THE PROBLEMS OF PARTICLE TRANSMISSION IN A HIGH-INTENSITY PROTON ACCELERATOR COMPLEX

C.S. Taylor European Organization for Nuclear Research Geneva, Switzerland

ABSTRACT

This paper discusses some of the general problems of high-intensity proton accelerators, that is the efficiency of the accelerator for the transmission of particles, and the allied questions of the optimization of the many variables which can affect the intensity, and the statistical behaviour of the machine. Some comments are made on operational aspects of high intensity machines, and on the feasibility of approaching phase space density conservation in proton accelerators.

Introduction

The purpose of this paper is to consider in a general way the problems of high intensity proton accelerators in series. The detailed behaviour of the beam in an accelerator is a complex matter and must take account of the 6-dimensional distribution of the particles in the input beam and its time variations, the topology and time variations of the electric and magnetic fields in the accelerator in the presence of the beam charge, the degree to which the mean values of these fields have been tuned or optimized to produce the highest output intensity possible given the input beam quality, and the natural fluctuations about these mean values. In the course of this paper, an attempt will be made to reduce this complexity to simpler terms without losing too much of the physical reality.

This study was inspired by the CERN working party $\frac{1}{2}$ which earlier this year investigated the effects of pre-injector improvements on I.S.R. interaction rates via the Linac-Booster-P.S. - I.S.R. complex.

The emphasis on optimization on this paper arises from the assumption that some empirical tuning around design values is required 1) at high intensities because of the action of non-linear space charge forces in machines designed primarily on linear theory, and 2) at any intensity if the electric and magnetic field topologies are not known exactly.

881

We shall first consider some beam descriptions and representations which seem appropriate to the high intensity situation.

The Beam Description

Since a particle requires the co-ordinates of positition and momentum, horizontal and vertical, and the axial energy and phase for the definition of its motion, the assembly of particles in a beam must be considered as a 6-dimensional population.

At present, experimentalists are able to measure distributions in the radial and vertical directions, but are not able to say very much about the longitudinal distribution. This is partly because of a more evident need for transverse quality but partly also due to the time resolutions required in direct measurements of phase distributions, calling for several GHz of bandwidth in the detector.

From the experimental work on the CERN-PS and Linac over the past few years, one generality which has emerged is that beams have near-Gaussian distributions of charge in the transverse planes. This has been observed at all energies except pre-injector energies (from slit and lens measurements in the range 10 to 50 MeV 2 , from ionisation beam scanner (I.B.S.)³ observations from 50 MeV to 25 GeV, and from target measurements⁴).

More generally still, it has been found that at those energies where space charge density distribution could be measured, i.e. at pre-injector energy and at 50 MeV, the current integrated out to constant phase space density contours in 2 dimensions follows closely an exponential "time constant" law of enclosed current against contour area^{5,6}. This is expected from a bivariate Gaussian distribution, but it was not obvious that this integral would remain valid for the pre-injector distributions which are quite far from Gaussian, or for the 50 MeV distributions whose contours are sometimes far from ellipses $^{\text{2}}$

This generalisation has led to the definition of an emittance constant or e-folding emittance $\frac{5}{5}$ in normalised units^{*}, which enables one to write down the current i within a given density contour of emittance ε for a total current I₀ as follows :

$$
i = I_0 (1 - e^{-\epsilon/\epsilon})
$$
 (1)

Further properties of this function are given in the Appendix.

If invariance applied to the 2-dimensional motion, then this figure ε_{α} should remain constant throughout acceleration. On the contrary it has been found experimentally that ϵ increases through the system of accelerators, and that the total current I aecreases. $\overline{\text{Area}}$ $\beta \gamma$

Let us now define the transmission efficiency n of an accelerator or section of an accelerator as the ratio of I^{\dagger}_{0} at the output to I^{\dagger}_{0} at the input, and a blow-up factor ξ as the ratio of ε [†] at the output to ε _o at the input.

