ON EMITTANCE GROWTH IN LINEAR ACCELERATORS*

R. A. Jameson and R. S. Mills
Accelerator Technology Division
Los Alamos Scientific Laboratory
Los Alamos, NM 87545

Summary

Factors affecting emittance growth in linear accelerator applications requiring high beam quality and low in-machine beam losses are explored. A generalized method is developed for matching the average properties of an arbitrary particle distribution to an accelerator structure in the presence of space-charge and other perturbations. The effects of the particle distribution and the accelerator parameters on the preservation of input emittance and other measures of beam behavior are investigated, to study optimum performance under carefully matched conditions, and to show the degradation of performance in the presence of mismatch, steering and other errors. The concept of detailed matching invoking desirable correlations is discussed briefly.

Introduction

Growth in the effective emittance of linear accelerator beams has been observed in all operating machines, and has been the subject of much study. Gluckstern[1] pioneered in the analytical treatment of K-V distributions. R. Chasman[2] did numerical simulation experiments that demonstrated lower limits to output emittance as input emittance was reduced, which identified the importance of longitudinal-transverse coupling. P. Lapostolle[3] and coworkers at CERN did numerical and analytical work with cylindrically symmetric, continuous beams that demonstrated the presence of violent growth modes under certain parameter conditions, leading to severe filamentation of phase space.

Recent work with continuous beams has clarified a number of points. Laslett, Smith and Hofmann[4,5] have catalogued the characteristics of parametric oscillations in K-V distributions and have discussed unstable modes in terms of hydrodynamic models. Numerical calculations[6,7] agree with the theory and have addressed some questions concerning numerical effects in the codes.

The general scalability of the problem over the parameter range is beginning to be better understood;[6,8] Reiser's important work in this area demonstrates that the scaling laws do not have a single simple form, but depend on the constraints imposed on the problem by the designer, and on the zero-intensity phase advances. In the examples below, all detailed variables were deliberately submerged and are described only in terms of phased advances. These are widely scalable, and are a very useful simplification.

Adding bunching and acceleration complicates the problem substantially. Lysenko[9] extended the equilibrium distribution work to 2-D, and is presently working on 3-D. Crandall[10] elaborated on the effect of rf gaps on emittance growth. Another example of the impact of the constraints on the design problem was shown in the somewhat surprising result[9,11,12] that, for given current, input emittance is best preserved by raising the frequency of the machine. Development of the radio-frequency quadrupole structure[13] has required that attention be paid to the optimum rate of change from a continuous beam to a bunched, accelerated beam, to minimize transverse emittance growth.

Comparison of code models to actual machine performance has also progressed, with detailed analysis of measurement techniques,[14] and several examples of exacting modeling studies[15,16,17] that resulted in agreement with experimental results to a few per cent or better.

All of this work lends confidence to the further use of numerical simulation models for more detailed exploration of the causes and effects of emittance growth. The PARMILA code was used in the work below; it is a full 6-D simulation including nonlinear effects. The space-charge subroutine uses a ring model on an r-z area-weighted mesh. Five hundred macroparticles were used for most runs; salient points were checked with 5000. The random number generator was restarted for each run, except for specific checks of the effect of randomness.

Generation of A Matched Linac

It was desired to generate linacs that would be well matched under various conditions, and with arbitrary particle distributions, eventually including measured ones.[18] For the initial studies, the phase advances along the linac were specified. First, an adequate measure was needed, over each structure period, of the phase advance, σ, and the ellipse parameters, α and β, that characterize the nearly periodic system. A least-squares solution in each plane using all particles gives values that work well for matching. The space-charge parameter, $\mu = 1 - (\sigma/\sigma_0)^2$, also is available. Using the desired particle distribution, a linac can be generated with a given prescription for the phase advances, using an iterative, least-squares procedure. The quadrupole strengths are adjusted to get the proper transverse phase advance. A constant longitudinal phase advance, for example, requires the quantity $E_o T \sin \phi_s/\beta$ to

*Work performed under the auspices of the U. S. Department of Energy.

