BEAM OPTICS IN THE CERN NEW 50 MeV LINAC

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Summary

A comprehensive treatment of the beam optics, including bunching, matching, and acceleration, is presented for the new 50 MeV Linac. It is based on a combined use of analytical approaches and computations, firstly using optimization techniques with linearized forces and then numerical beam simulation with non-linearities included. The matching between different parts of the accelerator and the choice of operating conditions are shown to be important in both transverse and longitudinal phase planes, especially if a wide range of beam intensities (up to 150 mA) is specified. For the description of the global beam characteristics r.m.s. values of phase-space coordinates are used.

Introduction

In another paper presented at this conference, the design of the Linac structure has been treated with emphasis on a procedure (use of computational tools) which leads to mechanical dimensions[1]. This paper, in contrast, deals solely with the dynamics of the problem which not only supports the structure design but also determines the operating conditions, particularly the matching parameters, and gives us information on the expected performance.

Applying first an analytical approach we obtain general results and trends, limited by the simplifications in the model (especially for space charge) but nevertheless showing the fundamental relations between parameters. Most of the formulae (and here we include stability diagrams) have been in general use for some years[2] and can be more easily applied since the advent of inexpensive computers.

With the linear treatment applied in optimization computer programs, the stress has been on the "machine aspects" of the problem, such as the determination of optimum operating parameters for given beam conditions. Thus we have optimized focusing structures (including 'weak' longitudinal focusing) to obtain matching conditions, quadrupole settings, and bunching voltages, subject to the condition that all restoring forces, including space charge, can be linearized.

In the non-linear treatment, applied finally, the stress is on the beam itself; thus the beam simulation checks the validity of the machine settings obtained via the linear treatment and also reveals the effects which were previously not considered. These include the non-linear RF potential well, non-uniform space-charge distributions, and other effects which can lead to emittance increase.

Given the three approaches, analytical, linear and non-linear simulation, the design will be conditioned by the guide-lines which we take in order to meet the output beam specifications. The following sections deal with these guide-lines in more detail as they generally apply to both the main systems under consideration here, namely the bunching (at 0.75 MeV) and acceleration from 0.75 MeV to 50 MeV.

General Design Principles

Matching

We take it as axiomatic that the beam should be matched to the accelerator and remain matched in the course of acceleration. Arguments on beam stability, aperture limitations, and performance of subsequent accelerators, which are valid for low currents, assume much greater significance at higher currents when coupling between phase planes, enhanced by mismatches, can give rise to appreciable emittance increase.

Matching in Six Dimensions. This implies that we treat each of the phase planes (x,x'), (y,y') and $(z,z'$ or $\Delta\phi,\Delta W)$ in identical ways, and as is shown later this leads to some difficulties in the longitudinal plane. Matching into acceptance ellipses is desired for all planes. Note that a mismatch in one plane automatically gives rise to a mismatch in another phase plane when space charge is present (even in the linear treatment). We expect coupling between phase planes to be significant if the emittances differ by a large factor.

Emittance Ellipses. The approximation of elliptical emittance boundaries is acceptable in the transverse planes, where nearly elliptical forms coming from ion sources can be further trimmed by acceptance limitations to approach ideal shapes in the accelerator. Longitudinally the argument is more tenuous, as a continuous beam has to be formed into discrete bunches, which can approximate to ellipses only with a complicated bunching system. The accelerator itself has an asymmetric acceptance in the longitudinal plane ("fish" or "golf club" shapes have been calculated).

