BEAM DYNAMICS IN A PROTON LINEAR ACCELERATOR FOR A NEUTRON SPALLATION SOURCE

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Abstract

The frequency choice for a 600 MeV - 100 mA pulse - 10 mA average proton linac is discussed. A 108-MHz Alvarez accelerates the protons from 500 keV to 100 MeV; a 324-MHz high energy structure follows. Space charge effects play a dominant role in the beam dynamics. This choice of frequencies insures large longitudinal and transverse acceptances at injection, small longitudinal emittance blow-up in the Alvarez, and a large enough longitudinal acceptance of the high energy structure. The transverse emittance at injection must not be too small in order to alleviate transverse matching, and to avoid longitudinal emittance blow-up, which would lead to particle loss after transition.

Introduction

For a neutron spallation source,[1] a proton linear accelerator[2] with about 600-MeV final energy,[2] and 10-mA average current has been proposed. The current in the beam pulse has been limited to 100 mA, since otherwise the installment cost for the rf power sources gets high, and the space charge problems in the accelerator severe. Lowering the pulse current increases the power bill. The dc injector voltage should not be larger than 500 kV in order to insure a reliable operation with the relatively high average beam current. From experience with existing machines, the Alvarez frequency ought to be in the 100 to 200 MHz range, and the frequency of the high energy part between 300 and 800 MHz. The availability of large rf power sources for a high duty factor strongly favors frequency choices around 108 MHz and 350 MHz, for which similar demands exist presently at GSI - Darmstadt and CERN (LEP). The total investment cost for the rf power sources is smaller for a 108 MHz - 324 MHz frequency choice compared to 200 MHz - 600 MHz. This more than compensates for the higher construction cost of the accelerator structures at a lower frequency.

In the following the frequency choice will be studied from the beam dynamics point of view. A major concern is to minimize particle loss in order to keep the activation of the machine low. To this end, the transverse emittances will have to be tailored at definite locations along the linac, at which a higher activation will have to be tolerated. At transition between the Alvarez and the high energy structure, the longitudinal phase acceptance is reduced by the frequency ratio (space charge effects neglected). Hence, special emphasis has to be given to insure a large enough phase damping in the Alvarez, and to have a well-defined longitudinal emittance without tails at transition.

Bunching

The space charge influence will be estimated during bunching for a single buncher. The analysis can be carried through analytically for two limiting cases. First, neglecting space charge forces, one can estimate the longitudinal focal length d, and the bunching efficiency. Second, in the highly space charge limited case, one can calculate the distance L, after which a small longitudinal velocity modulation produced by the buncher becomes zero in the beam center.[3] Comparing d and L one gets an idea of the importance of space charge effects during bunching. d << L is required. It is seen that:

$$\frac{d}{L} \approx 5\times10^{-4}\,\frac{W}{\Delta W}\sqrt{\frac{I/\mathrm{Amps}}{\beta^3}} \qquad (1)$$

W is the injection energy, β the corresponding relative velocity, I the beam current, and ΔW half the maximum energy spread imparted to the particles by the buncher. In order to achieve longitudinal matching in the phase-energy phase space at the linac input, ΔW is set equal to the energy acceptance of the Alvarez (space charge included). Note that ΔW is larger at the lower frequency. Taking this as a criterion, the result is that bunching for a 400 keV - 108 MHz system is comparable with respect to space charge problems to a 750 keV - 202 MHz one, and for a given injection energy the lower frequency is more advantageous (Fig. 1). Neglecting space charge, the bunching efficiency is independent of frequency. Also, the option of using the rf quadrupole structure[4] as an accelerating buncher looks more promising the lower the Alvarez frequency is chosen.

Acceptances at Injection

The acceptances at injection have been studied analytically as a function of injection energy and Alvarez frequency.[2,5] Linear transverse quadrupole focusing (FODO), rf defocusing, and space charge forces (the bunch is a uniformly-charged ellipsoid) are assumed. The non-linearity of the longitudinal motion is kept to second order terms. The rapid increase of the acceptances due to acceleration is neglected. The result is that all of the basic beam dynamic parameters are the more favorable the higher the injection energy. Further, the acceptances of a 750 keV - 202 MHz linac are similar to those of a 400 to 500 keV - 108 MHz one.