The current i' within an emittance ε' at the output becomes

$$
i' = \eta I_0 (1 - e^{-\frac{\epsilon'}{\xi}} e_0)
$$

or more generally

$$
i' = \eta(I_0) I_0(1 - e^{-\frac{\epsilon^2}{\zeta(I_0)\epsilon_0}})
$$
 (2)

 \blacksquare

since both n and ϵ may be intensity dependent. At operational intensities, e.g. with 100 mA injected single turn, yielding around 1.5 x 10^{12} protons per pulse at 25 GeV, the value of η is in the region of 25-35% for the Linac and for the P.S. and ξ is about 3 in both cases.

It should be noted that although eq. (2) is a 2-dimensional description of an accelerator, the efficiency n includes the longitudinal trapping as well as the transverse input matching and losses within the accelerator, and the blow-up factor ξ might be expected to be sensitive to the input bunch length as well as to the transverse properties of the input beam. The expression is therefore a reasonably comprehensive sunnnary of the behaviour of the accelerator.

Eq. (2) gives the current within a given emittance contour at the output. In order to obtain the current input - current output or "transmission characteristic" from this expression, it simplifies matters if we assume that there will be a point ^A within the accelerator beyond which there is little loss, although perhaps continued blow-up. We then apply eq. (2) to the section from the input to the output considered to be at A. This will normally be near injection (where the transverse losses are mainly vertical in an A.G. synchrotron and isotropic in a linac).

If the normalised acceptance at point A is ε' , then a matched beam will be scraped down to the density contour of $\frac{\text{area}}{\pi}$ $\beta \gamma = \varepsilon'$ and eq. (2) will give the intensity at this point and, from our assumption, the intensity at the output of the accelerator, so that we can now consider the input/output or transmission characteristics for various conditions of n_1, ξ and ϵ'/ϵ .

There appear to be four main cases to be distinguished :

- l) No loss and no blow-up, i.e phase space density is conserved so that $\eta = 1, \xi$ and ε [']>> ε ₀. Linear transmission characteristic of slope = 1.
- 2) Lossy but no blow-up, i.e. η = constant <1 and ξ = 1 or emittance is conserved. Linear characteristic, slope given by η and $\varepsilon'/\varepsilon_{\eta}$.
- 3) Lossy with constant blow-up, i.e. $n = constant < 1$, $\zeta = constant > 1$; emittance not conserved. Linear characteristic, slope given by η and ε'/ε ₂.
- 4) Lossy and with current-dependent blow-up
	- a) if $n = constant$ and $\varepsilon' >> \xi \varepsilon_0$ characteristic will be linear, slope given by $n =$
	- b) if η = constant, ε' is not >> $\xi \varepsilon_0$ and ξ increases linearly with current, i.e. $\xi = (1 + kI_0)$, characteristic will approach exponential as kI₀ becomes \rightarrow 1,
	- c) if η decreases with current while ξ increases, the trend will be as in 4 b) modified by n .

In the latter cases, 4b) and 4c), one will find the curve of diminishing returns, or in the limit, "saturation", familiar from accelerator observations or numerical computations with space charge.

Experimentally, this curve presents many difficulties. In the case of the P.S., by using sieves of varying transparency at the output of the Linac one can vary the amount of charge transmitted without significantly changing the emittance constant or the energy spread immediately after the sieve, but the matching and energy spread further downstream at the P.S. input will depend on the charge, as will the adjustments of the multi-turn injection parameters and the P.S. lenses and corrections for best performances, so that the whole system should be carefully re-tuned at each value of injected current, a lengthy procedure.

This process would have to be carried out if one wished to identify the accelerator's behaviour with one of the cases described above. However, in what follows we shall be asking whether the sensitivity of the output fluctuations to the input fluctuations is related to the changing slope and the flattening-off, and so a simpler experiment, in

i.e. the normalised emittance which will fit in the acceptance. If the machine limits the acceptance equally in the vertical and horizontal planes, then the

 $(1-e^{-\epsilon^{\dagger}/\epsilon}$ ₀) term should be squared, assuming no correlation (see Appendix).

which the P.S. was set up at full intensity and left untouched at lower injected intensities, has been carried out with the results shown in Fig. 2.

