EMITTANCE MEASUREMENT IN A 200-MEV BEAM TRANSPORT SYSTEM*

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ABSTRACT

In this paper we consider optimum positions for beam profile monitors, to be used for beam emittance measurement in the high energy beam transport (HEBT) system between the 200-MeV conversion linac and the AGS at Brookhaven National Laboratory. We also consider some characteristics of possible profile monitors.

Introduction

This paper is based on the assumption that one knows what emittance to expect under matched conditions anywhere in the system. Metzger[1] pointed out that if the beam emittance is calculated from successive (in time or space) beam-width measurements, the best result is obtained if these width measurements define a regular polygon in phase space which envelops the true emittance.

Applying this idea to emittance measurement in the HEBT, we note that the focusing system there is nearly periodic, consisting of mirror symmetric cells which cause a betatron phase shift of $\pi/2$ rad per cell for each of the two transverse motions. Therefore beam-width measurement in homologous points of adjacent cells produces a rectangle in phase space which circumscribes the true beam emittance. Width measurement at two other homologous points in the same cell yields another rectangle which is rotated with respect to the first one. If one is limited to these four measurements, Metzger's criterion requires that in a properly scaled phase space the two rectangles become squares rotated $\pi/4$ rad with respect to each other.

It is easy to show that in the HEBT this criterion may be met by performing the four width measurements in points midway between successive quadrupoles. A matched beam has equal widths in each of these points. If the horizontal and vertical emittances are equal and matched, the beam cross sections there are circles of equal radii. This property leads to a simple criterion for beam adjustment; if all beam widths are equal, the beam is matched.

Calculations

We first calculate the characteristics of matched emittance ellipses, using the thin lens approximation and disregarding the departure from exact periodicity.

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Then we calculate the characteristics of an arbitrary ellipse from measured beam widths. We find expressions for the case that the ellipse departs only slightly from a matched one and we use these for an accuracy estimate.

Transfer Functions

The transfer function for a half cell of the HEBT, going from one midplane between quadrupoles to the next one (see Fig. 1) is

$$M(a \rightarrow b) = \begin{bmatrix} 1 \pm \frac{1}{2} /2 & \frac{1}{2}\ell\ (2 \pm \frac{1}{2} /2) \\ \pm /2/\ell & 1 \pm \frac{1}{2} /2 \end{bmatrix} \tag{1}$$

The transfer function for a full cell, starting from the same midplane, is

$$M(a \cdot a') = \begin{bmatrix} \pm /2 & 3/2\ell \\ -2/\ell & \mp /2 \end{bmatrix} \tag{2}$$

and for a cell and a half, again starting from the same midplane, it is

$$M(a \rightarrow b') = \begin{bmatrix} -1 \pm \frac{1}{2} /2 & \frac{1}{4}\ell\ (4 \mp /2) \\ \mp /2/\ell & -1 \pm \frac{1}{2} /2 \end{bmatrix} \tag{3}$$

In these expressions, ℓ is the distance between adjacent midplanes (= half-cell length). The upper signs apply if one enters a defocusing quadrupole first, the lower signs if one enters a focusing quadrupole first. Of course, the quadrupole strength Q is given by

$$Q\ell = \frac{1}{E_o \beta\gamma}\ c\ \frac{\partial B}{\partial r}\ \ell_{quad}\ \ \ell = \pm /2 \ .$$

E_o = rest energy of proton, c = velocity of light, $\partial B/\partial r$ is tesla/m, and ℓ_{quad} is effective length of quadrupole.

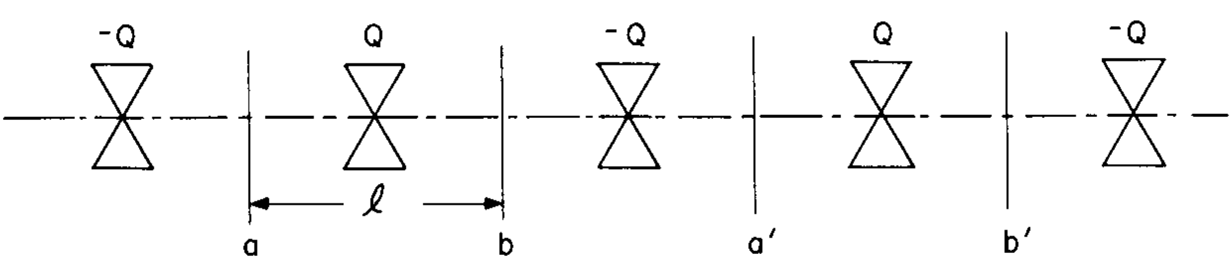

Fig. 1. The HEBT quadrupole cell structure.