The emittance ellipses are defined by r.m.s. values of beam coordinates in a phase plane and the linear analysis concentrates on their evolution, or, alternatively, on the evolution of the derived (CLS) parameters α, β, and γ:

$$\alpha = \frac{\overline{xx'}}{E_{rms}}\ , \quad \beta = \frac{\overline{x^2}}{E_{rms}}\ , \quad \gamma = \frac{\overline{x'^2}}{E_{rms}}\ , \qquad (1)$$

with

$$E_{rms} = \left[\overline{x^2}\ \overline{x'^2} - (\overline{xx'})^2\right]^{1/2} .$$

The emittance ellipse,

$$E = \gamma x^2 + 2\alpha xx' + \beta x'^2\ , \qquad (2)$$

where E is the marginal emittance, is deduced from E_{rms} and belongs to an "equivalent beam", which is of uniform density distribution and used in all linear computations. [$E = 4E_{rms}$, $5E_{rms}$ for unbunched and bunched beams, respectively, and analogously for marginal values of coordinates, for example, $\hat{x} = 2(\overline{x^2})^{1/2}$, $\sqrt{5}(\overline{x^2})^{1/2}$.] The fact that an equivalent beam can be representative of a real beam having the same r.m.s. values is shown in Sacherer[3].

Linear Forces. In the linear treatment the self forces (space charge) are assumed to have their "spring force constant" dependent only on beam envelopes in the three phase planes and this can be shown to apply with surprisingly good accuracy even for distributions which are not uniform provided they are of ellipsoidal type[3].

Linac Focusing. The advantages of the + - focusing in the Linac are well known and lead to the matched beam having an approximately circular cross section in the gap, with its average envelope:

$$\hat{x}_{gap} \simeq \hat{y}_{gap} \simeq r_{av} = \left(\frac{2\beta_r \lambda E}{\mu}\right)^{1/2} \quad , \quad \beta_r = \frac{v}{c} \; . \quad (3)$$

In order to control r_{av} we have to adjust μ, the phase advance per focusing period. Thus μ is an important design parameter which will be treated later in more detail.

Choice of Synchronous Phase, ϕ_s

The synchronous phase at injection into the Linac is the important parameter which determines the intensities which can be accelerated, by defining the longitudinal acceptance ($\Delta\phi$, ΔW plane). In contrast to the transverse phase planes, where the quadrupole focusing can be adjusted to cope with the space-charge effect, in the longitudinal plane we have to set the acceptance *a priori* and ensure that we have a sufficient margin for all desired currents.

Beam Current. The importance of the parameters involved can be seen in the following formula[4]:

$$I_{max} = 2r_{av}\varepsilon_0\beta_r c\bar{E}T \cos\phi_s |\tan\phi_s| \; \sigma_{opt}\widehat{\Delta\phi}^2_{max} \; , \quad (4)$$

with σ_{opt} the optimum value of the space charge parameter ($\simeq 0.45$) being obtained by maximizing the product $\sigma\widehat{\Delta\phi}^2$. Note that $\widehat{\Delta\phi}_{max} \propto |\phi_s|$ and hence approximately $I_{max} \propto |\phi_s|^3$, so that ϕ_s is much more significant than the accelerating rate ($\bar{E}T \cos\phi_s$) in determining the limiting current. This formula is generally pessimistic as it treats the non-acceleration case.

Limitations on ϕ_s. It would appear from the above that one could accelerate high intensities merely by increasing ϕ_s, but limits are found if we insist on matching properly in the $\Delta\phi$, ΔW plane. The linearized formula for the energy spread matched to $\widehat{\Delta\phi}$ is

$$\widehat{\Delta W} = \widehat{\Delta\phi}\left(\frac{mv_s^3 e\bar{E}T \sin|\phi_s|}{\omega}\right)^{1/2} (1-\sigma)^{3/2} \; , \quad (5)$$

which means that

$$\widehat{\Delta W} \propto |\phi_s|^{3/2} \qquad \text{(without space charge)}$$

or

$$\widehat{\Delta W} \propto |\phi_s|^{3/2}(1-\sigma)^{3/2} \qquad \text{(with space charge)} \; .$$

This $\widehat{\Delta W}$ can quickly go beyond the possibilities of present bunching systems and, in addition, the excessive bunching voltages require short distances between buncher and Linac which poses space problems for the housing of the transverse matching quadrupoles.