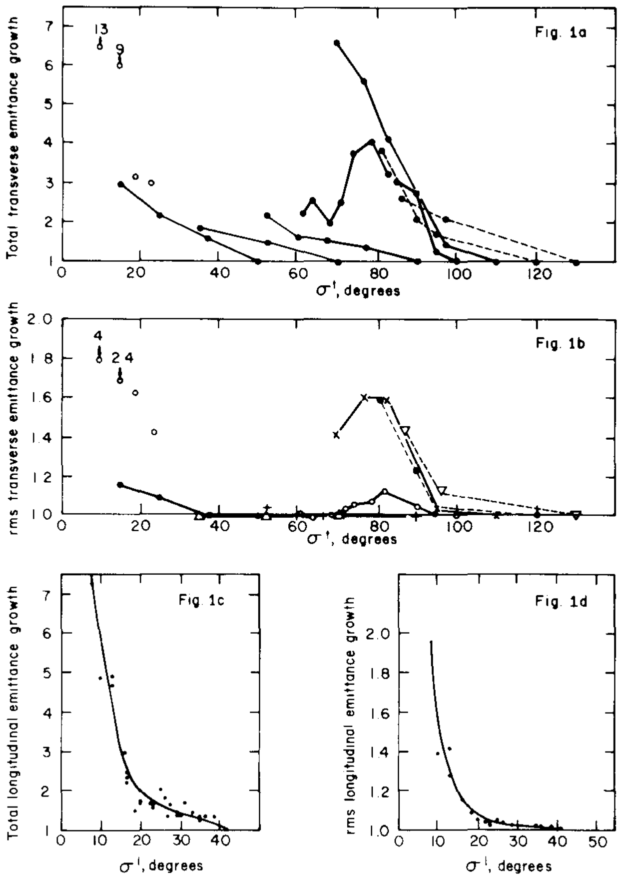

Fig. 1. Emittance growth after 20 cells, as a function of tune shift from various initial zero-intensity phase advance per transverse focusing period.

remain constant. The procedure converges more slowly as the tune is depressed, but is quite effective. Starting values are obtained from the envelope equations.

Emittance Growth with Matched Beams

The parameter space was explored by selecting a range of zero intensity, transverse-phase advances and depressing the tune of each by increasing the current in steps. As stated above, the detailed parameters are submerged, they are in fact quite ordinary and well within the scaling range of the proposed Fusion Materials Irradiation Test (FMIT) linac to the Clinton P. Anderson Meson Physics Facility (LAMPF), and CERN class of machine. Figure 1 shows the growth in the rms emittances and in the effective emittance for 100% of the beam at the end of 20 cells,* for a set of runs where the initial transverse emittance and longitudinal beam size are the same. An input distribution approximately uniform in real space

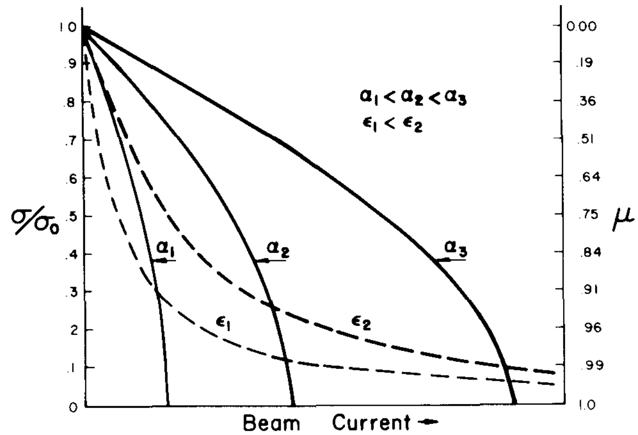

Fig. 2. Behavior of matched solutions from envelope equation, for fixed emittance, ε, or fixed average beam radius, a. For example, for a given radius and current, a certain emittance is required for a matched beam. A fixed accelerator and no emittance growth are assumed.

was used. The quads were set to give constant transverse phase advance, $\sigma_0{}^t$, at zero current; this resulted in σ^t remaining approximately constant also. The synchronous phase was held constant. The accelerating gradient started from the same value, but was raised linearly with β, along with adjustment of the initial energy spread to keep the longitudinal phase advance nearly constant for each case.