Each point represents the mean of 100 pulses, the data having been collected by the STAR acquisition system and processed on-line by the IBM 1800 control computer. The points are labelled with correlation coefficients between the observed input and output values.

In order to understand better what is implied by this transmission characteristic it is necessary to consider further the associated problems of optimization, and the pulse-to-pulse reproducibility and statistical behaviour of an accelerator.

Optimization and Reproducibility

The number of parameters which can affect the output beam in a machine such as the P.S. is very large. The effects of anyone variable will in general depend on the values of the others, and the value of the output intensity will be a function of the n variables can be thought of as an n-dimensional surface. The practical problem is to find and stay at the set of n values which gives the maximum intensity either by manual tuning or by computer search⁸. The transmission efficiency η will therefore depend on where one is placed on the n-surface, or how well the machine is tuned. It may be looked upon as a statement, rather more complete than the intensity alone, of how the machine is behaving at the moment of measurement, and may change from run to run or within the run. In the linac, n is affected by the quality of the input beam and by adjustments of the accelerator under the control of the operators, whereas the P.S. is vulnerable also to physical changes in the ring which can influence the magnetic fields near the orbit.

The significance if the pulse-to-pulse reproducibility is as follows: For any optimization process which depends on detecting a change in the mean value of the output intensity when we change one of the parameters, including manual tuning, we are interested in obtaining a good estimate of the mean in as few pulses as possible. It can be shown that the number of readings required to get wi thin a given percentage of the true mean with a given probability is proportional to the square of the coefficient of variation, i.e. to the dimensionless ratio of standard deviation to mean, e.g. if a fairly reproducible machine with, say, 2% coefficient of variation in the output current degenerates to 6% it will take 9 times as long to optimize.

A secondary aspect of pulse-to-pulse variation in high intensity operation is that the effectiveness of programmed compensation or correction (beam loading, closed orbit, etc.), set presumably for the mean values, will be reduced if instantaneous values of the beam intensity depart very widely from the mean and conceivably this effect might even magnify the original variations under some conditions.

Statistical Behaviour

Correlations

One question which one would like to have answered for a series of accelerators is whether the fluctuations in intensity which occur from pulse to pulse at the output of an accelerator tend to build up from stage to stage. We shall ask first whether these output fluctuations are correlated with the input intensity fluctuations. In the simple model in which the noise at the output is considered to result from the input noise plus some independent noise generated within the accelerator, it can be shown that if $(\sigma_1)^2$ is the variance of the input noise, $(\sigma_2)^2$ is the variance of the noise generated in the accelerator and the output current I_{0}^{t} is a function $f(I_{0})$ of the input current (e.g. Fig. 2), then the currelation between the observed values at the output and the input is given by (3)

corr (input, output) = $\frac{1}{100}$ in the linear approximation

where f' is the first derivative of the function $f(I_0)$. This predicts that the correlation will be small if the internal noise (σ_2) is large with respect to the input contributed noise $(f' \circ_i)^2$ and conversely will approach unity if the internal noise is small with respect to $(f^{\dagger} \circ_1)^2$. In general one expects a smaller correlation as the function flattens off at high intensity than in the steeply rising region, but this does not always happen. The simple model assumes the $(\sigma_0)^2$ fluctuations in the accelerator to be independent of $(\circ_1)^2$, but $(\circ_2)^2$ is also affected by jitter in the input beam energy and energy spread, and in the transverse position etc., properties which themselves may be strongly correlated with the input intensity fluctuations.