Matched Emittance Ellipse

Describing an arbitrary emittance ellipse with

$$\begin{aligned} &\gamma y^2 + 2\alpha y y' + \beta y'^2 = E \\ &\beta\gamma - \alpha^2 = 1 \quad , \end{aligned} \tag{4}$$

we find for a matched ellipse in the first midplane

$$\begin{aligned} \alpha &= \pm\sqrt{2} \\ \beta &= 3/2\ell \\ \gamma &= 2/\ell \end{aligned} \tag{5}$$

by requiring α, β and γ to remain unchanged after transfer through a full cell. From this it is clear that if the beam is matched, the ellipses for the horizontal and for the vertical direction have equal eccentricities but mirror symmetric orientations.

From (4) one calculates for the beam width:

$$W = 2\sqrt{\beta E}$$

so that

$$\beta = W^2/4E \tag{6}$$

Arbitrary Ellipses

Transforming an arbitrary ellipse from the first midplane (a) to the three following ones (b, a′ and b′) one finds

$$\begin{aligned} \beta_a &= \frac{W_a^2}{4E} = \beta \\ \beta_b &= \frac{W_b^2}{4E} = 1/8\ (9 \pm 4\sqrt{2})\ \gamma\ell^2 - \tfrac{1}{2}(5 \pm 3\sqrt{2})\ \alpha\ell + \tfrac{1}{2}(3 \pm 2\sqrt{2})\ \beta \\ &= \begin{matrix} 1.832106 \\ 0.417893 \end{matrix}\ \gamma\ell^2 - \begin{matrix} 4.621319 \\ 0.378680 \end{matrix}\ \alpha\ell + \begin{matrix} 2.914213 \\ 0.085787 \end{matrix}\ \beta \\ \beta_{a'} &= \frac{W_{a'}^2}{4E} = 9/4\ \gamma\ell^2 \mp 3\sqrt{2}\ \alpha\ell + 2\beta \\ &= 2.75\ \gamma\ell^2 \mp 4.242639\ \alpha\ell + 2\beta \\ \beta_{b'} &= \frac{W_{b'}^2}{4E} = 1/8\ (9 \mp 4\sqrt{2})\ \gamma\ell^2 + \tfrac{1}{2}(5 \mp 3\sqrt{2})\ \alpha\ell + \tfrac{1}{2}(3 \mp 2\sqrt{2})\ \beta \\ &= \begin{matrix} 0.417893 \\ 1.832106 \end{matrix}\ \gamma\ell^2 + \begin{matrix} 0.378680 \\ 4.621319 \end{matrix}\ \alpha\ell + \begin{matrix} 0.085787 \\ 2.914213 \end{matrix}\ \beta \end{aligned} \tag{7}$$

where W_a, W_b, $W_{a'}$ and $W_{b'}$ are measured beam widths.

It is easy to verify from (7) that for each of the two possible cases

$$\beta_a + \beta_{a'} = \beta_b + \beta_{b'} \tag{8}$$

or

$$W_a^2 + W_{a'}^2 = W_b^2 + W_{b'}^2 \tag{9}$$

Thus a necessary condition for the emittance to be an ellipse is that (9) is satisfied. System (7) together with (4) is overdetermined: five expressions for four unknowns.

This is a consequence of our assumption of an elliptic emittance. If the emittance is known to be an ellipse, three rather than four width measurements, i.e., a hexagon rather than an octagon in phase space, are sufficient to determine it.

