Proceedings of the 1979 Linear Accelerator Conference, Montauk, New York, USA

THE SIGNIFICANCE OF BEAM OPTICS CONCEPTS AS APPLIED TO THE NEW CERN LINAC

M. Weiss
CERN, Geneva, Switzerland

Summary

The beam dynamics equations are in general valid only for ideal situations, i.e. ideal machines and ideal beams. To deal with reality, certain assumptions and procedures have to complement the theory so as to form with it a logical or "reasonably" logical entity. The beam dynamics approach used for the new CERN linac is reviewed and important points are underlined. Computer calculations justified the procedure at the start of construction; comparison with measurements and machine performance should now evaluate the true "figure of merit" of the whole approach.

Design Methods

Beam optics problems can be approached in various ways, each approach being usually a combination of analytic and computer treatment. The analytic treatment gives an insight into the importance and interdependence of various machine and beam parameters, but to solve the equations, several simplifications have to be made. The computer treatment is more precise and is usually divided in two parts; one where linearized optimization programs are used to determine the machine and matched beam parameters, the other where beam simulation programs are applied to check the validity of the established settings.

The CERN beam optics treatment consisted of the following steps:

1) beam model
2) analytic considerations
3) optimization programs
4) simulation programs.

These topics will be treated in some detail; in particular the analytic considerations, which are the basis for the subsequent computer treatments.

Beam Model

The beam is represented by a hyperellipsoid in either a four-dimensional phase space (unbunched beam), or a six-dimensional one (bunched beam). The space-charge forces are computed from the projection of the beam density distribution in the two- or three-dimensional real space. It has been shown that for distributions with ellipsoidal symmetry, the evolution of the rms beam envelope depends almost exclusively on the linearized part (least square method) of the self-forces[1]. This important fact is extensively used in analytic calculations and linearized optimization programs, where the real beam is replaced by an "equivalent" one, having the same rms values of coordinates but being of uniform distribution.

Via rms coordinates, one can compute the matching parameters for beams with different density distributions; the results will be the more significant the more the density distribution approaches an ellipsoidal one ("well-behaved beams"). It should be noted that no direct indication is obtained about the evolution of the marginal beam envelope, which is only estimated from the rms envelope and the real beam density distribution.

Analytic Considerations

Analytic considerations are essential for choosing the basic parameters of the accelerator under space-charge conditions. It is convenient to analyse the betatron and synchrotron motion separately.

Transverse Beam Dynamics

The most useful equation to start with is the envelope equation, and it is sufficient for our purpose to analyse only the mean "smooth" envelope. Assuming that this envelope is essentially the same, over a period, for both the transverse directions x and y, one can write

$$\bar{x}'' + \bar{K}\bar{x} - \frac{E^2}{\bar{x}^3} - \frac{kI}{\bar{x}} = 0 \ ,$$

where $\bar{x}$, $\bar{K}$, E, I, k represent the smooth beam envelope, the mean outer focusing, the equivalent beam emittance, the beam current, and the space-charge factor, respectively. The condition for a matched beam is $\bar{x}'' = 0$ and the mean focusing per period must satisfy the expression

$$\bar{K} = \frac{E^2}{\bar{x}^4}\left(1 + k\,\frac{\bar{x}^2}{E}\,\frac{I}{E}\right) = \bar{\Omega}_\beta^2\,(1+\sigma_t) \ ; \qquad \sigma_t = k\bar{\beta}\,\frac{I}{E} \ ,$$

where σ_t is the transverse space-charge parameter, and $\bar{\Omega}_\beta$ and $\bar{\beta}$ are the smooth betatron frequency and amplitude function, respectively. If $\sigma_t \gg 1$, the mean focusing is mainly "space charge determined":

$$\bar{K} \cong k\,\frac{I}{\bar{x}^2} \ .$$

Some typical values of σ_t in the new CERN linac are:

	σ_t
LEBT	∿ 20 (unbunched beam) > 10 (bunched beam)
LINAC	3-5
HEBT	2.5-5

The reliability of computed settings usually drops with σ_t; in the LEBT, for example, it is essential that the beam is "very well behaved" in order to apply the computed settings.