This reminds one that the output intensity is a function of n variables, so that the concept of the n-dimensional surface^{*} of the optimization discussion should form a hetter basis for understanding. The input current now becomes one of the n variables. If the machine is optimized at one intensity, then by definition the trimming of the mean values of the n variables has conducted one to the peak of the n-surface. In practice this could also be a subsidiary peak. Here the regression line of the output intensity on anyone variahle will have zero slope and the correlation will be zero. Therefore at each measured point on an optimized transmission characteristic $(p.5)$ one should find a zero correlation coefficient between the input and output intensities. Away from the peak, the scatter diagram of output intensity against input intensity will depend on the local slopes of the other n-1 parameters, which will include any couplings between these and the input intensity, and in principle a range of values of the correlation coefficient between input and output is possible. Therefore on the unoptimized

The surface is determinate in the sense that if the values of all variables affecting intensity were known, the output intensity would be exactly predictable.

curve of Fig. 2 the low coefficient at full intensity could be a confirmation of the original optimization, whereas the coefficients at lower intensity have no particular significance in this context.

A provisional answer to the original question then is that if each accelerator in a chain is optimized, the fluctuations at each input will contribute little to the output, whereas off-maximum a wide range of possibilities exists .

One additional observation must be mentioned. Print-outs of the cumulative sum (CUSUM) i.e. the difference between successive observations and the mean value summed arithmetically, show that there can be a time structure on the mean values, e.g. the linac input might hold a steady mean value of 300 mA for 30 pulses, then jump to 305 mA and so on. It seems that not only does the intensity wander around the n-surface from pulse to pulse, but it also takes little jumps every now and then, as occurs when the mains voltage varies.

Intensity Distributions

When the number of beam pulses having a certain intensity is plotted against the intensity, one obtains a histogram or intensity distribution. It has been found that this distribution at working intensities is usually skewed towards the high values at 550 keV, 10 MeV, 50 MeV and 25 GeV. This observation was in fact one of the reasons for the explorations of input-output correlations, as high correlations would tend to preserve skewness. Since however the correlations at working intensities were normally small, an alternative hypothesis that skewness is associated with optimization was investigated. This is not unreasonable, as optimization might Le thought of as the process of forcing the mean value towards the maximum value possible.

W. Eadie⁹ has now shown for the 2-dimensional case that there is an association between the distribution and the state of optimization. The model taken was a 2 dimensional Gaussian hill for the performance index or intensity, and a 2-dimensional Gaussian for the variations of the control variables about their means, centred on the maximum of the hill. If the standard deviations (assumed equal) of the control variables with respect to the standard deviation of the hill is σ , the distribution P(Y) of the intensity Y is

$$
P(Y) = \frac{1}{\sigma^2} \cdot Y^{\left(\frac{1}{\sigma^2} - 1\right)}
$$
 (4)

which piles the values up towards the maximum when σ is small and towards the minimum when \circ is large. Next, the case of an off-centred control point was taken, using a

non-centrality parameter λ defined as the distance off-centre divided by σ^2 . This leads to the distribution P(Y) for the non-centred case

$$
P(Y) = \frac{1}{\sigma^2}, Y \begin{pmatrix} \frac{1}{\sigma^2} - 1 & e^{-\lambda/2} \\ e^{-\lambda/2} & I_0 \end{pmatrix} \begin{pmatrix} \frac{2\lambda}{\sigma^2} & 1 & Y \\ 0 & 0 & 0 \end{pmatrix}
$$
 (5)

where I_{α} is the modified Bessel function of order zero; which can be generalized to n degrees of freedon as (6)

$$
P(Y) = \frac{e^{-\lambda/2}}{\sigma^2 - \lambda^2} \left[\frac{-2\lambda}{\sigma^2} \ln Y \right]^{\frac{n}{2} - 1} \left(\frac{1}{Y^{\sigma^2}} - 1 \right) I_{\frac{n}{2} - 1} \left(\frac{-2\lambda \ln Y}{\sigma^2} \right)
$$

where $I_k(x)$ is the modified Bessel function of order k.

So far numerical experiments using the CDC 6600 and Tektronix display have been carried out for 2 dimensions. For a given ratio σ , one finds that moving the control point away from the centre changes the distribution from the piled-up at the maximum condition, through a skewed-to-the right state to a normal distribution (Fig. 3).

