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FOCUSING BY LONGITUDINAL MAGNETIC FIELD IN A HIGH-ENERGY PROTON LINEAR ACCELERATOR

by

B. I. Bondarev, V. V. Kushin, B. P. Murin,
L. Yu. Solovyev, A. P. Fedotov

Radiotechnical Institute, USSR Academy of Sciences,
Moscow, U.S.S.R.

ABSTRACT

In the main part (beyond 100 MeV) of a 600 to 1000 MeV ion linear accelerator,[1,2] particle focusing can be achieved by means of superconducting solenoids. Expressions are presented that enable one to calculate parameters of a channel with sparse solenoids. This axially symmetrical focusing is shown to result in decrease of aperture and as a consequence in reduced rf power requirements. Superconducting solenoid focusing also leads to less stringent tolerances on channel parameters.

I. PARTICLE DYNAMICS

The transverse particle motion in the longitudinal magnetic field can be described by the radial equation,

$$\frac{d^2 r}{dt^2} + \Omega^2 r - \frac{M}{r^3} = 0 \qquad (1)$$

In order to determine the beam dimensions, it is enough to calculate the phase volume of particles which have no initial rotation (M = 0).[3] Then equation (1) reduces to (in dimensionless coordinates z and ρ)

$$\frac{d^2\rho}{dz^2} + P(z)\rho = 0 \qquad (2)$$

where

$$z = \int \frac{\beta_s c\,dt}{L} \quad ; \quad \rho = \sqrt{\frac{P_s}{P_{so} L_o L}}\, r$$

Here, β_s is the ratio of the synchronous particle velocity to the velocity of light, C. Index 0 denotes the synchronous particle pulse, P_s, and the focusing period, L, at the beginning of the considered part of the accelerator. In dimensionless coordinates the focusing period equals 1, and the function P(z) in the accelerating section limits is given by the formula

$$P(z) = -A = -\frac{\pi e E_m (1-\beta_s^2)^{3/2} L^2}{m_o c^2 \beta_s^3 \lambda} \sin\varphi \quad (3)$$

and in the lens limits by

$$P(z) = \Lambda^2 = \frac{e^2 H_z^2 (1-\beta_s^2) L^2}{4 m_o^2 c^4 \beta_s^2} \quad (4)$$

Here, e and m_0 are the proton charge and mass respectively, E_m is accelerating wave amplitude, φ is the particle phase relative to accelerating wave maximum, H_z is the magnetic field strength.

The stability diagram (in A,Λ^2 coordinates) of equation (2) is similar to that of a quadrupole magnetic channel.[3] In both cases A is the defocusing parameter and Λ is the instantaneous dimensionless transverse oscillation frequency in the lens limits. Here the parameter, Λ^2, is always positive in distinction from the case of the quadrupole focusing where it changes its sign periodically.

Figure 1 shows the stability diagram for the case when $\mathcal{E}$ = 0.8 ($\mathcal{E}$ is the part of the focusing period occupied by the accelerating section). Curves of characteristic parameters, Cos $_\mu$, ν_{min}, and γ are shown in Fig. 1. These parameters depend on the parameters A and Λ as follows:

$$\cos\mu = \cos\Lambda(1-\mathcal{E})\,\mathrm{ch}\sqrt{A}\mathcal{E} - \frac{\Lambda^2 - A}{2\Lambda\sqrt{A}}\sin\Lambda(1-\mathcal{E})\,\mathrm{sh}\sqrt{A}\mathcal{E}$$

$$\nu_{min} = \Lambda\sqrt{\frac{2\sqrt{A}\Lambda\sin\Lambda(1-\mathcal{E})\mathrm{ch}\sqrt{A}\mathcal{E} - [(\Lambda^2+A)-(\Lambda^2-A)\cos\Lambda(1-\mathcal{E})]\,\mathrm{sh}\sqrt{A}\,\mathcal{E}}{2\sqrt{A}\Lambda\sin\Lambda(1-\mathcal{E})\mathrm{ch}\sqrt{A}\mathcal{E} + [(\Lambda^2+A)+(\Lambda^2-A)\cos\Lambda(1-\mathcal{E})]\,\mathrm{sh}\sqrt{A}\,\mathcal{E}}}$$