Law of Variation for ϕ_s. The larger than normal value for $|\phi_s|$ required at the Linac input to accept the specified beam intensities does not need to be maintained throughout the accelerator and is undesirable from dynamical as well as RF economy standpoints. If $|\phi_s|$ decreases (below 10 MeV, for example) the phase damping is less and hence the matched ΔW is smaller; this reduces the longitudinal mismatch inevitably occurring between Linac tanks. The procedure for finding an acceptable law of variation of ϕ_s involves treating several cases with both linearized and multiparticle programs (described in detail in another paper[1]).

Choice of μ

We have tended to choose μ as large as possible consistent with the limiting quadrupole magnet gradients, the ϕ_s, and the maximum specified current at Linac input. With higher μ the working point on the transverse stability diagram (with space charge included) is more central with respect to the limits at $\mu = 0$, $\mu = 180°$. In addition, the resulting small beam diameter avoids non-linear regions, both in RF accelerating gaps and in quadrupoles, and makes corrections of the focusing, due to space charge, relatively less as shown in the following: from the smooth envelope equation

$$r''_{av} + \bar{K}^2 r_{av} - \frac{E^2}{r^3_{av}} - \frac{kI}{r_{av}} = 0 \; , \quad (6)$$

($\bar{K}$ = average focusing, k = space-charge proportionality factor) we get for a matched envelope $r''_{av} = 0$. If I = 0, then

$$r_{av} = \left(\frac{E}{\bar{K}}\right)^{1/2} = \left(\frac{2\beta_r\lambda E}{\mu_0}\right)^{1/2} . \quad (7)$$

When $I \neq 0$ there are two approaches:

a) Keep $\bar{K}$ constant and let r_{av} increase.

b) Keep r_{av} constant by increasing $\bar{K}$ (quadrupole gradient, $G \propto \bar{K}$).

In case (a) for a small envelope increase Δr:

$$\frac{\Delta r}{r_{av}} \simeq \frac{k}{2}\,\beta_r\lambda\,\frac{I}{\mu_0 E} \; . \quad (8)$$

Similarly for case (b) the increase in quadrupole gradient is given approximately by

$$\frac{\Delta G}{G_0} \simeq k\;\beta_r\lambda\,\frac{I}{\mu_0 E} \; . \quad (9)$$

In our design we have found it advantageous to maintain μ constant throughout the Linac (as well as for varying currents), which has an additional advantage of keeping constant mean beam radius when comparing different cases and assessing the amount of mismatch or emittance increase.

Choice of Beam Space-Charge Parameters

The Linac specification demands correct operation over a wide range of currents up to 150 mA. In previous sections, the difficulties arising from high current operations have been emphasized, but other intensities have to be checked to make sure that over the whole range of currents we have proper acceptance and acceleration. We can show that (especially for a fixed μ) a significant space-charge parameter is the ratio current to transverse emittance, I/E_t. In the linear approach the beam bunch is represented by a uniformly filled ellipsoid; the space-charge force constants are

$$K_{sct} \propto \frac{I}{r^2\Delta\phi}\left[1 - f\left(\frac{\Delta\phi}{r}\right)\right], \quad K_{sc\ell} \propto \frac{I}{r^2\Delta\phi} f\left(\frac{\Delta\phi}{r}\right), \tag{10}$$

with $f(\Delta\phi/r)$ being a form factor. Keeping $\hat{\Delta\phi}_{in}$ constant ($= |\phi_s|$) for all currents, the force constants depend, apart from the form factor, on the ratio I/r^2, or, which is equivalent, on I/E.

In our analysis we have chosen a value of transverse emittance corresponding to a nominal 100 mA accelerated current. Keeping this emittance constant when analysing the dynamics for different currents increases the space-charge difficulties at higher currents ($\rightarrow$ 150 mA) and poses more problems for bunching (matched ΔW increases) for lower currents. In this way we have investigated a larger effective operating range than if we had kept I/E_t constant.

Description of Beam Optics Between 0.75 MeV and 50 MeV

Matching Conditions at Linac Input

The matching parameters, α and β (defined in a previous section), are required in the three phase planes at the input to the Linac before one can finalize the design of the bunching and matching system at 0.75 MeV.