To keep σ_t small (for a given I) becomes increasingly difficult for beams with a small emittance; one ends up with very narrow beams, requiring more focusing strength and possibly creating longitudinal space-charge problems.

The situation in the linac can be analysed a bit further: the constants of all linearized forces in the smooth approximation satisfy the expression

$$\bar{\Omega}_\beta^2 = \bar{\Omega}_Q^2 - \bar{\Omega}_{RF}^2 - \bar{\Omega}_{SC}^2 \ ,$$

with $\bar{\Omega}_Q^2$, $\bar{\Omega}_{RF}^2$, $\bar{\Omega}_{SC}^2$ being the mean force constants of the quadrupole focusing, rf defocusing, and space-charge defocusing. Taking into account that the FD focusing in the linac is in fact a sequence of quadrupole doublets[2], one can write for the quadrupole gradient

$$G \propto \bar{\Omega}_Q = \bar{\Omega}_\beta \left(1 + \bar{\Omega}^2_{RF}/\bar{\Omega}^2_\beta + \sigma_t\right)^{1/2} .$$

Inserting in this expression figures of a "nominal" acceleration with the new linac (I = 150 mA), one obtains roughly

$$G \cong \bar{\Omega}_\beta \ (1 + 1 + 3)^{1/2} .$$

The increase of quadrupole gradients in the linac due to space-charge is

$$\frac{G}{G_0} = \left(\frac{\bar{\Omega}^2_Q}{\bar{\Omega}^2_Q - \bar{\Omega}^2_{SC}}\right)^{1/2} = \left(\frac{5}{2}\right)^{1/2} \cong 1.6 ,$$

a figure confirmed also by more detailed computations (optimization programs). It is interesting to note that the ratio of betatron frequencies of the beam center and an average particle is

$$\frac{\bar{\Omega}_{\beta_0}}{\bar{\Omega}_\beta} \cong \left(\frac{\bar{\Omega}^2_Q - \bar{\Omega}^2_{RF}}{\bar{\Omega}^2_Q - \bar{\Omega}^2_{RF} - \bar{\Omega}^2_{SC}}\right)^{1/2} = 2 .$$

Longitudinal Beam Dynamics

The basic set of equations governing the phase and energy differences relative to the synchronous particle is

$$\frac{d}{dt}\Delta\phi = -\frac{\omega}{mc^2\beta_r^2\gamma^3}\Delta W$$

$$\frac{d}{dt}\Delta W = e\bar{E}T\beta_r c\left[\cos(\phi_s + \Delta\phi) - \cos\phi_s - \sigma_\ell|\sin\phi_s|\Delta\phi\right] .$$

The meaning of the symbols is the usual one, σ_ℓ being the longitudinal space charge parameter for the equivalent beam:

$$\sigma_\ell \cong \frac{I}{2\bar{r}\varepsilon_0\beta_r c\bar{E}T|\sin\phi_s|\overline{\Delta\phi}^2} ,$$

with $\overline{\Delta\phi}$: smooth phase amplitude, and $\bar{r}$: smooth beam radius. It is assumed that ΔW is small compared to the energy of the synchronous particle.

To solve the set of differential equations some assumptions are necessary, depending on the question one wishes to answer.