There appears to be an underlying growth pattern, plus an anomolous feature[3,11] in the transverse growth, which will be discussed later. Much of this "underlying" growth is attributable to the immediate shearing effect of the rf longitudinal-transverse coupling[12], and to the steadily accumulating effect of the series of rf gaps.[10] For each $\sigma_0{}^t$, the current was increased until it became quite difficult to find a solution because of the loss of adequate longitudinal focusing in the initial few cells. For tune depressions, $\sigma^\ell/\sigma_0{}^\ell < 0.4$, the longitudinal growth increased rapidly. For the $\sigma_0{}^t = 50°$ case, the electric field was raised to keep some longitudinal focusing, and the current was increased to depress the transverse tune further. The transverse growth also increased rapidly for $\sigma^t/\sigma_0{}^t < 0.4$ (open circles, Fig. 1). The envelope equations indicate that for a fixed accelerator and fixed emittance, a matched solution should be attainable at any current by choosing the proper beam size, as indicated in Fig. 2. The emittance and space

*Total effective emittance is found by fitting ellipses with the rms emittance ellipse parameters through each particle and taking the largest.

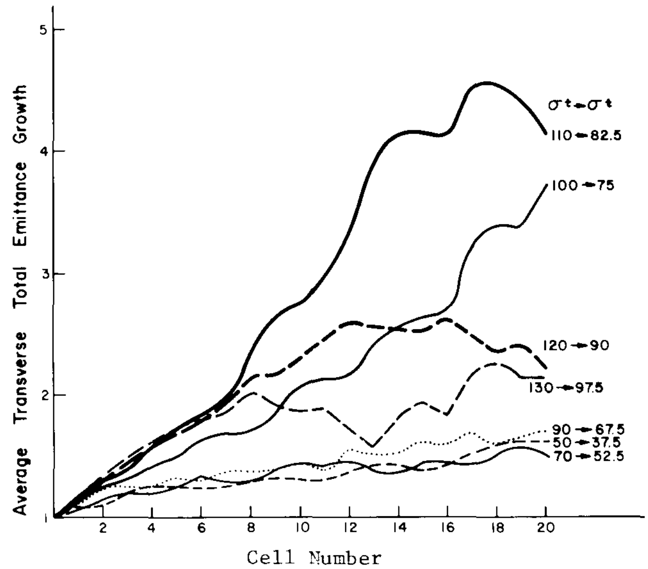

Fig. 3. Growth in average transverse total emittance vs cell number for σ^t/σ_0^t = 0.75.

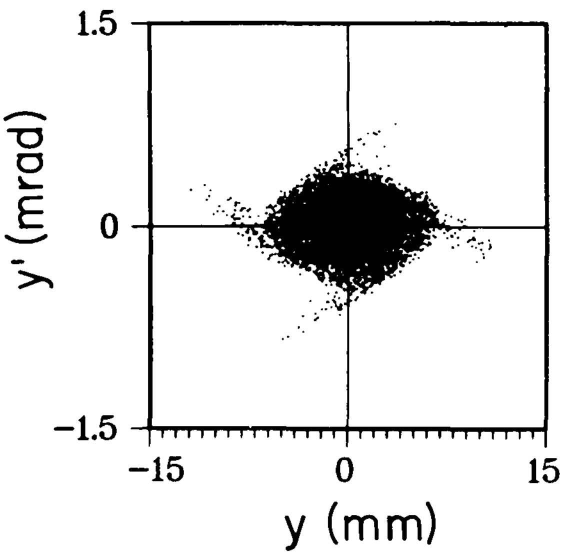

Fig. 4. The y-y' phase-space at Cell 20 for the σ^t/σ_0^t = 78°/100° case. 5000 particles were computed and plotted.