Transverse Acceptance and Matching. The beam radial excursions are limited by the drift tube apertures, defining a transverse acceptance which we calculate for an equivalent beam having the same form factor as the real input beam. With beam and machine parameters currently obtainable, this acceptance A_t is much greater than E (80π mm mrad is our target value) as shown in Fig. 1a; A_t is given as a function of current, with μ as fixed parameter. If we specify μ and E, then the average beam size in the Linac follows directly [Eq. (3)]. The parameters of practical importance, α and β at the assumed input plane of the Linac, depend also on the current and are output from the linearized programs.

Longitudinal Acceptance and Matching Parameters. The role of "aperture" in the longitudinal sense is taken by the phase extent of the separatrix, which in the non-linear zero space-charge case is approximately $3|\phi_s|$ and can be identified with a maximum phase excursion $2\hat{\Delta\phi}$. For the highest currents to be accelerated in our Linac there is a reduction in acceptance and we therefore take $\hat{\Delta\phi} = |\phi_s|$ for the definition of the linearized longitudinal acceptance A_ℓ. In contrast to the transverse case we fill up this longitudinal acceptance ($A_\ell = \hat{\Delta\phi}\hat{\Delta W}$) with our equivalent beam. As $\hat{\Delta\phi}$ is constant, the other matching parameter $\hat{\Delta W}$, given in Fig. 1b as a function of current (μ as a constant parameter), is also proportional to A_ℓ. It follows from the essentially weak nature of the longitudinal focusing and our choice of input plane that the matched emittance ellipse is nearly in principal axes ($\alpha_\ell = 0$) for all currents.

Bunching and 0.75 MeV Beam Transport

Prior to being injected into the Linac, the beam undergoes a bunching process which forms the longitudinal beam emittance. The choice of bunchers[5], their disposition, and settings have been determined by the optimization of the low-energy beam transport and matching to the Linac in six dimensions[6].

Bunching System. The bunching system is composed of a double buncher with cavities operating on the fundamental and the second harmonic of the RF, respectively, and a single buncher on the fundamental frequency, close to the Linac input. The double buncher (DDHB) is a reasonable solution for filling the longitudinal emittance efficiently and uniformly, see Fig. 2a; the distance between the two cavity gaps and the ratio between their respective voltages have been optimized to this end (d_{12} = 150 mm, $V_2T_2/V_1T_1 \simeq 0.4$). The single buncher, B3, helps to achieve a correct longitudinal matching over the whole current range; it changes the shape of the emittance, hardly affecting its size. Also it helps to reject the non-trapped particles quickly by giving them excessive energy modulation as in Fig. 2b. Bunching efficiency as computed by beam simulation[7] is always between 75% and 80% of particles in our specified longitudinal acceptance ellipse.

Bunched Beam Transport. In Fig. 3 the beam dynamics in the bunching region is schematically presented with a typical set of parameter values. The six quadrupoles ensure the transverse matching with additional constraints of limited beam diameters in the buncher gaps.

Acceleration from 0.75 MeV to 50 MeV

The procedure for obtaining the machine parameters which are important for the longitudinal dynamics, for example ϕ_s, dW/dZ, and cell lengths, has been dealt with in some detail for the structure design[1], and in Fig. 4 we present graphs of some of these dynamics parameters.

Aspects of Beam Dynamics above 0.75 MeV. The beam optics in the acceleration process is initially analysed by linear optimization programs[8], which compute matching parameters and quadrupole gradients for the useful range of values of μ and of beam current. The results are checked with multiparticle programs[9]. To simulate the beam in the linear treatment we use an equivalent beam (uniform density in an ellipsoid) having transverse emittance constant for all currents (80π mm mrad) and a longitudinal emittance as required by the proper longitudinal matching with $\hat{\Delta\phi} = 35°$. For the multiparticle beam we assign coordinates statistically to a large number (about 600) of particles, so that there is a more realistic phase-space distribution but with (statistically) identical matching parameters to the linear case[8]. In both linear and nonlinear cases the beam is injected a few degrees away from ϕ_s so that there are minimum phase oscillations of the beam centroid (the axial particles are not the "average" particles of a finite radius beam, nor is the potential well symmetric in the general case).