To resolve this dilemma one might disregard one of the four equations in (7) after having checked via (9) that the assumption of ellipticity is compatible with the measurements. Doing so reduces the accuracy because Metzger's criterion is no longer met. Therefore, we form four groups of three equations, solve α, β, γ and E from each group and average the results. In this way, each width measurement contributes equally to each of the unknowns. We find

$$\begin{aligned}
\alpha &= \pm \frac{1}{4E\ell} \left(0.43773\, W_a^2 - 0.192869\, W_b^2 + 0.033672\, W_{a'}^2 + 0.664274\, W_{b'}^2\right) \\
\beta &= \frac{1}{4E} \left(0.75\, W_a^2 + 0.75\, W_b^2 - 0.75\, W_{a'}^2 + 0.75\, W_{b'}^2\right) \qquad (10) \\
\gamma &= \frac{1}{4E\ell^2} \left(0.047619\, W_a^2 - 0.474789\, W_b^2 + 0.619048\, W_{a'}^2 + 1.141455\, W_{b'}^2\right) \quad .
\end{aligned}$$

Using $\beta\gamma - \alpha^2 = 1$, we have for $4E\ell$

$$\begin{aligned}
4E\ell = \Big[& 0.422902\, (W_a^2 W_{a'}^2 + W_b^2 W_{b'}^2) + 0.286448\, (W_a^2 W_{b'}^2 + W_b^2 W_{a'}^2) - \\
& - 0.155896\, (W_a^4 + W_b^4 + W_{a'}^4 + W_{b'}^4) - 0.175336\, (W_a^2 W_b^2 + W_{a'}^2 W_{b'}^2) \Big]^{\frac{1}{2}} \quad . \qquad (11)
\end{aligned}$$

With $4E\ell$ known from (11) and the half-cell length ℓ, one may first calculate E and then solve (10) for α, β and γ.

Check

In order to check the expressions (10) and (11) we recollect that a matched beam as specified by (5) should produce equal beam widths in the four measuring stations. Therefore, we substitute

$$W_a = W_b = W_{a'} = W_{b'} = W$$

into (10) and (11) and find

$$\begin{aligned}
\alpha &= \pm\, 1.414213 = \pm \sqrt{2} \\
\beta/\ell &= 1.5 \\
\gamma\ell &= 1.999999 = 2 \\
E\ell &= 0.666666\, (W/2)^2 = 2/3\, (W/2)^2 \quad .
\end{aligned}$$

Slight Mismatch

Obviously, the characteristics of any elliptical emittance may be determined from expressions (10) and (11). For small departures from the matched condition, the departures from the matched values may be linearly related to the departure from equality of the beam dimensions W. It is easy to show that, then, the widths W of a matched beam with an emittance area equal to that of the actual beam follows from the dimensions of the actual beam via

$$W = 0.25\ (W_a + W_b + W_{a'} + W_{b'}) \quad . \tag{12}$$

Using this, one finds for α, β and γ in the case of slightly unequal beam dimensions to first order:

$$\alpha = \pm \left(1.414213 + 1.31319 \frac{\delta W_a}{W} - 0.578607 \frac{\delta W_b}{W} + 0.101016 \frac{\delta W_{a'}}{W} + 1.992822 \frac{\delta W_{b'}}{W} \right)$$

$$\beta/\ell = 1.5 + 2.25 \frac{\delta W_a}{W} + 0.75 \frac{\delta W_b}{W} - 0.75 \frac{\delta W_{a'}}{W} + 0.75 \frac{\delta W_{b'}}{W} \tag{13}$$

$$\gamma\ell = 2 + 0.142857 \frac{\delta W_a}{W} - 1.424367 \frac{\delta W_b}{W} + 1.857144 \frac{\delta W_{a'}}{W} + 3.424365 \frac{\delta W_{b'}}{W} \quad ,$$

where

$$\delta W_a = W_a - W$$

$$\delta W_b = W_b - W$$

$$\delta W_{a'} = W_{a'} - W$$

$$\delta W_{b'} = W_{b'} - W$$

Accuracy

Differentiating (11) one obtains for the error δE in the emittance area as a consequence of errors δW in the beam-width measurements:

$$\frac{\delta E}{E} = 0.5 \left(\frac{\delta W_a}{W} + \frac{\delta W_b}{W} + \frac{\delta W_{a'}}{W} + \frac{\delta W_{b'}}{W} \right) \quad . \tag{14}$$

If the measurement errors are statistically unrelated we may expect $\delta E/E \approx \delta W/W$, i.e., a 10% precision in emittance requires a 10% accuracy in beam-width determination.

