MODELLING OF BIPERIODIC SLOW-WAVE STRUCTURES

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Summary

Two models are considered for defining the electrical parameters of biperiodic slow-wave structures (BSWS) with beam loading. The first model is represented by a chain of coupled cavities and the second one by a chain of lumped-parameters circuits. The feeding waveguide is accounted for in both models. Two computing programs which use a finite difference method on a square mesh were written. Some computational results are given.

Introduction

A number of requirements in radiography and medicine, strongly demand development of compact standing-wave electron linacs. Theoretical and experimental research on the performance at low and high power levels, at biperiodic slow-wave linacs, made it possible to start commercial production of these machines.

The design of standing-wave electron linacs requires the development of computer programs. In the modelling of biperiodic slow-wave structures (BSWS), two models were considered. In the first model, the BSWS was represented by a chain of coupled cavities; in the second a chain of coupled lumped circuits was used. In both models the rectangular feed guide was taken into account, and considered to be essential for the design of a standing-wave linac with a 3 db hybrid junction rf feed scheme.

Biperiodic Slow-Wave Structures As A Chain Of Coupled Cavities

In this model all rf and accelerating systems are divided into two regions: the biperiodic slow-wave structure proper, and the feeding waveguide (Fig. 1). This separation is quite practical because it allows regular and irregular regions to be considered independent of each other. The boundary lies on the surface of the coupling iris. The thickness of the separating wall is assumed to be negligibly small. Considering the tangential component of the field at the boundary to be known, one can evaluate the electromagnetic fields in regions I and II. Indeed, to preserve continuity of magnetic field as one goes through the opening in the iris, the following equation must be valid:

$$[\vec H^{I}(\vec j^{\,I}_{ex},\vec E_{tg})\vec n_{21}]+[\vec H^{II}(\vec j^{\,II}_{ex},\vec E_{tg})\vec n_{12}]=0 \quad . (1)$$

Here H^{I} and H^{II} are boundary magnetic field vectors in regions I and II. The fields are excited by some external sources and by a tangential electric field, E_{tg}. $\vec n_{21}$ and $\vec n_{12}$ are normals to the boundary surface directed correspondingly into regions I and II. Assume first there is no current in the BSWS. Then, according to the superposition principle, and taking into account that $\vec j^{\,II}_{ex}=0$, for a linear system it is found:

$$[\vec n_{21}\vec H^{I}(0,\vec E_{tg})]+[\vec n_{12}\vec H^{II}(0,\vec E_{tg})]=[\vec H^{I}(\vec j^{\,I}_{ex},0)\vec n_{21}] \quad . (2)$$

The Ritz-Galerkin method[2] can be used to solve equation (2). In this method, the field is represented as an expansion in the full system of coordinate functions :

$$\vec E_{tg}=\sum_p e_p\vec{\mathcal{E}}_p \quad . (3)$$

Substituting (3) in (2), and after the usual transformations, yields a system of linear equations with respect to unknown coefficients e_p (the order of the coefficients is determined by the number of the coupling slots and the number of coordinate functions taken into account):

$$\sum_p e_p\int_{S_{slot}}\vec{\mathcal{E}}^*_m[\vec n_{21}\vec H^{I}(0,\vec{\mathcal{E}}_p)]\,dS+\sum_p e_p\int_{S_{slot}}\vec{\mathcal{E}}^*_m[\vec n_{12}\vec H^{II}(0,\vec{\mathcal{E}}_p)]\,dS=\int_{S_{slot}}\vec{\mathcal{E}}^*_m[\vec H^{I}(\vec j^{\,I}_{ex},0)\vec n_{21}]\,dS \quad , (4)$$

where $\vec{\mathcal{E}}^*_m$ is the complex conjugate of $\vec{\mathcal{E}}_m$, S_{slot} is the surface of the coupling slot. The accuracy of solving (4) depends on the number of terms in (3). For terms of (3), take a set of coordinate functions ψ_ℓ and ψ_k which have to satisfy the wave equation and homogeneous boundary conditions on the coupling iris:

