EFFECTS OF PERTURBATION IN LOW β PROTON ACCELERATING STRUCTURES*

by

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Summary

In the first tank of the LAMPF 201 Linac it is desired to have a linear field distribution. One tries to achieve this by perturbing the first and last cells of the tank. In this paper we consider how perturbations in cell geometry in a periodic structure affect the field distribution in structures which correspond to low to intermediate values of β. It is shown that a geometric perturbation in one cell couples to many cells and we show how to get the coupling distribution from the geometric model. Finally we discuss what is necessary to achieve the desired field distribution at LAMPF.

Introduction

The first tank of the 201 MHz drift tube accelerator at LAMPF is designed to have a field distribution which increases linearly from 1.6 MeV/m in the first cell to 2.2 MeV/m at the end of the tank. One tries to obtain this distribution by putting large geometric perturbations in the first and last cells of the tank. This perturbation is achieved by moving the first and last half drift tubes in such a way as to preserve the frequency of the tank, thereby increasing the gap in cell 1 and decreasing the gap in cell 31. Unfortunately, this perturbation does not yield exactly the desired distribution. Referring to Fig. 1, one sees that the field distribution fluctuates strongly in the first few cells. In fact, the field is 6% higher than desired in the first cell, correct in the second cell, and 6% lower than desired in the third cell. It is clear that this distribution can create beam dynamics problems.

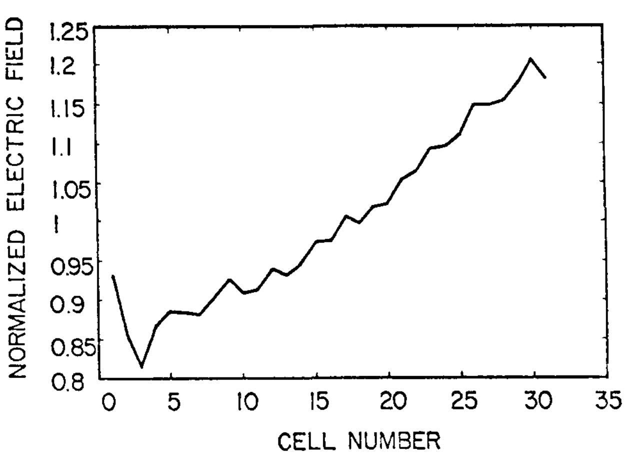

Fig. 1. Average Electric Field Distribution in Tank 1 of the LAMPF 201 Linac

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In order to be able to smooth this distribution, we have been studying the effects of this type of perturbation using a code developed at BNL[1] which uses an analytic solution to Maxwell's equations to obtain the gap fields in a multi-cell cavity. These investigations have shed new light on understanding physically what happens when a drift tube is axially misaligned, enabling us to determine how many cells are coupled to the perturbation. Hence we can construct any desired field distribution from a known set of frequency errors as we shall now describe.

Theory

In our model, the field in each gap is related to that in adjacent gaps by an expression of the form[2]

$$E_n = \sum_m B_{nm} E_m \tag{1}$$

Earlier simpler forms of the theory approximated the multi-cell structure by an equivalent circuit chain, in which the basic equation for the zero mode is of the form

$$E_{n+1} + E_{n-1} - 2E_n \simeq \frac{2}{k} \frac{\delta\omega_n}{\omega} E_n \tag{2}$$

Here $\delta\omega_n/\omega$ is the fractional frequency error in cell n, and k is the coupling coefficient between adjacent cells.

The qualitative considerations for obtaining a desired field shape have usually been the following.

1) There is a symmetry requirement at the end walls which is most simply represented by requiring $E_0 = E_1$, $E_{N+1} = E_N$. Because of this, the multi-cell structure will resonate at a frequency which makes $\Sigma\, \delta\omega_n = 0$.

2) A flat field requires each $\delta\omega_n = 0$, that is, exact resonance for each cell.

3) A linear tilt is obtained by making

$$\frac{2}{k}\frac{\delta\omega_1}{\omega} = \varepsilon\ ,\quad \frac{2\delta\omega_N}{k\omega} = -\varepsilon\ ,\quad \delta\omega_n = 0 \text{ for } n \neq 1,\ N \tag{3}$$

giving, for $\varepsilon N \ll 1$,

$$E_n \simeq E_1[(1 + \varepsilon(n-1)] \tag{4}$$

(4) If a particular geometrical configuration leads to a field distribution different from

that desired, this can be corrected by

a) calculating the second differences of the desired field changes

b) inserting frequency changes corresponding to (2) in appropriate cells.

In order to check this model, we used the BNL code for six-cell periodic structures (Fig. 2.A), each corresponding to values of β from $\beta = 0.07$ to $\beta = 0.56$. In this structure, a central drift tube was axially displaced which gives rise to equal and opposite perturbations in two adjacent cells. If the frequency error distribution looks like that shown in Fig. 2.B, equation (2) implies that the gap electric field distribution will be as shown by curve 1 of Fig. 2.C. However, the calculated electric field distribution is the one shown by curve 2 of Fig. 2.C.

