MESSYMESH - AN IMPROVED VERSION

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Introduction

MESSYMESH, the MURA program to calculate electromagnetic fields for an Alvarez-type linear accelerating cavity, has been revived as an operational program, presently residing on a CDC-6600 computer at Fermilab. It has been modified so that one may now start the over-relaxation computations from an analytically derived initial load, instead of the simple Bessel function load which was previously used. The effect of this new loading is to significantly improve the running time for the program and also to allow one to arrive at a truly converged solution to the field quantities for a given mesh size. These two features are of particular interest in calculations associated with low-β structures, which have previously been quite hard to do with MESSYMESH.

The Analytical Load

In order to arrive at an initial load which more closely represents an actual drift tube cavity than the TM_{010}-type field of a hollow cylindrical cavity previously used, the drift tube geometry has been approximated as a square step geometry, shown in Figure A, where the step dimensions have been optimized to best simulate the rounded outer corner.[1] General equations for the magnetic field and its derivative are then written for the three regions shown as follows:

Region I $0 \leqslant r \leqslant A,\ -G/2 \leqslant Z \leqslant G/2$

$$H_\theta = a_o J_1(kr) + \sum_{n=1}^{\infty} a_n I_1(t_n r)\cos(2n\pi Z/L)$$

$$\partial(rH_\theta)/\partial r = a_o kr J_o(kr) + $$

$$+ \sum_{n=1}^{\infty} a_n t_n r I_o(t_n r)\cos(2n\pi Z/L)$$

Region II $A \leqslant r \leqslant SD/2,\ -H \leqslant Z \leqslant H$

$$H_\theta = q_o J_1(kr) + Q_o Y_1(kr) +$$

$$+ \sum_{p=1}^{\infty}[q_p I_1(\mu_p r) + Q_p K_1(\mu_p r)]\cos(2p\pi Z/L)$$

$$\partial(rH_\theta)/\partial r = q_o kr J_o(kr) + Q_o kr Y_o(kr) +$$

$$+ \sum_{p=1}^{\infty} \mu_p r[q_p I_o(\mu_p r) - Q_p K_o(\mu_p r)]\cos(2p\pi Z/L)$$

Region III $SD/2 \leqslant r \leqslant D/2,\ -L/2 \leqslant Z \leqslant L/2$

$$H_\theta = c_o F_1(kr) + \sum_{m=1}^{\infty} c_m G_1(s_m r)\cos(2m\pi Z/L)$$

$$\partial(rH_\theta)/\partial = c_o kr F_o(kr) -$$

$$- \sum_{m=1}^{\infty} c_m s_m r G_o(s_m r)\cos(2m\pi Z/L)$$

where $k = 2\pi/\lambda$; $s_m{}^2 = (2m\pi/L)^2 - k^2$, $\mu_p{}^2 = (2p\pi/H)^2 - k^2$, $t_n{}^2 = (2n\pi/G)^2 - k^2$;

$$F_o(kr) = Y_o(kr) - [Y_o(kD/2)/J_o(kD/2)]\cdot J_o(kr)$$

$$F_1(kr) = Y_o(kr) - [Y_o(kD/2)/J_o(kD/2)]\cdot J_o(kr)$$

$$G_o(s_m r) = K_o(s_m r) -$$

$$- [K_o(s_m D/2)/I_o(s_m D/2)]\cdot I_o(s_m r)$$

$$G_1(s_m r) = K_1(s_m r) -$$

$$- [K_o(s_m D/2) I_o(s_m D/2)]\cdot I_1(s_m r);$$

$J_{o,1}$, $Y_{o,1}$, $I_{o,1}$, and $K_{o,1}$ being the regular and modified Bessel functions. Here the c's, q's, Q's, and a's are coefficients to be determined. The step dimensions have been taken to be

$$A = \frac{SD}{2} - \frac{2}{3}\,RC$$

$$H = G/2 + \frac{1}{2}\,RC.$$

In order to solve these equations, the summation indices must be terminated at finite values. Taking the values m = 50, n = p = 8, one can sufficiently accurately determine the resonant frequency. Matching the values of H_θ and $\partial(rH_\theta)/\partial r$ at the boundaries r = A and r = SD/2, one arrives at a set of linear, homogeneous equations for the coefficients q's and Q's. Solving in this region for the resoant frequency one can then determine all but one of these coefficients. The other coefficients, the c's and a's are then linear combinations of the q's and Q's, and the one remaining unknown coefficient can be determined by specifying the voltage across the gap.

Once the coefficients are known, the field at each mesh point can be calculated from the above equations and put into MESSYMESH as the starting load for the over-relaxation calculations. The typical improvement in this load can be seen by comparing the frequencies determined for a particular geometry, as shown in Table I.

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Results

The advantages of using this initial loading are manifest in both the total calculational time needed for convergence, and the accuracy of the final solution. MESSYMESH, starting with the Bessel function load, runs until some specified convergence criteria are reached. It then stops at a solution of some particular frequency. If one subsequently reruns the program, changing neither the geometry, mesh size, nor convergence criteria, but using the previously calculated fields as the loading, the calculation continues and a somewhat lower frequency is obtained. Subsequent reiterations continue to produce progressively lower frequencies, with the incremental frequency changes becoming steadily smaller. Some attempts have been made to start from the Bessel function loading and reiterate the program until this progression converged, but these attempts have been unsuccessful due the amount of physical time required. Thus, there always remained a question in using MESSYMESH as to how far one should go before deciding to stop this progress, and relatedly, how far from a final answer one was.