Fig. 4 shows the histograms measured at 550 keV, 10 MeV and 50 MeV after careful setting-up of the Linac. Since the optimization criterion is the 50 MeV intensity for everything including the ion source, the 550 keV and 10 MeV intensities are not necessarily at their maxima, and this is seen from the increase in skewness factor from 1.0 at the input to 7.0 at the output.

We shall now leave these problems to look briefly at some operational aspects of accelerators running in series at high intensities.

Transfer Instrumentation

One can assume that the user of an accelerator, which in a complex includes practically everyone except the ion source man, is best served by a steady beam which can be interrupted on demand and then returns immediately to its previous value. Departures from this ideal are usually expressed by the fault rate, i.e. the time off due to faults as a percentage of the scheduled beam time. The fault rate, together with the mean intensity and standard deviation, form a reasonable criterion for the efficiency of operation of an accelerator. At the transfer points between accelerators though it is useful to have some indications of quality in addition to intensity.

It is sometimes said that a machine itself is the best analyser of the beam quality at its input. This is true if the machine is working normally, but when the machine cannot be properly tuned up it is necessary to determine quickly whether **it** is the input beam that has deteriorated in quality, or the machine, or both.

At the present moment one can rather easily, at least at energies up to hundreds of MeV, include in the beam transfer system an arrangement of apertures and lenses which define a known rectangular window in each transverse phase plane, scaling these window acceptances to be approximately equal to the acceptances of the downstream machine. The input beam pulse will now contain only the transverse motion which can be accepted and will be sensitive to variations in the density, emittance orientations, and position produced by the upstream machine, from pulse-to-pulse and within the pulse. This arrangement, effectively a 4-dimensional average density indication, can also provide a more stringent performance index for the optimization of the previous machine.

The extension to 5 or 6 dimensions is a good deal more difficult, and one must at present rely on the pulse-to-pulse reproducibility and carry out the quality measurements off-line on intermediate pulses. This introduces uncertainties in principle should one be looking for correlations between output intensity and input energy spread for example, but the hardware solutions of bending magnets, corrections, deflecting cavities and so required by 6-dimensional collimators seem impractical at the moment. on

General Comments

Eq. (2) was expressed in current mainly because this is what one measures in Linacs. For a circular machine, the current formulation can be retained by converting the circulating beam charge back into single-turn current at injection energy.

Applying this to the P.S., one finds that for multi-turn injection the central phase-space density I_0/ϵ is reduced through the accelerator to around 1/8th of its input value, assuming the value of 3 for the blow-up (recently measured for multiturn injection⁴ but not for single-turn). A very similar situation prevails in the Linac where the factor is about 1/7.5. This means that the 25 GeV central density at 25 GeV is 1/60th of the pre-injector central density.

One way of appreciating this loss is to consider the present pre-injector beam to be accelerated to 25 GeV without blow-up or loss, which would result in beam diameters at 25 GeV of 3.6 x 2.7 mm for 95% of the beam with an intensity $\,$ of 1.7 x 10^{13} ppp, with corresponding small diameters at injection. As a practical aim, this performance seems unrealistic, but it may be a valid direction in which to go in order to reduce magnet apertures and costs.

With Linacs, there is something to be gained by raising the pre-injector energy as shown by a recent study 10. At 1.5 MeV the computations showed a more linear transmission and an increase in intensity of 70%, although with diminishing returns (Fig. 5)

 \mathop{c} Encouraging results with high transmission efficiencies have recently been reported from Brookhaven (Batchelor et al. , this Conference). '

and the question of apertures and structure design would need also to be re-examined.

In the case of circular machines, improved designs in the future will depend a lot whether we can achieve a balanced view of what is limiting to-day's machines, the Serpukhov P.S., the B.N.L. A.C.S. and the CERN-P.S., to name the largest. That is, we need to be able to assess the importance of Q-shift effects, compared with other effects which are energing such as emittance 11 and energy spread growth 12 and problems at transition¹³, and eventually synthesize the approaches of dynamic instabilities and quasi-irreversible process. In the author's view, we also need to know to what extent automatic control could be used to keep the Q values constant and independent of intensity during the acceleration cycle, to minimize the closed orbit amplitudes and to reduce the effect of resonances.