$$\nabla^2\psi_{\ell,k}+\varkappa^2_{\ell,k}=0 \qquad \frac{\partial\psi_k}{\partial n}=0 \qquad \psi_\ell=0 \quad ,$$

where $\vec n$ is the normal to the surface. In this case

$$\vec E_{tg}=\sum_k e_k\vec{\mathcal{E}}_k+\sum_\ell e_\ell\vec{\mathcal{E}}_\ell=\sum_p e_p\vec{\mathcal{E}}_p$$

and

$$\vec{\mathcal{E}}_k=[\nabla\psi_k\vec n] \qquad \vec{\mathcal{E}}_\ell=\nabla\psi_\ell \quad .$$

The next step is to find H^{I} and H^{II} in (4). The Kysun'ko method[2] can be used for the determination of the magnetic field H^{I} in the feeding waveguide. According to this method, the electromagnetic field is represented by a series in eigenvector functions

$$\vec{E}=\sum_{a} U_a \vec{\mathcal{E}}_a \qquad \vec{H}=\sum_{b} I_b \vec{\mathcal{H}}_b \quad , (5)$$

where unknown coefficients U_a, I_b are derived from the so called waveguide equations. To find the first term in the left side of (4), $H^{I}(0,\vec{\mathfrak{J}}_p)$ must be determined (the value of the magnetic field which is excited in the rectangular waveguide by the component $\vec{\mathfrak{J}}_p$ at the iris). Then, in accordance with (5), and taking into account that in (4) the unknown field is a part of the vector product, it will suffice to derive only the field transverse components in region I at the coupling iris.

$$\vec{H}^{I}(0,\mathfrak{J}_p)=\sum_{h}\frac{1}{Z_h}\int_{S_{slot}}\mathfrak{J}\vec{\mathcal{E}}_h\, dS\,\vec{\mathcal{H}}_h \quad . (6)$$

For the ultimate solution of (6) it is necessary to write eigenvector functions $\vec{\mathcal{E}}_h, \vec{\mathcal{H}}_h$ and coordinate functions $\vec{\mathfrak{J}}_h$ in explicit form. Eigenvector functions $\vec{\mathcal{E}}_h$ and $\vec{\mathcal{H}}_h$ are formed in a similar manner to $\vec{\mathfrak{J}}_p$. $\vec{H}^{I}(\vec{j}_{ex}^{\,I},0)$ in the right-hand side of (6) is defined likewise, but the source here is j^{I}_{ex}, a microwave generator and the coupling iris is metal coated, i.e. $E_{tg} = 0$. In this case

$$\vec{H}^{I}(\vec{j}^{\,I}_{ex},0)=\sum_{h}\frac{1}{Z_h}2A_h\vec{\mathcal{H}}_h \quad , (7)$$

where A_h depends on magnetic wave power (with index h) flowing into the iris:

$$P_h=\frac{1}{2}\frac{A_h^2}{Z_h}\int_S \vec{\mathcal{H}}_h^{\,2}\, dS \quad .$$

The last term in (4) must be defined. This problem is similar to the definition of fields in a BSWS with a side hole in one of its cells. The structure is excited by tangent field E_t or its component $\vec{\mathfrak{J}}_p$. For the solution Slater's equation[3] will be used with the method described in Ref. 4, 5. As shown in Ref. 6, a set of equations with respect to complex amplitudes U_n and I_n for a chain of cavities coupled by slots, has the following form:

$$(\omega_{mn}^2-\omega^2)U_{mn}=\omega^2(1-j)\frac{1}{Q_{mn}}U_{mn}-\frac{\omega_{mn}}{\sqrt{\varepsilon\mu}}\int_{S_{slot}}\left[\vec{E}_{tg_n}\vec{\mathcal{H}}_{mn}\right]\vec{n}\,dS; \quad (8)$$
$$I_{mn}=j\omega\varepsilon 2\pi C\omega_{mn}U_{mn} \, .$$