$$\nu_{max} = \sqrt{A\,\frac{-2\Lambda\sqrt{A}\cos\Lambda(1-\mathcal{E})\mathrm{sh}\sqrt{A}\mathcal{E} + [(\Lambda^2+A)+(\Lambda^2-A)\mathrm{ch}\sqrt{A}\mathcal{E}]\sin\Lambda(1-\mathcal{E})}{2\Lambda\sqrt{A}\cos\Lambda(1-\mathcal{E})\mathrm{sh}\sqrt{A}\mathcal{E} + [(\Lambda^2+A)-(\Lambda^2-A)\mathrm{ch}\sqrt{A}\mathcal{E}]\sin\Lambda(1-\mathcal{E})}} \qquad (5)$$

$$\Upsilon = \sqrt{\frac{\nu_{max}}{\nu_{min}}}$$

The thin lens approximation can be used for estimation. Then the point on the stability diagram that corresponds to $A = 0$ and $\cos\mu = 0$ will be characterized by the following dependence of the parameters Λ, ν_{min}, and Υ upon $\mathcal{E}$:

$$\Lambda = \sqrt{\frac{2}{1-\mathcal{E}}} \qquad (6)$$

$$\nu_{min} = \sqrt{2-\mathcal{E}}$$

and

$$\Upsilon = \sqrt{\frac{1}{(1-\frac{\mathcal{E}}{2})[1-\frac{\mathcal{E}^2}{2} - \frac{2\mathcal{E}(1-\mathcal{E})}{3}]}}$$

The amplitude coefficient ν and the channel transverse admittance increase with increasing in distinction from the quadrupole focusing case. In a wide range of $\mathcal{E}$ values, the amplitude coefficient $\nu = 1.0 - 1.4$. It means that, with the longitudinal magnetic field focusing, the transverse admittance is 1.5 - 2 times as large as that with the quadrupole focusing (where $\nu = 0.6 - 0.8$). In the 600-MeV meson factory project[2] chosen are: $\mathcal{E} = 0.8$ and $\nu = 0.6$. With the same value of $\mathcal{E}$ the longitudinal magnetic field can provide $\nu = 1.0$. The increase of ν by 70% at fixed beam emittance results in a decrease of the beam radius by 30% and, as a consequence, in a decrease of the rf power by 20%.

The quoted parameters show that the radial motion of the particles with no initial rotation in a channel with sparse solenoids resemble, by its parameters, the particle motion along one of the transverse coordinates in a strong focusing channel. The transverse oscillation frequency is much higher than the longitudinal one and thus the solenoid focusing should not be called a "weak" one.

From (4) we can determine the magnetic field value necessary for the creation of a strong focusing channel by solenoids,

$$H_z = \frac{m_0 c^2}{e}\;\frac{2\Lambda\beta_s}{L\sqrt{1-\beta_s^2}} \qquad (7)$$

The magnetic field strength is proportional to the synchronous particle pulse when Λ and L are constant. When $\mathcal{E} = 0.8$ and the acceleration rate is equal to the one chosen in the project,[2] parameter A equals 0.8 at the energy of 100 MeV. With energy rise up to 600 MeV, this parameter decreases by 10 times. If the parameters $\mathcal{E}$ and A are as mentioned above and $\nu = 1$, then $\Lambda = 3.3$ when the energy is 100 MeV and $\Lambda = 2.7$ when it is 600 MeV. The focusing period length changes from 220 to 350 cm along the second part of the meson factory. With these parameter values the magnetic field strength is 44 kgs at the beginning of the second part, and it increases up to 63 kgs at the end of the accelerator.

II. TOLERANCES ON THE FOCUSING CHANNEL PARAMETERS

Solenoids rotating around the longitudinal axis will not result in motion disturbance. The main errors which result in transverse oscillation disturbance are due to solenoid inclinations and random deviations of magnetic field values.

Random solenoid inclinations result in a beam axis displacement, the rms value of which is

$$\sqrt{\overline{(\Delta\tau)^2}} = \frac{\Lambda^2(1-\mathcal{E})}{\nu}\sqrt{n\,D(\Delta)} \qquad (8)$$

where n is the number of focusing periods and D(Δ) is the dispersion of Δ, which can be determined as the product of solenoid inclination angle and its length. For $\mathcal{E} = 0.8$, $\nu = 1$, $\Lambda = 3$ (the mean value of the two mentioned above), and $D(\Delta) = 0.15$ mm, the calculation by (8) gives the rms beam axis displacement value 2.9 mm. The same displacements in case of the quadrupole focusing are achieved with the tolerances three times more stringent.