charge contribute quadratically to the required size, and at σ/σ_0 = 0.4 (0.25), the emittance term has 0.8 (0.5) the weight of the space charge term. In those cases where the tune is depressed in the first few cells to less than 0.4, there is an immediate growth that tends to push the tune back up until a balance is achieved. The beam has not yet been followed

	1	x	x^2	x^3	x^4
1			1.4	.9	2.2
y			3.6	8.1	7.9
y^2	1.3	4.1	1.3	.8	1.7
y^3	16.4	2.3	140.9	2.6	6.2
y^4	2.6	14.9	1.9	7.0	1.8

	1	x	x^2	x^3	x^4
1			1.4	.9	2.2
x'			.9	2.2	8.4
x'^2	.7	.8	1.5	2.2	3.4
x'^3	2.7	1.6	3.4	1.5	20.7
x'^4	.6	1.9	1.7	7.6	5.6

Fig. 5. Absolute ratio of average cross-correlation products at Cell 20 to those at Cell 2 for σ^t/σ_0^t = 78°/100° case.

enough to determine the asymptotic behavior, although it would be unrealistic to sustain the ramped accelerating gradient indefinitely.

This series of cases was repeated with the accelerating field held constant at the value used above in the initial cell. The results are very similar to Fig. 1, except there is less longitudinal growth (see Fig. 8). This observation, plus the fact that the longitudinal growth in Fig. 1 is a smooth curve, indicates that the anomalous transverse growth for the tunes with σ_0^t in the range 100° - 120° is essentially a transverse phenomenon.

This drastic growth behavior appears similar to an unstable mode reported in Refs. 4-7 for the K-V distribution. Figure 3 shows the growth in average transverse total emittance for the σ^t/σ_0^t = 0.75 tune depression for each σ_0^t case. Some evidence of beat frequencies is apparent, and the growth rate is extremely fast. Figure 4 shows a y-y' scatterplot for σ^t/σ_0^t = 78°/100° at Cell 20, with the four arms characteristic of this instability mode. Figure 5 shows the ratio of average cross-correlation products at Cell 20 to those at Cell 2, out to fourth order in each axis. If the size of terms is compared as the tune is depressed, a pronounced peak is found in the vicinity of the instability, in y^2x, y^3, y^4x and a number of the x'-x terms, particularly $x'x^4$, x'^2x^4, x'^3x^4, x'^4x^3 and x'^4x^4. Other terms show a steady increase (or both ramp and peak), especially y^2x^2, y^2x^4, y^3x^4, y^4x^2, y^4x^4, x'^2x^4, x'^4x^3, and x'^4x^4. This mode may be, in fact, the Rayleigh-Taylor instability[19] arising from the alternating-gradient focusing system; the x-y nature, and the fast growth would fit this hypothesis. Theoretical development of this approach is in progress.

Figure 6 shows a histogram of the phase advance of particles in the bunch, in the frame of the least-squares parameters derived for the Cell 17-18 period. The second peak indicates that a group of particles is persistently rotating at a separate rate. This group appears to be a result of the "underlying" mechanisms and not of the instability mode. It consists of particles originating at transverse radii out to