Linearized equation (analysis of the evolution of small synchrotron oscillations with time): for $\Delta\phi \ll 1$, one can linearize with respect to $\Delta\phi$ and get the set of equations in the form:

$$\frac{d}{dt}\Delta\phi = -b^2(t)\,\Delta W$$

$$\frac{d}{dt}\Delta W = a^2(t)\,\Delta\phi ,$$

a(t) and b(t) being slowly varying functions of t. At a given moment t_0, the motion (smooth) in the phase plane is given approximately by an ellipse with the axis ratio:

$$\frac{a_0}{b_0} = \left(-\frac{\Delta W}{\Delta\phi}\frac{d\,\Delta W}{d\,\Delta\phi}\right)^{1/2} = \frac{\overline{\Delta W}}{\overline{\Delta\phi}} .$$

This ratio varies during the acceleration as

$$\frac{\overline{\Delta W}}{\overline{\Delta\phi}} \cong (\beta_r\gamma)^{3/2}\,(1 - \sigma_\ell)^{1/2} .$$

The set of linearized first-order equations can be transformed into a second-order equation:

$$\frac{d}{dt}\left[\beta_r^2\gamma^3\frac{d}{dt}\Delta\phi\right] + \frac{2\pi e\bar{E}T\beta_r|\sin\phi_s|}{m\lambda}(1 - \sigma_\ell)\,\Delta\phi = 0 .$$

Assuming σ_ℓ = const and multiplying by $\beta_r^2\gamma^3$ and substituting,

$$\beta_r^2\gamma^3\frac{d}{dt} = \frac{d}{du} ,$$

this equation can be solved for the independent variable u by the BKW method; returning to the variable t, the solution is written as:

$$\Delta\phi(t) = C\left[\beta_r^2\gamma^3\,\bar{K}(t)\right]^{-1/4}\sin\left[\int_0^t\left(\frac{\bar{K}(t')}{\beta_r^2\gamma^3}\right)^{1/2}dt' + \phi_0\right] ,$$

where C is an integration constant and

$$\bar{K}(t) = \frac{2\pi e\bar{E}T\beta_r|\sin\phi_s|}{m\lambda}(1 - \sigma_\ell) .$$

One sees from the solution that the smooth synchrotron frequency is given by

$$\bar{\Omega}_s = \left[\frac{2\pi e\bar{E}T|\sin\phi_s|}{m\beta_r\lambda\gamma^3}(1 - \sigma_\ell)\right]^{1/2} = \bar{\Omega}_{s_0}(1 - \sigma_\ell)^{1/2} .$$

$\bar{\Omega}_{s_0}$ is the frequency in the absence of the space-charge. During acceleration $\bar{\Omega}_s$ varies as

$$\bar{\Omega}_s \propto \beta_r^{-1/2}\,\gamma^{-3/2}\,(1 - \sigma_\ell)^{1/2} .$$

The amplitude of small phase oscillations is damped in the course of acceleration; for $\bar{E}T|\sin\phi_s|$ = const one has

$$\overline{\Delta\phi} \propto (\beta_r\gamma)^{-3/4}\,(1 - \sigma_\ell)^{-1/4} .$$

The parameter σ_ℓ is, in fact, not constant during acceleration, but varies (see its formula):

$$\sigma_\ell \propto \beta_r^{-1}\,\mu^{+1/2} \quad \text{for } \overline{\Delta\phi} = \text{const}$$

$$\sigma_\ell \propto \beta_r^{1/2}\,\mu^{+1/2} \quad \text{for } \overline{\Delta\phi} \propto (\beta_r\gamma)^{-3/4} .$$

In most cases, σ_ℓ increases along the linac.

The described analysis, valid for $\Delta\phi \ll 1$, is usually also applied in optimization programs for bigger $\Delta\phi$. Several precautions are, however, necessary when a parabolic potential function (linear motion) replaces the true one.

Non-linearized adiabatic equation (analysis of synchrotron oscillations with large amplitudes): to solve the non-linear equations, one must assume adiabaticity ($\beta_r\gamma$ = const); the second-order adiabatic equation is

$$\frac{d^2}{dt^2}\Delta\phi + \bar{\Omega}^2_{s_0}\left[\frac{\cos(\phi_s + \Delta\phi) - \cos\phi_s}{|\sin\phi_s|} - \sigma_\ell\Delta\phi\right] = 0 ,$$

and due to the assumption $(d/dt)\bar{\Omega}^2_{s_0} = 0$, one can obtain the energy integral as:

$$\frac{d}{dt}\left[\frac{1}{2}\dot{\Delta\phi}^2 + \bar{\Omega}^2_{s_0}\,V(\Delta\phi)\right] = 0 ,$$

with the potential function

$$V(\Delta\phi) = \frac{\sin(\phi_s + \Delta\phi) - \Delta\phi\cos\phi_s - \sin\phi_s}{|\sin\phi_s|} - \sigma_\ell\frac{\Delta\phi^2}{2} .$$