Using the method described in Ref. 4 one can find that the beam dimensions will increase by times due to focusing field fluctuations

$$\left.\begin{aligned} \bar{\Theta} &= \sqrt{\left(\frac{\overline{S_p M}}{2}\right)^{n} + \sqrt{\left(\frac{\overline{S_p M}}{2}\right)^{2n} - 1}} \\ &\text{where} \\ \frac{\overline{S_p M}}{2} &= 1 + \frac{2\Lambda^4 (1-\mathcal{E})^2}{\nu^2}\left(\overline{\frac{\Delta H_z}{H_z}}\right)^2 \end{aligned}\right\} \quad (9)$$

If we take $\sqrt{(\overline{\Delta Hz/Hz})^2} = 2\%$, and if $\mathcal{E} = 0.8$, $\Lambda = 3$, $\nu = 1$ and $n = 110$, then $\Theta = 1.5$ without taking into account adiabatic damping and $\Theta = 1.1$ with it. To get the same value of Θ in the case of the magnetic quadrupole focusing, the rms tolerances must be 0.1% for the magnetic field value and 20' for the angle of the quadrupole lenses median plane rotation.

III. ABOUT COMPARISON OF THE FOCUSING SYSTEMS

Stranded niobium-titanium superconducting wire (trade mark НТБ-1) manufactured by our industry can be used for superconducting solenoids with magnetic fields from 50 kgs to 60 kgs. The experimental solenoid made of such superconductor has effective current density about $2 \cdot 10^4$ A/cm^2 when the magnetic field and the temperature are 50 kgs and 4.2^0 K, respectively. A 600-MeV linac requires about 5 kg of this Nb-Ti alloy for solenoids with parameters: magnetic field, 55 kgs; length, 60 cm; inner and outer diameters, 4.5 and 9.2 cm, respectively. The cost of such focusing channel (including cryostats) is 2.5 or 3 times as large as that of conventional magnetic quadrupole doublets. But the cost of the whole superconducting focusing system, including the refrigeration system and helium pipe line, plus 10 years of operation cost, is about equal to or even less than that of a channel with quadrupole doublets. Taking into account essential decrease in required rf power, the application of superconducting solenoids for proton linear accelerator focusing is justified.

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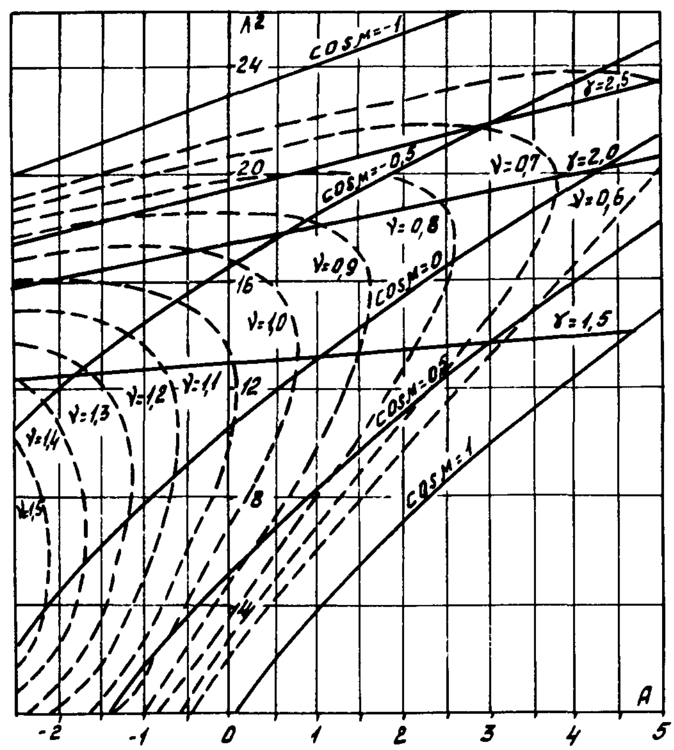

Fig. 1 Stability diagram

DISCUSSION

Teng, NAL: You compared superconductors to normal iron but indicated a maximum 2.3 kG/cm gradient.

Bondarev: That is all that is required for this type of accelerator, so superconducting magnets are not needed.