In Fig. 2,3, and 4 the acceptances and the required quadrupole field at 500 keV injection energy are shown as a function of the normalized transverse input emittance $\beta\gamma E$ (100% of beam). Having fixed the transverse focusing strength (betatron phase advance per focusing period, $\mu = 40^0$, at a 0.1 A beam current) the average beam radius[6] is $\sqrt{E2\beta\lambda/\pi\mu}$

In order to stay well inside the region of transverse stability, μ is kept constant when varying the emittance. This means that a smaller transverse emittance leads to a smaller average beam radius, hence to larger space charge forces, which in turn have to be compensated by larger focusing forces. For normalized emittances smaller than 1 π mm-mrad, the longitudinal acceptance gets extremely small and the required quadrupole fields can no longer be obtained for a 202-MHz Alvarez.

A too small transverse emittance is undesirable especially for a multi-stage linac system operating close to the longitudinal space charge limit. After injection the bunch would rapidly expand until the longitudinal rf focusing force and the defocusing space charge force are equal. The phase damping would be reduced, and at transition to the next higher frequency section the phase emittance might be larger than the phase acceptance. On the other hand, a too large transverse emittance results in a too large beam radius, and, because of that, in a too large dependence of the transit time factor on a particle's radial position. The influence of the transverse motion on the longitudinal one would get too large. The maximum beam radius that can be tolerated from this point of view is set to 0.75 $\beta\lambda/2\pi$ or 3/4 of the drift tube aperture at injection. At this radius a particle would already get a 15% higher energy gain than an on-axis one. The transverse acceptance in Fig. 3 is calculated accordingly. For a given transverse emittance, Fig. 3 shows a larger transverse acceptance for the lower frequency. Also, a relatively smaller fraction of the drift tube aperture is filled by the beam. It follows that the radial dependence of the longitudinal motion and the resulting longitudinal emittance blow-up is less at the lower frequency for a given transverse emittance. The transverse emittance blow-up is affected similarly by the radial non-linearities.

It is concluded that 108 MHz is favored over 202 MHz from the beam dynamics aspects at injection: the acceptances are larger, longitudinal space charge problems are less severe, there exists a range of transverse input emittances, for which both longitudinal and transverse motion are stable (between 4 and 7 π mm-rad), and the required quadrupole fields can more easily be realized.

Phase Damping

The phase damping along the linac can be calculated analytically using the above-mentioned approximations, and in addition, linearizing the longitudinal motion.[5,7] The transverse motion is coupled to the longitudinal via the space charge forces. It can be expressed by a single parameter, the ratio $a_{xo}a_{yo}/a_{x1}a_{y1}$ of beam cross-sections at the entrance and exit of the linac. Then the result is

$$\frac{\Delta\phi_1}{\Delta\phi_o} = \frac{\Delta W_o}{\Delta W_1} = \left[\frac{E_o T_o \sin\phi_{so} \beta_o^3 \gamma_o^3 (1-\mu_{\ell o})}{E_1 T_1 \sin\phi_{s1} \beta_1^3 \gamma_1^3 (1-\mu_{\ell 1})} \right]^{1/4} \qquad (2)$$

$$\mu_\ell = \frac{-15\Omega\ (I\lambda)\ \beta\lambda}{\pi\sqrt{a_x a_y}\ b^2 ET \sin\phi_s} \qquad (3)$$

$$\frac{1-\mu_{\ell o}}{1-\mu_{\ell 1}} = \left[\frac{\mu_{\ell o} D}{2} + \sqrt{1-\mu_{\ell o}+\left(\frac{\mu_{\ell o} D}{2}\right)^2} \right]^2 \qquad (4)$$

$$D = \left[\frac{E_o T_o \sin\phi_{so} \beta_1 \gamma_1^3 a_{xo} a_{yo}}{E_1 T_1 \sin\phi_{s1} \beta_o \gamma_o^3 a_{x1} a_{y1}} \right]^{1/2} \qquad (5)$$

$$\Delta\phi_{max} = \frac{3}{2}\,\phi_s\ (1-\mu_\ell) \qquad (6)$$

$$\Delta W_{max} = q\left[-\frac{2\lambda}{3\pi}\,\frac{mc^2}{q}\,ET\ (\beta\gamma\phi_s(1-\mu_\ell))^3 \right]^{1/2} \qquad (7)$$