Typical results for beam envelopes obtained for high currents and $\mu = 39°$ are given in Figs. 5 and 6 for linear and multiparticle treatments, respectively.

When there is no emittance growth the envelope results can be summarized as follows: for $\mu = 39°$ (constant throughout Linac and for all currents) the "wiggle" factor defined by $w(W) = 0.5\ (r_{max} - r_{min})/r_{av}$ has typical values:

for I = 0 mA: $w(0.75) = 0.20$, $w(50) = 0.17$

for I = 150 mA: $w(0.75) = 0.29$, $w(50) = 0.20$.

Neglecting the envelope oscillations, the longitudinal phase dampings can be expressed as

$$\widehat{\Delta\phi}(\beta_r) \propto \beta_r^{-x} ,$$

where x depends on current and takes values 0.73 and 0.60, respectively, for I = 0 and 150 mA (N.B. $\widehat{\Delta Z} \propto \beta_r^{1-x}$).

Discontinuities and Mismatches. The Linac is a quasi-periodic structure and its non-periodicity is further increased by discontinuities unavoidably present in the practical Linac design. All discontinuities cause mismatches, which can start in any of the phase planes but usually manifest themselves in the others as well via the coupling processes.

Transverse discontinuities are due to

a) abrupt changes in quadrupole dimensions (the quadrupoles are in five batches, four of which are in the first tank);

b) several quadrupoles connected to the same power supply;

c) inter-tank spaces.

In general, the effect of all these discontinuities can be kept under control by a suitable (computed) setting of quadrupole gradients.

The longitudinal discontinuities arise from

a) drift tube aperture changes causing jumps in T (and dW/dZ in Tank I, where one cannot change the electric field correspondingly);

b) inter-tank space.

In practice, the effects of longitudinal discontinuities cannot be annulled by adjustments of operating levels and have therefore to be kept in reasonable limits by a proper accelerator design. All discontinuities mentioned so far have been included in the computations presented in Figs. 5 and 6. The only visible effects are the oscillations of the longitudinal beam envelope caused by inter-tank spacings (which are at their minima of 150 and 200 mm, respectively). Mismatches also occur if the matching requirements at the Linac input are not satisfied. All matching parameters are applied as computed by the linear program with the exception of the longitudinal amplitude function, β_ℓ, which is increased by $\simeq$ 7% when used in multiparticle test runs; this increase (empirically determined) is due to the actual non-uniform longitudinal potential well into which the beam bunch is placed asymmetrically.

An example of a mismatched beam is given in Fig. 7, where at Linac input an error of 25% in β_x has been introduced.

Summarized Results of Beam Simulation. Multiparticle beam simulation has been done for the complete Linac with emphasis on the settings of α, β, and $\widehat{\Delta W}$ dictated by the linear programs and for $\mu = 39°$. Except for longitudinal oscillations the motion was essentially matched in all cases given in Table 1 (as in Fig. 6). Note that with our strong focusing the transverse emittance shows negligible growth (< 20%), whereas the longitudinal emittance tends approximately to the same value for all currents in spite of rather different values at input. Modulation of $\widehat{\Delta\phi}$ generally increases with current, although for a favourable phase of the envelope function at the inter-tank the amplitude can be reduced.

Runs of the multiparticle program[9] have been made also with the more realistic distribution coming from the bunching simulation program[7]. After the particles outside the longitudinal acceptance ('tails') have been lost (below ∿ 5 MeV) the beam behaviour is not significantly different from that of the idealized beam.