In the high energy beam transport system between the new 200-MeV injection linac and the AGS, one expects a span of emittance values from 0.2 mrad·cm to 2 mrad·cm. At the linac exit, the corresponding matched beam widths should range from 5 mm to 15 mm in the matching section just before the injector from 7 mm to 21 mm. Therefore, for an emittance measurement accuracy of about 10%, one would require an accuracy of the beam-width measurement of the order of 0.5 mm.

Emittance Representation

If we are reasonably certain that the emittance is elliptic [relation (9) satisfied], its representation is not difficult. If (9) is not satisfied it may be better to reconstruct the polygon in phase space that envelops the emittance than to guess what the emittance might be like. The polygon representation would give an immediate impression of the credibility of the measurement and would allow variation of the number of measuring stations.

Requirements for Beam-Width Monitors

Desirable characteristics for beam-width monitors are:

a) Nondestructive (i.e., not causing beam interference), in order to keep the measured beam usable for injection into the AGS.

b) Sensitive, so that low intensity beams may be measured as well as small changes in beam width.

c) Linear, so that there exists a linear relationship between the actual beam width and the output signal and also to prevent dependence on the transverse position of the beam axis.

d) Speed of response, so that variations during a single beam pulse may be detected.

e) Reliable, taking into consideration the consequences of hitting the device directly with the beam.

f) Absence of electronics close to the beam channel in order to prevent radiation damage and promote ease of servicing.

g) Low cost and simplicity of device and associated gear.

h) Shortness in the beam direction.

Beam-Width Monitors Under Consideration

Two beam-width monitors are being considered, the ionization profile monitor[2] and the multiwire secondary emission profile monitor.

The ionization profile monitor is practically nondestructive. However, its sensitivity is a function of the residual gas pressure. For reliable operation it requires a pressure of 5×10^{-6} torr, rather larger than the 5×10^{-7} torr or less for present installations.

The multiwire secondary emission monitor is very attractive for its shortness along the beam axis, its sensitivity and independence of the residual gas pressure. Its "destructive" effect can be reduced considerably by the use of thin wires. For instance, 0.001 in. tungsten wires with a pitch of 1 mm intercept less than 2.5% of the beam, scattering the beam about 4 mrad. This will appear on the next two monitors (minimum requirement for a single pulse emittance measurement) as a low density halo ($\sim$ 5%), which can be electronically eliminated.

A disadvantage is that it cannot be in the beam for a long time. In case of an accidental line focus of 2.5 mm width we can expect a temperature increase of the wire per pulse of 300°C, while the radiation flux is too small to compensate the fast temperature increase.

The two monitors under consideration each require electronic equipment close to the device in order to yield fast response and low noise levels. In a preliminary setup in the present 50-MeV linac injection line, no deterioration of the electronic gear was found during a period of about two months.

References

1. C. Metzger, "Mesure des emittances et du centrage des faisceaux dans la ligne de mesure '800 MeV' du PSB," Report CERN/SI/Int.DL/69-10 (1969).
2. W.H. DeLuca, "Beam Detection Using Residual Gas Ionization," IEEE Trans. Nucl. Sci. NS-16, No. 3, 813 (1969).

DISCUSSION

A. Citron (Karlsruhe): Where do you expect the beam actually to hit limiting aperture along the way?

T. J. M. Sluyters (BNL): In the center of the first bend. The most critical area is the first 18 degrees.

A. Citron: Will the inflector then get all the beam loss?

J. Claus (BNL): There is a copper shadow target in front of the inflector. The aperture in the inflector is 4×4 cm, whereas the beam is expected to be less than 2.5 cm in diameter. So, you would have beam loss at the inflector only if there is madadjustment.

C. D. Curtis (NAL): When you get the area of the ellipse from the wire profiles, say with three wires (four wires are better), do you have any way of estimating the percentage of the beam more accurately, say, by folding in the shape of the profile of the beam itself? How do you determine the percentage of the beam in the ellipse?

T. J. M. Sluyters: We have not considered that yet. We will try to do it with our destructive emittance device and compare it to the experimental results you have obtained on profile monitors.

C. D. Curtis: We were just hoping somebody had a solution.

R. P. Featherstone (Central Engineering): Will the switching magnet that can divert the beam down the other tunnel be a fast magnet that can be used to take out a pulse for analysis or other purposes?

T. J. M. Sluyters: Yes.