Conclusions

In this paper, the current distribution in the limiting transverse phase plane has been related to the input-output transmission properties of an accelerator, by means of a transmission efficiency η and a blow-up factor ξ . The nature of the transmission characteristic has been elaborated in terms of the optimization concept and the statistical behaviour of an accelerator. It has been concluded that input-output correlations will be minimised when the accelerator is optimized for maximum output intensity, and the relation between optimization and the resulting intensity distributions from pulse to pulse has been demonstrated. The requirements of transfer instrumentation have been discussed and some comments have been made on the problem of density conservation in accelerators as a practical aim.

Acknow ledgemen ts

The author wishes to acknowledge the invaluable help given by W.T. Eadie in the statistical studies, by :1.A. Nour Eldin of the Hybrid-Computer Centre E.T.H. Zurich for helpful discussions on optimization, by J.P. Benincasa for his aid in the experimental runs and in the discussion of their interpretation, by K.D.H. Pedersen for the analysis of emittance data and useful discussions,and by T.R. Sherwood for kindly checking the draft and making many clarifying suggestions.

Appendix

The geometrical interpretation of the relation

$$
i = 1_0(1 - e^{-\epsilon/\epsilon} \circ)
$$

is as shown in the sketch, where i is *the* current enclosed within a contour of emittance $\frac{\text{area}}{\pi}$ $\gamma = \epsilon$ and ε is the emittance constant. For a bivariate

Gaussian in (x, p_y) , ε_g encloses 63.21% of the total current and represents the product of $\sqrt{2}$ σ_{x} and $\sqrt{2}$ σ_{px} .

For reference, $2\varepsilon_0$ encloses 86.47% (20) and $3\varepsilon_0$ encloses 95.02% ($\sqrt[3]{6}$ σ).

The central density is given by the initial slope: I_n/ε_n .

When the horizontal and vertical motions are uncorrelated $\frac{2}{\lambda}$, we can write, for matched horizontal and vertical acceptance A_{μ} and A_{ν} respectively

$$
i = I_0 \t(1 - e^{-\frac{A_H}{\epsilon_0}}) \t(1 - e^{-\frac{A_V}{\epsilon_0}})
$$

For multiturn injection, assuming that A_H is shared equally between n turns¹⁴, 15

$$
i = n I_0 (1 - e^{-\frac{A_H}{n \epsilon}}).
$$

References

- C. Bovet et al. Evolution of Proton Density between Ion Source and ISR. \mathbf{I} . CERN/MPS-SI/Int. DL/70-7. May 1970.
- $2.$ W.T. Eadie and M. Martini. Statistical Population of the Linac Beam Transverse Phase Space. MPS/int. LIN 68-9. December 1968.
- C.D. Johnson and L. Therndahl. The CPS Gas-Ionization Beam Scanner. pp 909-913. 3. IEE Trans. Nuclear Science, Washington Conf. 1969.
- E. Brouzet, C. Johnson, P. Lefèvre. Mesures des dimensions verticales du faisceau 4. du PS. MPS/DL - Note 70-21. August 1970.
- 5. P.M. Lapostolle, C.S. Taylor, P. Têtu, L. Thorndahl. Intensity Dependent Effects and Space Charge Limit Investigation on CERN Linear Injector and Synchrotron. CERN 68-35.
- K.O.H. Pedersen. Private Communication. 6.
- 7 . M. Martini. Computer Simulation of Linac Beam Dynamics with Space Charge. CERN/MPS/LIN 69-20. December 1969.
- 8. A. Daneels. On-Line Optimization of the Proton Synchrotron Closed Orbit at Injection. CERN/MPS/CO 70-4.
- 9. W.T. Eadie. Report in preparation.
- 10. P. Bernard, J. Huguenin, U. Tallgren, M. Weiss. Proposal for a Higher Energy Pre-Injector. This Conference.
- 11. P.M. Lapostolle. Quelques propriétés essentielles des effets de charge d'espace dans des faisceaux continus. CERN/ISR/DI 70-36.
- 12. J. Bittner. Private Communication, BNL Linac Conf. 1968.
- 13. E.D. Courant. Some High-Current Effects on Particle Accelerators. 7th International Conf. On High-Energy Accelerators. Yerevan 1969.
- 14. Minutes of the 5th Meeting High Brilliance Working Party. MPS-SI/Min. DL 69-7. 23.10.1969.
- 15. C. Bovet, D. Lamotte. Numerical Analysis of the PSB Multiturn Injection. CERN/SI/lnt. DL 69/13. 30.12.1969.