It is assumed that the electric and magnetic fields to be determined in (8) can be expressed as

$$\vec{E}_n=\sum_{m}U_{mn}\vec{\mathcal{E}}_{mn} \; ; \; \vec{H}_n=\sum_{m}I_{mn}\vec{\mathcal{H}}_{mn} \, ,$$

where n is the cavity number, $\vec{\mathcal{E}}_{mn}$ and $\vec{\mathcal{H}}_{mn}$ are electric and magnetic field eigenvectors in cavities without slots. In (8), ω_{mn} is the frequency of the m-mode in n-cavity, Q_{mn} is the quality factor of the cavity, E_{tg} is tangent electric field on the surface of the coupling slot. In the following considerations, the fields in cavities of a BSWS will be represented by the first term of expansion in functions $\vec{\mathcal{E}}_m$ and $\vec{\mathcal{H}}_m$. In case of axially symmetric cavities, it is similar to considering the E_{010} mode (m=1). If the coupling slots are narrow enough and cut along the azimuthal coordinate near the side wall, they can be represented by a transmission line with a T-mode wave.[4,5] Then the integral over the slot between cavities n and n-1 looks like:

$$\int_{(S_{slot})_{n,n-1}}\left[\vec{E}_{tg_n}\vec{\mathcal{H}}_n\right]\vec{n}\,dS=j\omega L_0\left[I_n\mathcal{H}_n(r_s)-I_{n-1}\mathcal{H}_{n-1}(r_s)\right]\times\left[2\tan\frac{K\ell_s}{2}-K\ell_s\right]\cdot\frac{\mathcal{H}_n(r_s)}{K^3} \quad . (9)$$

Here r_s is the location of slot radius; ℓ_s is the slot length; $k = 2\pi/\lambda$. The equation set (8) is non-homogeneous, due to the term

$$\int_{S_{slot}}\left[\vec{E}_{tg_n}\vec{\mathcal{H}}_n\right]\vec{n}\,dS \quad . (10)$$

Since E_{tg} in (10) is identically equal to $\sum_p e_p\vec{\mathfrak{J}}_p$ the definition of the remaining term in (4), $\vec{H}^{II}(0,\vec{\mathfrak{J}}_p)$ is carried out in the following way. After substituting $e_p\vec{\mathfrak{J}}_p$ for E_{tg} in (10) and taking the integral, (8) must be solved with the right-hand side in the form of (10). The solution is carried out separately for every component of the tangent electric field E_{tg}. From (8) amplitudes I_n, V_n of the fields in cavities of BSWS with a feeding waveguide are obtained. The value of the magnetic field at the coupling slot is substituted into (4) for the ultimate integration. When all the terms of equation (4) are finally determined, it may be solved with respect to e_p by any appropriate method. The solution allows evaluation of the dispersion relation, input impedance, reflexion coefficient and the electric field distribution in the structure. The beam loading problem for the first model of BSWS is solved by the introduction of terms containing the beam current $\vec{J}_n$ into (8)

$$\frac{1}{\varepsilon}\frac{d}{dt}\int_{V_n}\vec{J}_n\vec{\mathcal{E}}_n\,dV \quad . (11)$$

To express (11) in explicit form, assume that the continuous beam entering BSWS has constant density.

Divide this beam at every time interval (0,T where $T = 2\pi/\omega$) into M elementary bunches. Each bunch can be represented by the first harmonic of the Fourier series. Then (11) becomes

$$\frac{1}{\mathcal{E}}\frac{d}{dt}\int_{V_n}\vec{J}_n\vec{\mathcal{E}}_n dV = \frac{j\omega}{\mathcal{E}M} I\left(\sum_{K=1}^{M}\frac{e^{-j\varphi_{1K}}-e^{-j\varphi_{2K}}}{\varphi_{1K}-\varphi_{2K}}\right)\int_0^{L_n}\mathcal{E}_n dz\, e^{j\omega t}, \qquad (12)$$

where φ_{1K} is the field phase when the bunch appears at the cavity entrance and φ_{2K} is the field phase when it leaves the cavity. I is the peak current. With the terms like (12) present, the equations set (8) and consequently (4) become non-linear, and an iteration method is used to solve them. At each iteration, the solutions of (8) and (4) were combined with the solution of the longitudinal motion equation for the bunches in BSWS. This method allowed evaluation of the longitudinal electron dynamics in BSWS, and takes into account the beam loading influence on the parameters of BSWS.