Table 1

Dynamics Results Summary

I (mA)	μ (°)	Trans. Emit. Inc.	Long. Emit. Inc.	Long. Emit. at 50 MeV (MeV°)	Modn. on $\widehat{\Delta\phi}$ (%)
0	39	1.1	1.0	1.7	2
50	39	1.0	1.3	1.8	6
100	39	1.1	1.4	1.7	8
150	39	1.1	2.3	1.9	18
100	25	1.3	1.3	1.6	13
150	25	1.1	1.6	1.7	14

Performance of the Linac

In this paper we have summarized the treatment of the beam optics from the bunching at 750 keV to the Linac output at 50 MeV. Beam simulation has confirmed the validity of our approach in choosing and setting machine parameters; how relevant are the figures quoted in Table 1 and concerning the Linac beam?

In the computations we have neglected some perturbing effects, including:

1. Geometrical errors (alignment errors, errors in electric and magnetic fields due to shape errors.
2. Adjustment errors (maladjustment of some parameters, variations in power supplies).
3. Statistical errors (fluctuations in the ion source beam during a pulse and from pulse to pulse).

These effects could measurably increase the beam emittances at 50 MeV. However, comparing

Table 1 with the specifications of the new Linac[10] we see that:

a) there is a safety factor of two in the transverse emittance, even for the maximum current of 150 mA ($E_t \simeq 12\pi$ mm mrad as compared to specified $\lesssim 25\pi$ mm mrad for I = 100 mA);

b) the energy spread is ∿ 150 keV at Linac output, i.e. before debunching, the longitudinal emittance being 0.15 MeV × 13° (the specification asks for an energy spread of ≲ 150 keV, after debunching, at booster input).

With the above, we consider that there is a sufficient safety margin to meet the specifications without major difficulties.

Acknowledgements

Many of the beam optics problems of the new Linac have been handed in cooperation with B. Bru (now with GANIL, France), whose contributions to the program development have been particularly valuable. P. Lapostolle (CNET, France) and M. Promé (GANIL) have collaborated with us on many aspects, as has, more recently, J. Guyard (CERN Fellow from the University of Nancy, France). C. Poinard has made numerous computer runs and classified the results. Our special thanks go to all of them.

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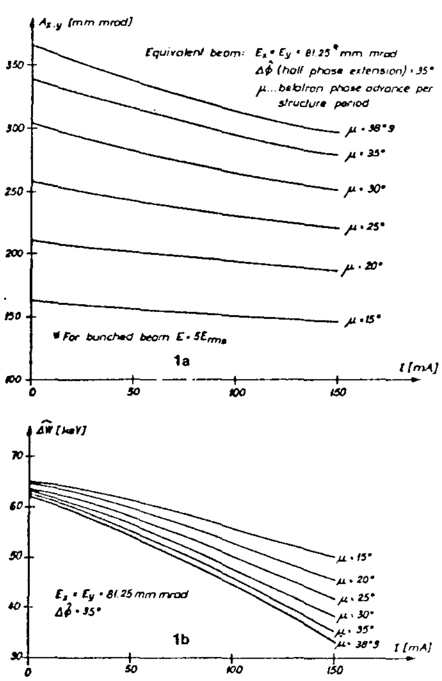

Fig. 1 Transverse acceptance A_t (1a) and $\widehat{\Delta W}$ (1b) as a function of I

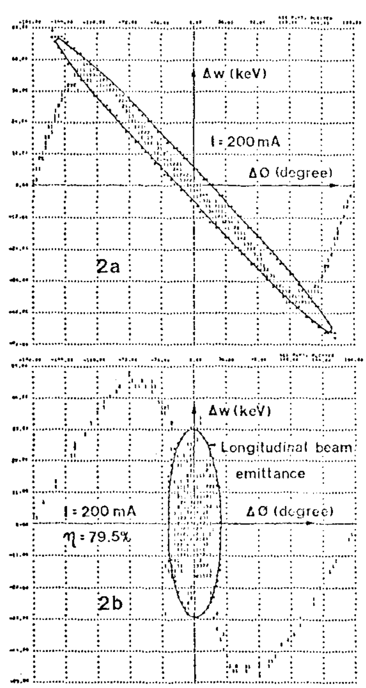

Fig. 2 Longitudinal emittance after DDHB (2a) and at Linac input (2b)