So the evidence from our calculations is that the procedure outlined above is imperfect, and the explanation is clearly that (2) is too crude an approximation. Equation (1) suggests coupling to cells other than the nearest. This can be simulated by writing

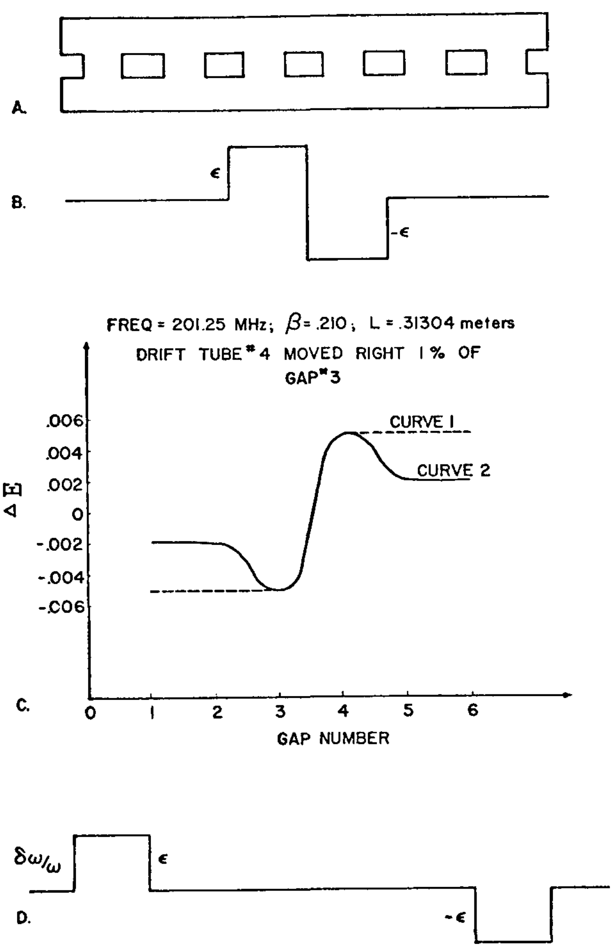

Fig. 2. Frequency Distributions and Field Distributions for a 6-cell periodic structure.

$$E_{n+1} + E_{n-1} - 2E_n \simeq \sum \alpha_{mn} \frac{\delta\omega_m}{\omega} E_m \qquad (5)$$

and by generating the coefficients α_{mn} from computation of actual fields resulting from frequency errors in individual cells.

Numerical Results

As an example of how the procedure works if the α's are known, consider a 6-cell structure where the coupling extends to the next nearest neighbor cells only. Also assume that the couplings are such that

$$\alpha_{nn} = 5 \qquad \alpha_{n\pm1,n} = -2$$

(where the α_{mn}'s are normalized with respect to $\delta\omega_n/\omega$). We want to introduce perturbations in all the cells such that we get a linear field distribution (which will be obtained if the right hand side of (5) yields a distribution as is shown in Fig. 2.D.) In constructing this distribution, it is necessary to include the image effects due to the end walls.

The procedure is to introduce the perturbation indicated above into the first and last cells and some proportion of that same error into adjacent cells.

First, we construct the error table below, assuming that E_m on the right side of (5) is approximately constant.

$5\delta\omega_1$	$-2\delta\omega_1$				
$-2\delta\omega_1$					
$-2\delta\omega_2$	$5\delta\omega_2$	$-2\delta\omega_2$			
	$-2\delta\omega_3$	$5\delta\omega_3$	$-2\delta\omega_3$		
		$2\delta\omega_3$	$-5\delta\omega_3$	$2\delta\omega_3$	
			$2\delta\omega_2$	$-5\delta\omega_2$	$2\delta\omega_2$
				$2\delta\omega_1$	$-5\delta\omega_1$
					$2\delta\omega_1$

In this table we have assured that the perturbations sum to zero. In order to get the distribution shown in Fig. 2.E it is only necessary to require that the 4 interior columns sum to zero. This gives two equations for the two unknown frequency perturbations $\delta\omega_2$ and $\delta\omega_3$. Solving these yields

$$\delta\omega_2/\delta\omega_1 = 14/31 \; ; \; \delta\omega_3/\delta\omega_1 = 4/31$$

This procedure can be extended to any number of periodic cells (even or odd).

Now we will look at the results of the six-cell studies using this model and try to construct a linear gap field distribution. Table I shows the relative α's calculated from a 1% perturbation in a single gap for 8 different structures, each corresponding to a different β. The scale factor gives

TABLE I

$\delta g/g = 0.01$

		GAP					
β	scale factor	1	2	3	4	5	6
.07	55	.05	.11	.35	-1	.36	.12
.14	93	.08	.12	.33	-1	.34	.13
.21	128	.12	.16	.27	-1	.28	.16
.28	173	.18	.21	.20	-1	.20	.21
.35	234	.24	.26	.11	-1	.11	.26
.42	325	.29	.31	.04	-1	.04	.31
.49	453	.33	.34	-.01	-1	-.01	.35
.56	622	.35	.37	-.05	-1	-.05	.38

the magnitude of the central perturbation. There is a slight asymmetry in the table due to the fact that the perturbation was not symmetrically produced.