Biperiodic Slow-Wave Structure As A Chain Of Coupled Lumped-Parameters Circuits

Figure 2 shows an equivalent circuit of a beam loaded BSWS with a feeding waveguide. To each cell in BSWS ascribe capacitance C, inductance L and resistance R. The interactions of the cells are due to the mutual inductance, which is characterized by coupling coefficient K_c. A microwave generator is simulated by a voltage source U_G with resistance W. A feeding waveguide is simulated by a transmission line with the wave impedance of the same value W. The coupling of the feeding waveguide with BSWS is simulated in the equivalent circuit by inductive coupling with transformation coefficient m. The power is fed into the g-cell at the terminals "a-a". The beam loading effect is taken into account by an ideal current source J_n. For the circuit of the type shown the Kirhgoff equations with respect to the mesh currents have the following form after some transformations:[7]

$$\begin{cases} X_0\Phi_0 + K_{c1}X_1 = J_0 \\ X_1\Phi_1 + \frac{1}{2}\left(K_{c1}X_0 + K_{c2}X_1\right) = J_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ X_g\Phi_g + \frac{1}{2}\left(K_{cg}X_{g-1} + K_{cg+1}X_{g+1}\right) = J_g + \frac{U_{aa}}{j\omega\sqrt{2L_g}} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ X_N\Phi_N + K_{cN}X_{N-1} = J_n \end{cases} \qquad (13)$$

Here $X_n = i_n\sqrt{2L_n}$, $\frac{1}{2}|X_n|^2$ is the stored energy in circuit n, $\Phi_n = 1-(\omega_n/\omega)^2 + \omega_n/jQ_n\omega$, Q_n is the circuit n quality factor, ω_n is the resonant frequency, U_{aa} is the voltage amplitude at "a-a" terminals. The second term in the right side of g equation (13) may be transformed to

$$\frac{U_{aa}}{j\omega\sqrt{2L_n}} = \frac{2S_{aa}X_g}{j\omega|X_g|^2}. \qquad (14)$$

Here $S_{aa} = \frac{1}{2}U_{aa}i_g^*$ is the complex power at the input circuit terminals. For the input voltage:

$$\frac{U_{aa}}{j\omega\sqrt{2L_n}} = \frac{Z_{aa}X_g}{j\omega 2L_g}, \qquad (15)$$

where Z_{aa} is the input impedance of the chain at the "a-a" terminals. Keeping in mind that the microwave generator is represented by an ideal voltage source with inner resistance W, an expression for complex power S_{aa} can be obtained in terms of the generator's power:

$$S_{aa} = \frac{4m^2WP_G}{|m^2W + Z_{aa}|^2}Z_{aa}. \qquad (16)$$

Combining (14), (15) and (16) one can get

$$\frac{8P_G}{|1+1/\beta|^2} = q\,\omega_g|X_g|^2. \qquad (17)$$

Here $q = m^2W/\sqrt{2L_g/C_g}$. Note that $\sqrt{2L_g/C_g}$ is g circuit characteristic impedance. $\beta = m^2W/Z_{aa}$ is the coupling coefficient between BSWS and the feeding guide at the operating mode. Divide each equation of (13) by X_g and introduce new variables $\chi_n = X_n/X_g$. These equations show that for deriving the unknown value, one has to solve two equations for the circuits adjacent to the one with the feeding guide. In case of the negligible beam loading, all J_n are equal to zero ($n=0\div N$) and a set of linear algebraic equations are obtained from which dispersion relation, BSWS parameters, fields distribution along the structure and their dependence on the generator power may be defined. The beam loading problem is solved similarly to the first model.

Electrical Parameters of an Axially Symmetric Cavity Excited in the Lowest Mode

For the solution of practical problems requiring the use of equations (8) and (13) one should know the electrical parameters of cells, such as frequencies, quality factors and shunt impedances. The values of these parameters were defined with the help of a computer program similar to LALA.[8] The program was written by converting the equation to finite difference forms using a square mesh. This approach uses relaxation method for the iterations.