From $V'(\Delta\phi) = [dV(\Delta\phi)/d\Delta\phi] = 0$, one gets one stability limit (unstable point) as

$$\Delta\phi_1 \cong 2|\phi_s|\ (1 - \sigma_\ell) \quad \text{(see Fig. 1)} .$$

The other stability limit can be obtained from $V(\Delta\phi) - V(\Delta\phi_1) = 0$ (in the easiest way by expanding this expression in a Taylor series):

$$\Delta\phi_2 \cong -|\phi_s|\ (1 - \sigma_\ell) .$$

The stability limits are not symmetric around the synchronous phase. To place more beam in the stable region, one injects off the synchronous phase, $\phi_s + \phi_0$, see Fig. 1. There are several points of view according to which ϕ_0 can be chosen. If one decides to inject in the middle of the stable region, then the stability limits are a function of σ_ℓ and ϕ_0:

$$\Delta\phi_1 = f_1\ (\sigma_\ell, \phi_0) , \quad \Delta\phi_2 = f_2\ (\sigma_\ell, \phi_0) ,$$

satisfying

$$\phi_0 = \frac{f_1 + f_2}{2} .$$

The maximum allowable phase amplitude is

$$\overline{\Delta\phi} = \Delta\phi_1 - \phi_0 .$$

The potential function $V(\Delta\phi)$ now has the form:

$$V(\Delta\phi) = \frac{\sin(\phi_s + \Delta\phi) - \Delta\phi\cos\phi_s}{|\sin\phi_s|} - \frac{\sigma_\ell}{2}(\Delta\phi - \phi_0) + C ,$$

and the integration constant C can be chosen so as to make the minimum of the potential function equal zero. The condition $V'(\Delta\phi) = 0$ gives $\Delta\phi_1$ [maximum, $V''(\Delta\phi) < 0$] and $\Delta\phi_0$ [minimum, $V''(\Delta\phi) > 0$], see Fig. 1. Proceeding as before, one computes $\Delta\phi_2$ and ϕ_0.

The space-charge parameter σ_ℓ has not yet been fixed; it can be chosen in such a way as to maximize the trapped beam:

$$I_{max} \cong 2\bar{r}\varepsilon_0\beta_r c\bar{E}T\cos\phi_s\ |\mathrm{tg}\,\phi_s|\ \sigma_{\ell\,opt}\ \overline{\Delta\phi}^2 .$$

To obtain $\sigma_{\ell\,opt}$ it suffices to differentiate

$$\frac{d}{d\sigma_\ell}(\sigma_\ell\overline{\Delta\phi}^2) = f(\sigma_\ell) = 0 \rightarrow \sigma_{\ell\,opt} = 0.425 .$$

Taking this $\sigma_{\ell\,opt}$ and the typical settings of the new CERN linac, one would get

$$I_{max} \cong 75\ \text{mA} ,$$

which is the optimum filling of the stationary bucket. In fact, at injection into the CERN linac one has

$$\sigma_\ell \cong 0.9 ,$$

and this means that the stable region is drastically reduced and that a good part of the beam lies initially outside of it. The beam starts to grow longitudinally, but the bucket length (in mm, not degrees) increases $\propto \beta_r$, and quicker than the beam; after ∿(10-15) cells, the σ_ℓ has dropped sufficiently for the beam to be now fully in the new bucket. Figures 2 and 3 show the initial potential $V(\Delta\phi)$ and bucket of the CERN linac for two values of σ_ℓ. Figure 4 (simulation program result) shows the phase damping of beams with I = 0 and I = 150 mA, respectively. In both cases, no particles are lost. The matching and machine parameters correspond to computed settings (optimization program results), and no attempt was made to annul the residual phase oscillations. The initially unstable beam gives rise to a large increase in the longitudinal emittance (∿ 2 computed, > 5 measured), but this does not adversely affect the accelerator (booster) downstream of the linac.