Where: $\Delta\phi$ = half phase width, ΔW = half energy spread, E = average axial electric field, T = transit time factor, ϕ_s = synchronous phase, λ = wavelength, b = half bunch length, $\pi\Delta\phi_{max}\Delta W_{max}$ = longitudinal acceptance, mc^2/q = mass/charge.

Evaluating these formulae for typical parameters[2,5] allows the following conclusions: 1. In linear theory, the phase width at the Alvarez exit depends only weakly on the parameters of the Alvarez. 2. The influence of the longitudinal space charge force along the linac is the larger the higher the frequency and the lower the injection energy is. 3. The phase acceptance at the Alvarez exit is reduced appreciably compared to the zero space charge phase acceptance. This reduction is less pronounced at the lower frequency and higher injection energy. 4. The longitudinal acceptance reserve gets extremely small at the Alvarez end if the average beam radius is not allowed to grow along the linac. These results have to be modified if the non-linear parts of the phase focusing force are considered. They yield, together with the space charge force, a strong asymmetric depression of the potential well for particles with positive phases (that is, trailing particles). For these the phase damping will be smaller than for particles sitting close to the bottom of the well, and the effect will be the larger the larger the ratio of space charge to rf force (μ_ℓ). As μ_ℓ is smaller for a lower frequency, the linear phase focusing region is larger at a lower frequency. A too small beam diameter (e.g. caused by a too small transverse emittance) would cause a larger than necessary longitudinal space charge force, and hence a too small phase damping in the non-linear phase focusing region.

Emittance Growth

Transverse emittance growth is mainly caused by a combination of the phase dependence of the rf plus space charge defocusing, and the fact that consequently the transverse matching can never be perfect for all parts of the bunch.[8] A single slice of the bunch will experience the phase dependence of the rf defocusing at the synchrotron oscillation frequency, the space charge forces then will change at twice the synchrotron frequency, and both together cause the beam radius to oscillate about its matched value at twice the betatron frequency. The combined effect will be a rapid transverse emittance increase after injection, settling down in the early sections of the linac, because the amplitudes of the oscillations about the matched solution depend only on the initial mismatches.[9] This effect can only be neglected if [2,10]

$$\frac{k_\ell^2|\sin\phi-\sin\phi_s|}{k_{t\rho}^2\,2|\sin\phi_s|} = \frac{q}{mc^2}\,\frac{4\pi\lambda\ ET|\sin\phi-\sin\phi_s|}{\beta\gamma^3\mu^2} \ll 1 \quad (8)$$

k_ℓ is the longitudinal wave number for zero current, and $k_{t\rho} \approx \mu/2\beta\lambda$ the radial one including the space charge effect. At injection, for parameters as in Fig. 2, the ratio (8) = 1.9; thus a large transverse emittance increase must be expected, being less severe at a higher frequency. The effect is more pronounced the smaller the emittance as reducing the emittance towards zero without changing the quadrupole gradients [11] means also $\mu \to 0$. On the other hand, if one keeps μ constant when scaling the emittance to zero, the beam radius will go down making the space charge forces and the tune shift with respect to zero current very large. As a real bunch is not a uniformly-charged ellipsoid, the radial space charge force depends on the longitudinal position being smaller at the ends of the bunch. This again causes a phase dependence of the transverse motion, and hence a larger transverse emittance blow-up the smaller the emittance is. Also this effect is more troublesome at a lower frequency, as the ratio of transverse space charge to focusing force and the tune shift is larger. [2]

Transverse emittance growth could also be caused by transverse bunch instabilities [12] or by temperature exchange between longitudinal and transverse directions, both being more probable at a lower frequency. [2] However, at a lower frequency the transverse acceptance is larger, and a larger emittance blow-up can be tolerated. The current carrying capability is larger at the lower frequency; if on the other hand a high brightness is the major design goal, a higher frequency choice looks more promising.