Since there are two unknown values (magnetic field H_φ and wave constant $K = 2\pi/\lambda$) the double iterative process for field and wave constant is used. To speed-up the convergence, the program provided iteration on the mesh succession, when the mesh length decreased from one iteration number to another. For the same purpose, smaller meshes were used when the field changed rapidly. For simplicity of use, the program provides automatic mesh superposition and boundary point specification, which reduce preparation time before computing. The program described has the following features: If the EC-1040 computer and 6000 mesh nodes are used, the computation time is about five minutes. The error of the wave constant computation appeared to be less than 10^{-3}.

Some Computational Results

Two computing programs, "CAMEA" for the first model, and "LUCH" for the second model, were written and executed by the EC-1040 computers. The development of these programs was stimulated by the design of standing-wave linacs and the study of their performance.[1] The use of a 3 db hybrid junction seems to be the main difference of the linacs under investigation from standing-wave linacs operating elsewhere. The function of the hybrid junctions are to isolate the rf generator from the resonant load and to increase the frequency stability. From this point of view, parameters of interest of BSWS are input impedance and reflection coefficient, and their dependence on frequency. Some results computed by the programs are presented herewith: Figure 3 shows the experimental dependence of the VSWR in the feeding guide, on the coupling iris width.[1] The accelerating guide here has 6 on-axis coupling cells. The solid points on the curve indicate the results obtained by the "CAMEA" program. They show a good agreement of the calculated and experimental data. Figure 4 shows the dependence of β (see eq. 17) module and phase on the frequency. The dependence is computed by "LUCH" program. As can be seen from (17) β module is proportional to the input susceptance, hence Fig. 4 may be interpreted as a frequency response characteristic of the accelerating guide. The accelerating guide under consideration consists of a regular BSWS with on-axis coupling cells. The respective values for accelerating cell and coupling cell Q factors are equal to $12x10^3$ and $3x10^3$; the coupling coefficient is equal to 0.24. The rf power is fed into the 6th cell. Curve 1 corresponds to the negligible beam loading case and curve 2 corresponds to the peak beam current I, equal to 100 mA. It is of interest, that without a beam, the input susceptance phase at the operating frequency is equal to zero, while with the beam of 100 mA peak current, it differs from zero. This effect shows that the guide is detuned due to the reactive beam loading, i.e. shifting of the bunch synchronous phase. This leads to the feeding wave guide coupling change: overcoupling $\beta = 2.1$ when $I = 0$ and overcoupling $\beta = 0.5$ when $I = 100$ mA.

Conclusion

The investigations carried out showed good agreement between the data calculated by means of the "Camea" and "Luch" computing programs and experiments. Thus the developed computing programs may be recommended for the design and adjustment of electron standing wave linacs using BSWS.

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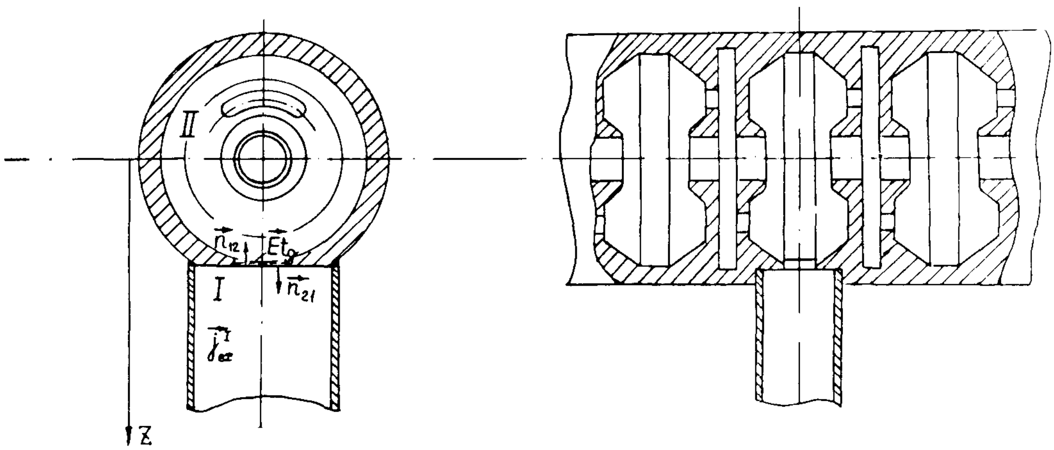