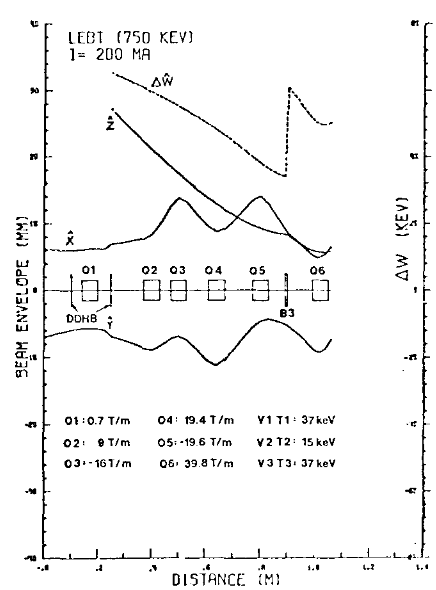

Fig. 3 Beam dynamics in the bunching region

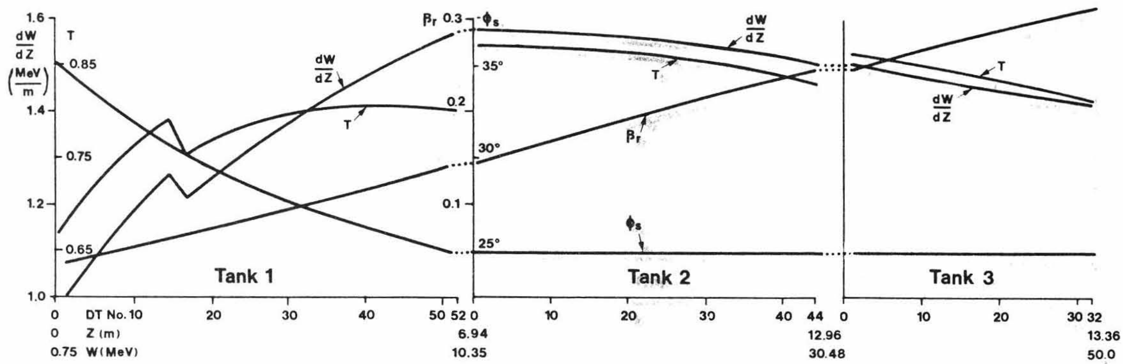

Fig. 4 Dynamics parameters variation along the Linac

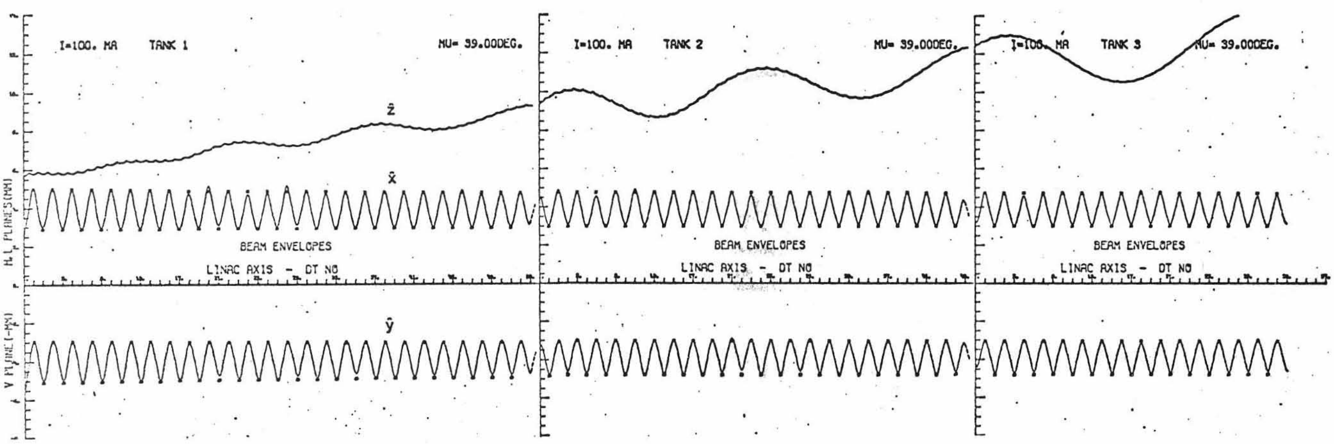