To study emittance growth in more detail, multi-particle beam dynamic simulations were performed for a 108-MHz Alvarez with parameters as at injection. Note that $\mu = 40^\circ$ at $I = 0.1$ A is kept constant along the linac for all runs. The computer programs [6,13] used are of CERN origin. A total of 4 x 2000 macro particles were traced. The cut-off distance for the space charge force was set equal to the average particle distance resulting in about 3 close collisions per particle per cell of length $\beta\lambda$. The mesh of the "space charge cage" was 60 × 60 × 60 fitted closely around the bunch. Typical rms values change by 10% when varying any of these program parameters by a factor of 2.

In Fig. 5 and 6 the emittance increase and phase damping along the Alvarez is shown for matched beams. The emittance at 100% of beam is defined e.g. as $E_{rms}\cdot\max(x)\cdot\max(x')/(\mathrm{rms}(x)\cdot\mathrm{rms}(x'))$. Although these emittances scatter a lot, it is clearly shown that they increase much more than the rms ones. (Comparing measurements and calculations one has to refer to the same percentage of beam!) This corresponds to the formation of tails, being especially dangerous in a high average current linac. Most of the growth has happened after one synchrotron oscillation (at about 2.3MeV), which fits into the picture given above. The growth rate is larger for a smaller initial emittance. [11] The phase damping is nearly the same for rms and maximum phase widths, and is only slightly less at the lower transverse emittance, as has been expected for our parameter choice. Figure 7 shows the rms emittances and the phase width at 100% of beam at the Alvarez exit (100 MeV). For these runs the input matching conditions and the quadrupole gradients are kept constant, equal to the matched values for a 4.65 π mm-mrad input emittance; hence all other runs were mismatched at injection. A large transverse emittance increase is noticed for small input emittances (for which good matching will be very difficult in a real linac). Due to this the longitudinal motion is not affected much, although a decrease of the phase damping at small input emittances is observed.

At 100 MeV, a transition to a 324-MHz high energy structure is proposed,[5] which has about a ± 27° phase acceptance, safely larger than the phase emittance of about 3 × 7°.

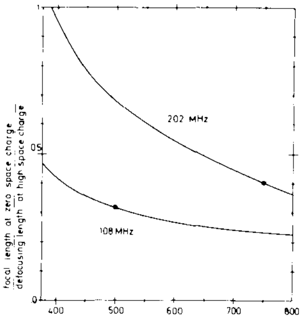

Fig. 1:

Influence of space charge effects during bunching for 0.1 A proton beam

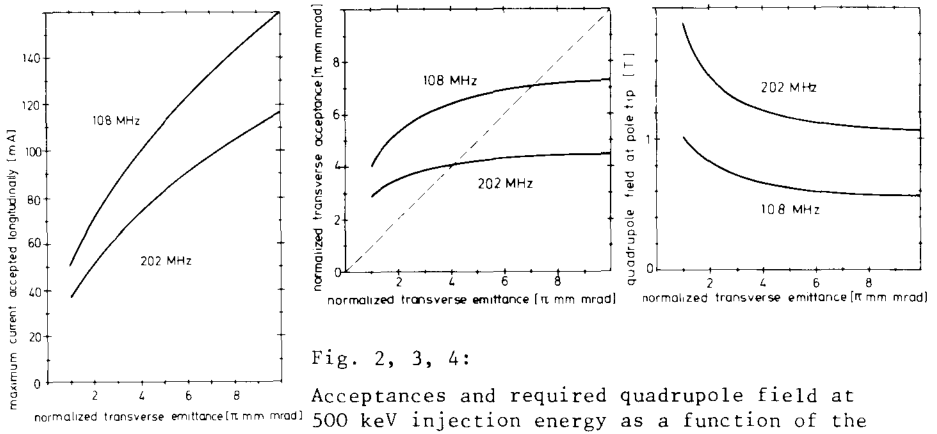

Fig. 2, 3, 4:

Acceptances and required quadrupole field at 500 keV injection energy as a function of the input emittance for $I = 0.1$ A, $\phi_s = -35^\circ$, $\Delta\phi = 35^\circ$, $E = 2$ MV/m, $ET = 1.45$ MV/m, $E_{peak} = 11.7$ MV/m, FODO focusing, $\mu(I = 0.1\ A) = 40^\circ$

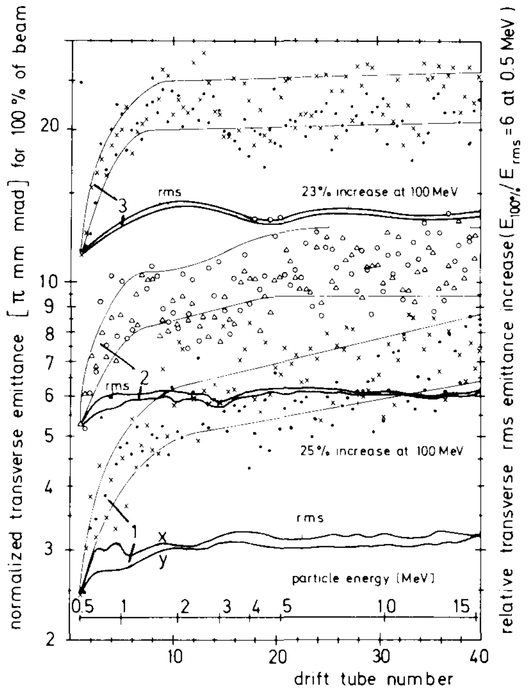

Fig. 5: Transverse emittance increase along the Alvarez for matched beams

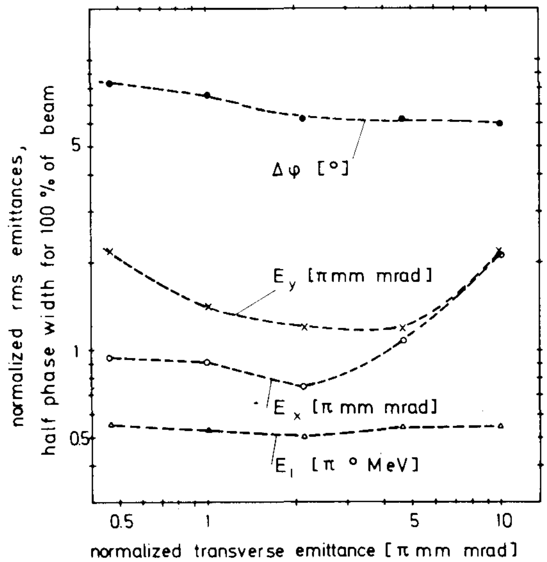

Fig. 7: Emittances and phase width at the Alvarez exit (100 MeV) as a function of the input emittance. Matching and quadrupole gradients for 4.6 π mm-mrad input emittance. Ratio of maximum to rms emittance assumed to be 5 at injection.

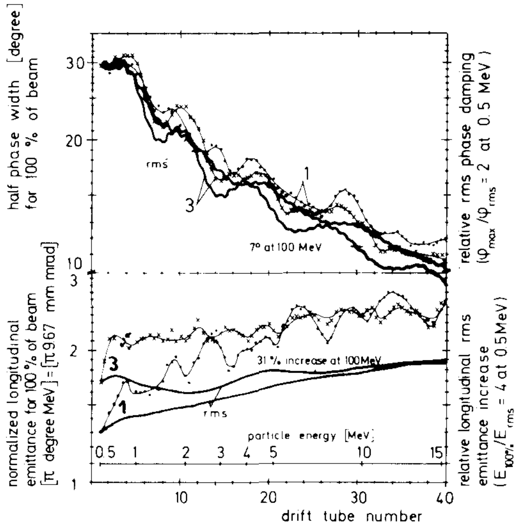

Fig. 6: Longitudinal emittance increase and phase damping along the Alvarez for matched beams

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