Linear Optimization Programs

All parts of the linear accelerator complex have been designed by using linearized optimization programs. Programs treating the LEBT and linac have been entirely developed and written at CERN[3] and subsequently improved. They apply matrix formalism, linearized Lapostolle-Schnizer gap equations[4], and have the following special features:

1. Space-charge forces are computed for uniformly filled, infinitely long cylinders (unbunched beam) or ellipsoids (bunched beam). In the bunching region, both models are used to account for the action of the subsequently trapped as well as non-trapped particles[3].

2. The longitudinal beam emittance is formed by the non-linear energy modulation of the beam in the bunching system. This is the only "non-linearity" contained in the optimization programs; it is also responsible for the transverse beam emittance growth in rf gaps.

3. The linac (quasi-periodic structure) is treated as a periodic structure, when computing matching parameters; space-charge forces are included.

4. The gap forces are linearized around an "average" beam radius r_{eff} (effective beam radius), which is related to the envelope of the equivalent beam as

$$\frac{r_{eff}}{\hat{r}} = \sqrt{\frac{2}{5}} .$$

The divergence theorem holds.

5. The synchronous particle is abandoned as the representative particle and replaced with particles lying on r_{eff} in the transverse median plane of the beam bunch. These particles have to get the nominal acceleration.

6. Emittance increase (non-linear effect) can be artificially introduced in all phase planes.

Linear programs having the above features also proved satisfactory for setting the running machine parameters; in particular the linac. Some corrections were necessary in the LEBT (quadrupole focusing, not bunching), where the high σ_t value, intensity oscillations during the beam pulse ("grass"), and the presence of non-protons, made the situation more difficult to handle.

Beam Simulation and Experiments

In the linac, important questions concerning the design[5] and operation[6] were settled by multiparticle programs. In particular, conditions minimizing the transverse emittance growth were analysed and a strong preference for rather narrow beams found.

However, some experiments have recently been performed confirming the importance of initial σ_ℓ values: a beam of 65 mA has been accelerated and, contrary to previous runs, a smaller μ value ($\sim 30°$) has been chosen, so as to have the beam longitudinally stable at injection. As expected, the longitudinal output emittance decreased (by a factor of 0.57), but surprisingly the transverse one was also improved (factors of 0.7 and 0.9 for 90% and 63% of beam, respectively). Perhaps this allows one to conclude that in each case an optimum combination of σ_ℓ and μ can be found.

Conclusion

The following may be said concerning the CERN beam optics approach:

1. It is complete in the sense that it contains procedures for determining all the parameters of the design;

2. All design options were subsequently proved correct;

3. Computed settings of accelerator parameters (focusing) are in general satisfactory, but corrections are necessary in areas of high σ_t values;

4. Phenomena such as emittance growth, in particular in the longitudinal plane, were underestimated by beam simulation programs, which, however, did not include steering errors. The qualitative dependence of these phenomena on machine and beam parameters was rightly foreseen.

Finally it should be noted that the CERN approach was developed for "classical" proton linacs, where variations in beam losses of the order of a percent are neglected.

References

1) F. Sacherer," RMS envelope equation with space-charge", CERN/SI/Int. 70-12

2) B. Bru and M. Weiss, "Tolerances for quadrupole focusing in the linac", CERN/MPS/LINP/Note 73-7.

3) B. Bru and M. Weiss, "Computational methods and computer programs for linearized analysis", CERN/MPS/LIN 72-4.

4) M. Promé, "Effets de la charge d'espace dans les accélérateurs linéaires à protons", thesis, Orsay, No. 761/1971.