Fig. 5 Linearized beam dynamics in the Linac

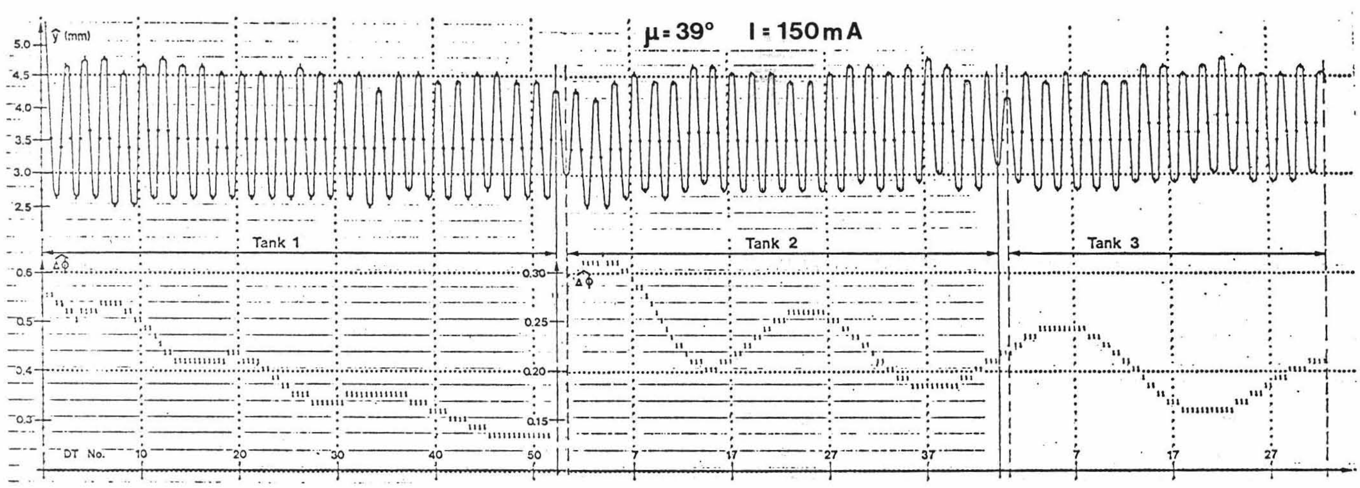

Fig. 6 Simulated beam motion in the Linac

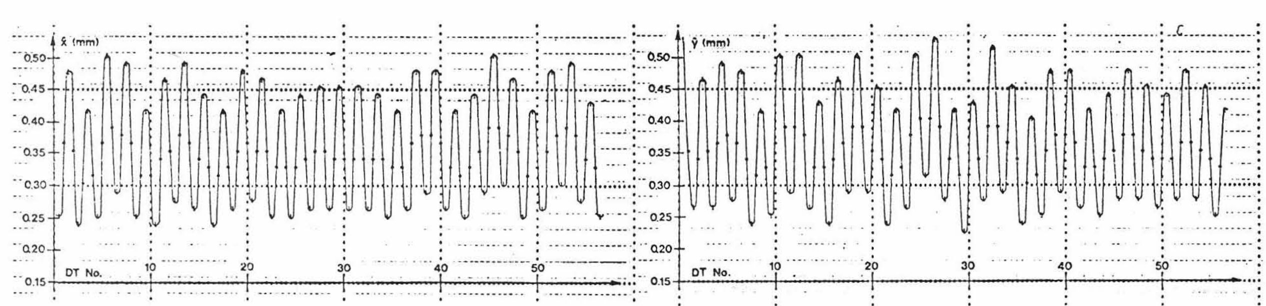

Fig. 7 Simulated unmatched beam in Tank I