As an example, we will choose the set of α's corresponding to β = .07. Going through algebra similar to our previous example, we obtain

$$\delta\omega_2 = 0.50\ \delta\omega_1 \quad \text{and} \quad \delta\omega_3 = 0.17\ \delta\omega_1.$$

Making the corresponding gap changes in our model produces the field distribution (curve 2) in Fig. 3. Shown there also are two other field distributions. One is obtained by perturbing only cells 1 and 6 (curve 1) and the second is obtained by adding one quarter of the curve 1 perturbation to the curve 2 perturbation (curve 3). The field distribution obtained from our prescription is linear in the interior 4 cells but deviates from the straight line by 5% in the first and last cells. The field distribution obtained from the combination of perturbations (curve 3) deviates from linearity by only 1.6%. (Note that it should be possible to extend the model to study non-periodic structures.)

Discussion

The model represented by equation (5), while not exact, gives a good first approximation

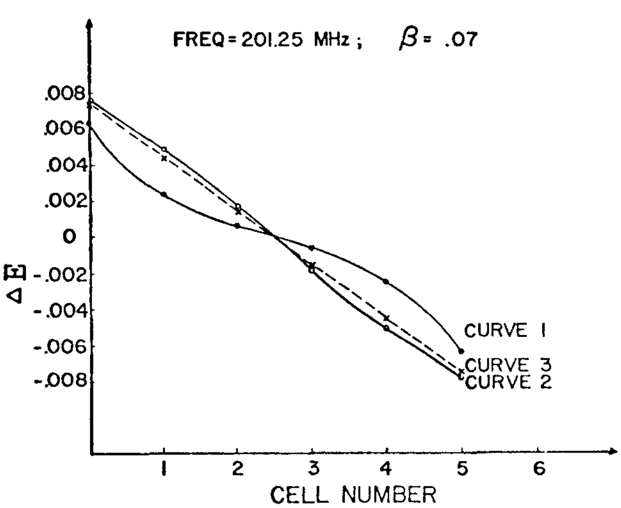

Fig. 3. Field Distributions for Various Frequency Distributions

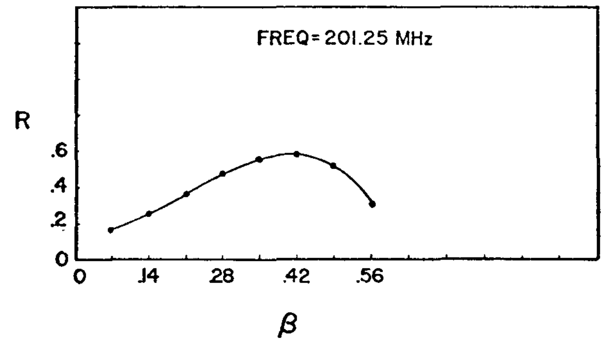

Fig. 4. Ratio Showing How Field Distribution Changes as a function of β.

to what is happening physically. In what follows, we will try to partially explain why the agreement is not better.

As β increases, one expects the calculated field distribution to approach that predicted by equation (1) since the error should extend over fewer cells. This distribution would yield a ratio of field in cell 6 to field in cell 4 equal to unity. This ratio is plotted in Fig. 4 as a function of β. The ratio starts to rise toward unity as β increases and then, beyond β = .42, decreases.

In Table I, note that the α's in the cells adjacent to the perturbed cell monotonically decrease and go through zero at β = .42. One effect which influences this behavior is the proximity of the second passband and the character of this passband. Using a different analytic formulation of the periodic cavity code,[2] one can incorporate the phase advance per cell explicitly in the calculation. This same formulation also yields the symmetry of the zero and π modes. Using this code to study the character of the first two passbands, it was found that these bands cross in the region $.14<\beta<.21$. This implies that the coupling constant that one would calculate from any dispersion curve obtained from equivalent circuit theory would not necessarily be a monotonic function of β. This coupling constant makes up part of the proportionality constant which relates E'' to $\delta\omega/\omega$. So, the behavior of the passbands probably strongly influences the cavity response to perturbations.

It is evident that this model shows us what one has to do in order to smooth out the field distribution in the first tank at LAMPF. Specifically, it is necessary to change several gaps at both ends of the tank. (This is accomplished by inserting several new drift tubes.) While this may not give an exactly linear field distribution it will remove the gross fluctuations and hence help solve the beam dynamics problems.

References

1) R.L. Gluckstern, M.J. Lee, R. Chasman, H.K. Peterson, "A Method For Calculating Resonant Frequencies, Mode Structure, and Field Flatness in an Alvarez Type Linac Cavity," IEEE Trans. Nucl. Sci. NS-16, 340 (1969).

2) W. Jule, R.L. Gluckstern, R. Chasman, "Band Closing in an Alvarez Type Periodic Structure," Proc. of the 1970 Proton Linear Accel. Conf., (343).