5) D. Warner, "Accelerating structure of the CERN new 50-MeV linac", Proc. 1976 Proton Acc. Conf., Chalk River.

6) D. Warner and M. Weiss, "Beam optics in the CERN 50-MeV linac", same proceedings.

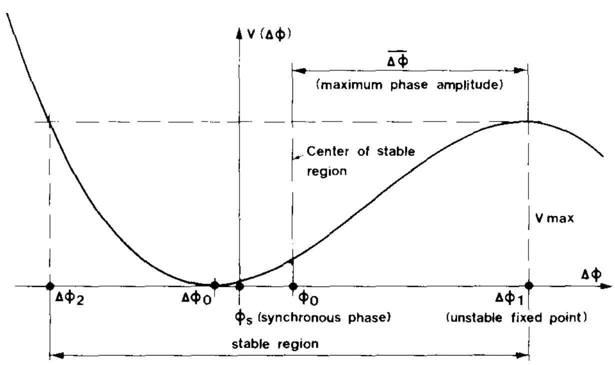

Fig. 1 Potential function for synchrotron motion with space charge.

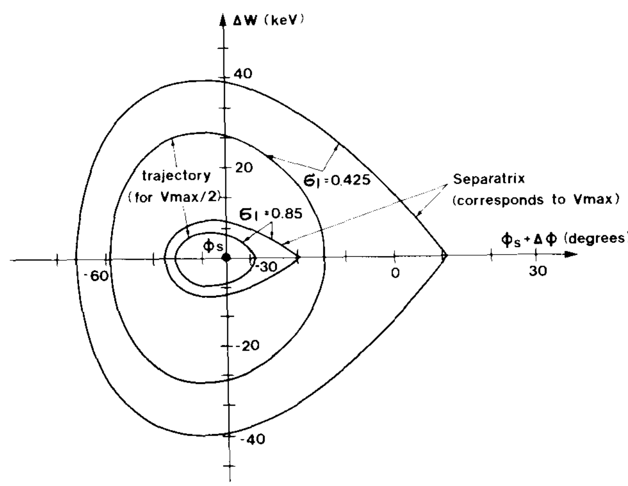

Fig. 3 Separatrix and trajectory in the $\Delta\phi$, ΔW plane with σ_ℓ as parameter.

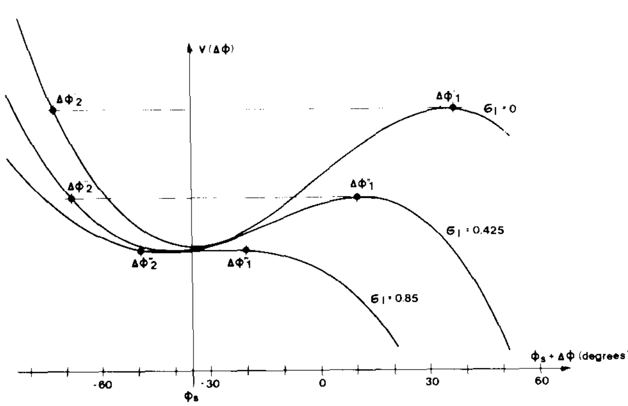

Fig. 2 Potential function with σ_ℓ as parameter.

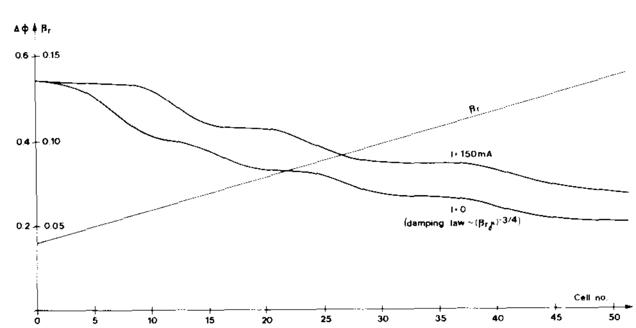

Fig. 4 Phase damping in tank 1 of the new linac.