Fig. 1 Biperiodic slow-wave structure with feeding rectangular guide

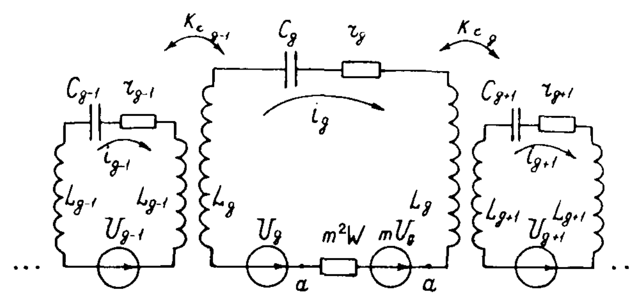

Fig. 2 Equivalent circuit of a beam loaded BSWS

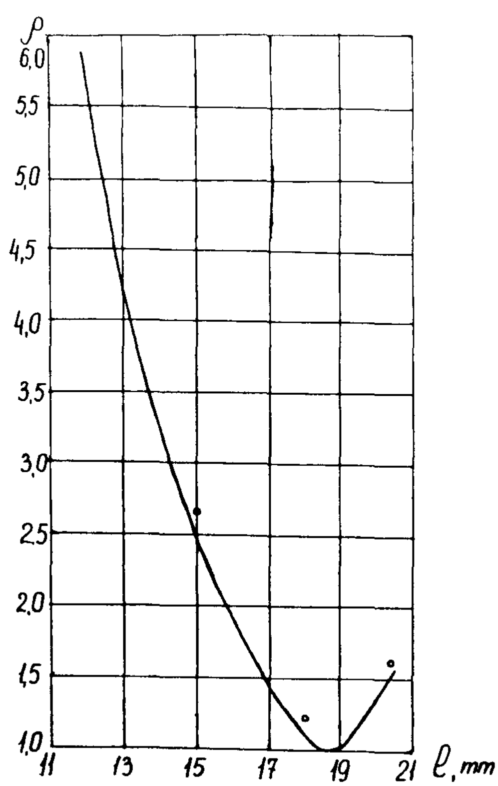

Fig. 3 Dependence of the VSWR in the feeding guide on the coupling iris width

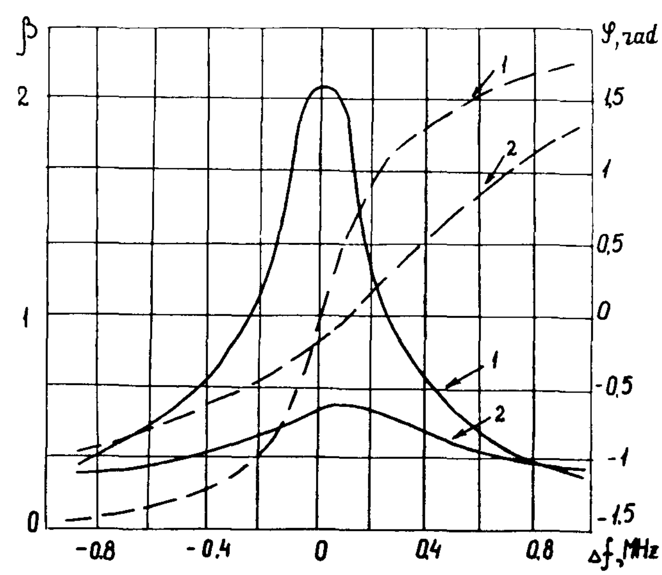

Fig. 4 Dependence of β module and phase on the frequency