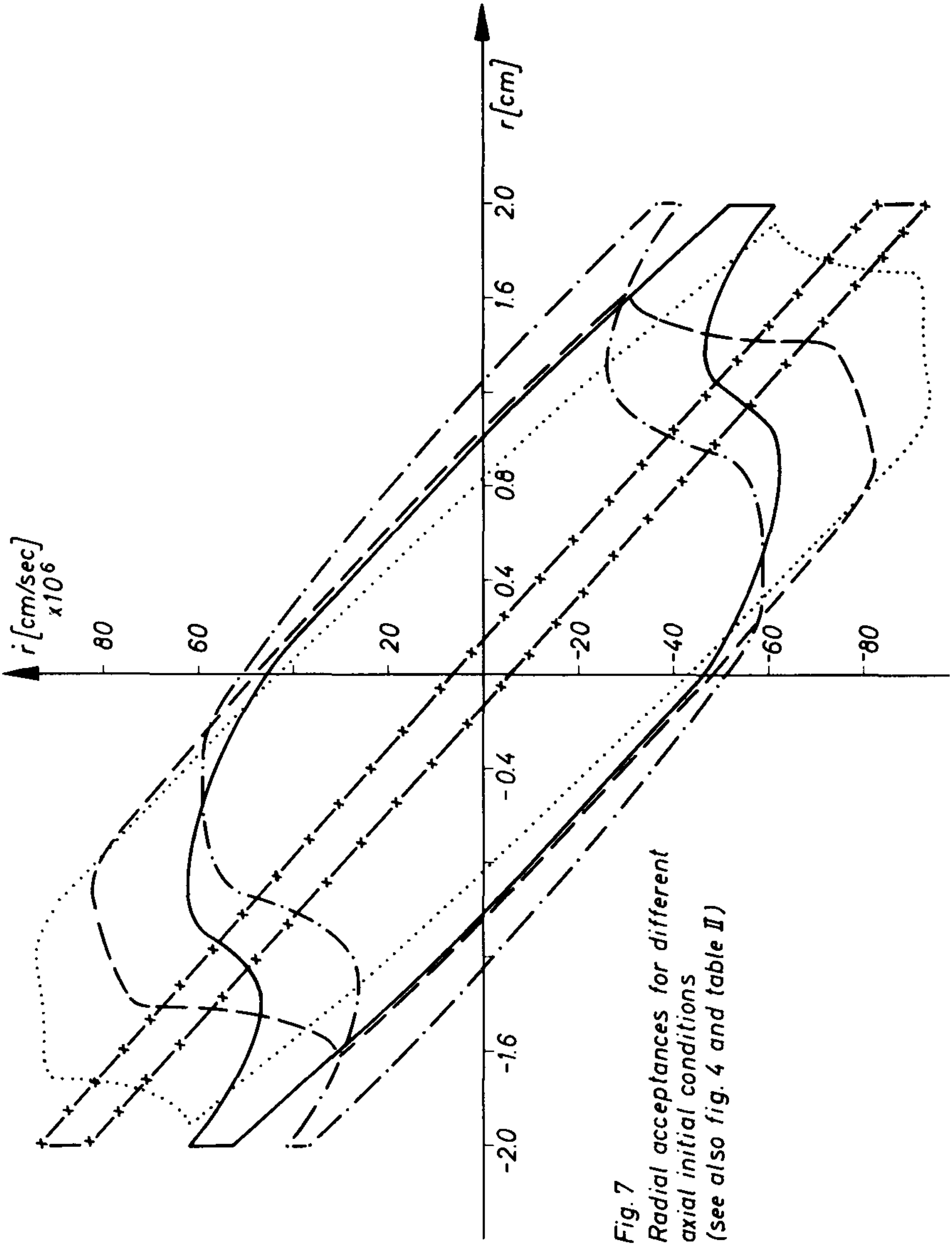

Fig. 7
Radial acceptances for different axial initial conditions (see also fig. 4 and table II)