Running MESSYMESH for some given geometry, starting from the step loading described above, the program not only arrives at an answer much faster than it does using a Bessel function load, but also, the value found does not change upon reiteration. Thus, one is able to specify any set of dimensions, not particularly limited to those which are close to geometries previously calculated, choose some appropriately small mesh size, and after one short run, have a believable frequency and field for that geometry.

Table II lists progressive calculations for both a Bessel function and the step loading for two particular geometries - the first and last cells of the present Fermilab linear accelerator. In this and subsequent tables, the mesh size and convergence criteria were used the same as those used in the old MESSYMESH calculations. In all cases, the first run is from the load specified and the subsequent runs use the results of the previous calculation as a starting point. The times listed are approximate running times, in seconds, on a CDC-6600 computer. The frequencies and betas listed at the top of the table are values previously found at MURA for these cells. Table III compares calculated quantities of interest for these same two geometries for the final solutions from these two leads. Finally, Table IV shows, for several cavities constructed and measured at MURA, a comparison of the MESSYMESH, and those obtained after the first run using the step load.[2]

References

1. S. Ohnuma, "A Simple Program to Calculate the Alvarez-Type Linear Cavity", Fermilab Internal Report, TM-611, 1 Oct 75.

2. P.F. Dahl, et at., "Linac Cavity Field Calculations", Proc. of 1966 Linear Accel. Conf., LASL, 1966.

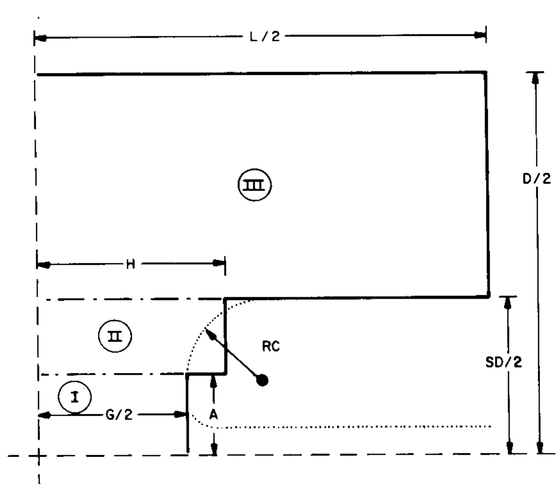

FIGURE A

TABLE I

	Frequency (MHz)
MESSYMESH #32102	201.344
TM_{010}	244
Step	200.434

A comparison of the final resonant frequency determined by MESSYMESH, and the initially-loaded frequencies using both a TM_{010}-type loading and a step function approximation to a drift tube loading.

TABLE II

MESSYMESH #32102 β = .0406, f = 201.344 MHz MESH = .25 cm				MESSYMESH #32440 β = .5658, f = 201.252 MHz MESH = .50 cm			
Bessel function		step		Bessel function		step	
frequency (MHz)	CPU (sec) (approximate)	frequency	CPU	frequency	CPU	frequency	CPU
202.333	360	201.194	162	201.335	833	201.236	465
282	30	201.195	25	.318	33	201.232	44
.237	20			.305	33		
.193	20			.292	34		
.151	20			.282	34		
.110	20			.274	33		
.071	20			.266	33		
.034	20			.259	32		
201.998	20			.254	35		
.964	20			201.249	35		
.932	20						
.901	20						
.872	20						
.844	20						
.817	20						
.792	20						
201.768	20						

A comparison of the frequencies and running times for successive iterations starting from initial TM_{010}-type loads and from an analytically derived step function approximation to a drift tube for two particular geometries for the Fermilab linear accelerator.

TABLE III

	MESSYMESY #32102		MESSYMESH #32440	
	Bessel	Step	Bessel	Step
Frequency	201.344	201.194	201.252	201.236
T	0.6411	0.6408	0.5543	0.5470
zTT	26.877	27.453	15.212	14.551
PW	540.54	538.47	6949.12	6822.59
PDT	606.36	589.32	10691.13	11140.31
PT	1146.90	1127.79	17640.25	17962.90
Q	82431.7	84598.2	65603.6	64685.1
ZS	65.39	66.86	49.50	48.64

A comparison of several calculated quantities of interest for each of the two loads for two particular geometries for the Fermilab linear accelerator: Frequency (MHz), transit time, zT^2, Power dissipated in the outer walls, on the drift tube, and total (watts), Quality factor, and shunt impedance (Megohms/meter).

TABLE IV

	L/2(cm)	3.350	4.039	4.717	5.565	6.304
	G/2(cm)	0.947	1.128	1.293	1.633	1.864
Frequency (MHz)	Measured	202.473	200.702	198.813	200.556	198.941
	MESSYMESH					
	Bessel load	203.062	201.159	199.238	200.188	199.204
	Step load	202.954	201.075	199.188	200.105	199.152

A comparison of measured and computed resonant frequencies for cavities constructed at MURA.