Standard deviation = 9.2 Distance of control point waries from 0.0 to 1.0

Fig. 3. Intensity histograms with varying off-centre parameter.

```
177.097.75 - 0.009.281.5011STAR ADR. THEAN ZALOF THIS SERVICE SEPTERESS
        u(0) 2 u(545539975+0) 7.538305288+00 - 1.001561745+00
           No. of Pulses
        217 122 13\label{eq:3.1} \hat{\mathbf{a}} = \mathbf{a} + \mathbf{a} + \mathbf{a} + \mathbf{a} + \mathbf{a}\sim 1612 - 14 - 16\sim1.3-20-222h218 +219 *****<br>220 *****<br>221 *
        222.........\frac{223}{224} *********
        725 + 11226 + \cdots227228 -*********
           ----------<br>*************
        229730Intensity
           231\frac{232}{255}. . . . . . .
            *********
       234235*************************
            7362.5.7
            ***************
   \vec{E}738*************************
                                                          \mathcal{A}_{\mathcal{A}}239**********
            ****************************
       2.9 - J2\,4\,1*************************
       742*************************
       243**************
                                                                     Fig. 4a
            *********************
       244550 keV Intensity Histogram
       245\bullet .
       246**************
       247......7\,4.8\ddot{\phantom{a}}249\bullet250
            \ddot{\phantom{0}}STAR ADR. MEAN VALUE 5THD, JEV.
                                           SKEL IFSS
    0003 1.07899993F+82 6.72638130F+00 - 5.423444745+80
        No. of Pulses
                                                                                                   2418 - 20\sim 2.2\mathbf{0}\sim 2 and \simContact Contact
    \begin{smallmatrix}8&8\8&9\end{smallmatrix}\ddot{\phantom{1}}30\,*****
    \begin{array}{c} 91 \\ 92 \end{array}\ddot{\cdot}93\bullet9<sub>h</sub>\bullet\frac{95}{96}...97......98
    99
       *********
Intensity
   100 + \cdots101102\,*********
        103
   104
   105
        **************************
        *********************
   106
        107
       --------------
   108
        109
   110
   111 ******************************
        112
   113 **************************
   Fig. 4b
                                                                           10 MeV Intensity Histogram
   116 - \cdots117 - \cdots118 +\frac{119}{120} :
```
 $\overline{5}$

 \star \star \star \star $\frac{8}{1} \frac{d}{dx}$

DISCUSSION

D. R. Machen (LASL): Do you have any plans for eventually using the PS control computer on this problem?

C. S. Taylor (CERN): The IBM-1800 control computer was used to collect these data and to work up the correlation coefficients.

L. C. Teng (NAL): I think the moral of your last remark applies very well to the $500-$ GeV machine. I think that what you are saying is that in order to optimize it at 500 GeV, one may not want to optimize it at 200 MeV , then at 8 GeV , and so on. Do you have a suggestion now how to do this? The usual procedure is to optimize at each step. You are posing a difficult problem

C. S. Taylor: One uses processes of iteration. The initial criterion for setting up a machine are not the same as the optimum ones found after a few weeks of operation, but they are a good starting point.