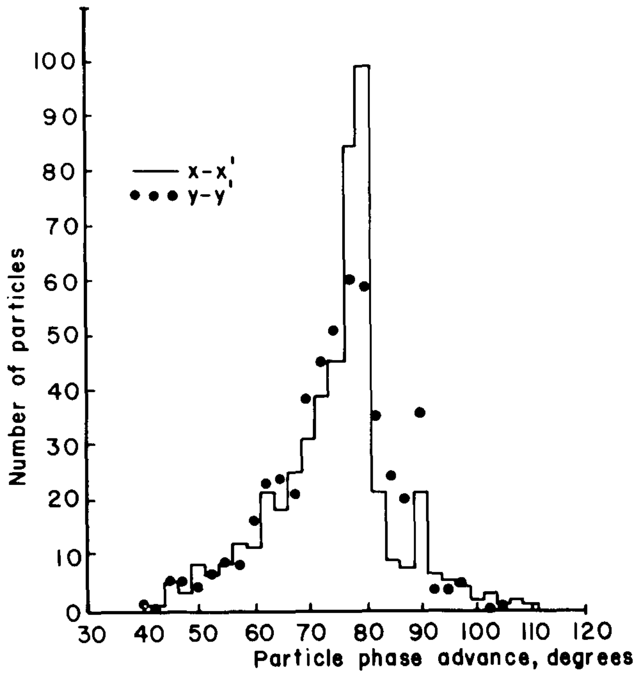

Fig.6. Phase advance histogram for $\sigma^t/\sigma_0{}^t = 78^o/100^o$ case, showing double peak. (Period from Cell 17-18).

0.4 r_{max}, is uncorrelated with particle phase at injection, and is within the 90% core of the beam in the transverse emittance projections.

Practical parameter choices for applications commonly result in $\sigma^\ell < \sigma^t$. A preliminary search was made for resonances of the $2\sigma^t = n\sigma^\ell$ type. Keeping $\sigma^t = 50^o$, E_0/β was adjusted for constant σ^ℓ with n from 2 to 8. No difference in emittance growth was seen out to 60 cells, which is beyond the point to which the E_o ramp could practically be sustained. Other possibilities should be studied.

In another slice of the parameter space, starting with the $\sigma^t/\sigma_0{}^t = 51.5^o/70^o$ case, and, keeping the current constant, reduced the input transverse emittance by factors of 2 and 4. The transverse emittance growth increased, and the longitudinal growth decreased for this example. Such transfers among the projections are commonly observed, including transfers between x-x' and y-y' if the initial emittances differ or during oscillations. This indicates that ratios of emittances are also important parameters, and suggests multi-dimensional matching with equal emittance areas, especially if the parameters change along the machine. It has been found, however, that full 6-D matching is not necessarily advantageous in all applications--another example of the influence of designer constraints on the scalings. Other sets of parameters typical

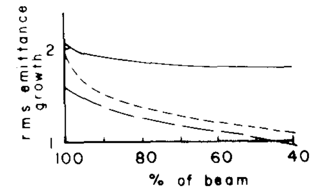

Fig. 7 Typical variation in rms transverse emittance growth with input distribution.
--- Uniform in 3-D real space
--- Uniform in 4-D transverse space, separate 2-D longitudinal
--- Gaussian in real space, truncated at 3σ

of various applications will be explored, in an effort to sort out some of the basic calculations.

Influence of Particle Distribution

The $\sigma_0{}^t = 100^o$ cases were rerun for input distributions uniform in 6-D, and Gaussian in 6-D (truncated at 3σ), keeping the rms emittances constant. The quadrupole strengths were those used to achieve a constant phase advance for the original distribution, approximately uniform in real space. The 6-D distributions grew more rapidly in the first two-to-three cells. From Cells 3-20, the growth in total emittance was very similar, but the rms growth for the 6-D cases was about double that of the 3-D case. The 6-D cases became somewhat mismatched as the beam progressed through the cells. The quads could be reset using the above procedure, for each particular distribution; it is expected that this would smooth but not necessarily reduce the growth--it may in fact increase (see below). Similar general influences of the distribution have also been observed for other choices of parameters. Figure 7 shows a typical redistribution of emittance.[17] It is concluded that the shape of the distribution does influence emittance growth, with greater effect as the beam brightness is increased, and with greater growth as the central density is increased. The unstable mode evidenced in Fig. 1 is not the result of a particular particle distribution.

Influence of Matching

Mismatched beams will be smeared by the action of nonlinear space-charge forces and eventually will assume an emittance congruent with the machine acceptance. Figure 8 demonstrates how emittance growth is affected by mismatching the input beam size up to a factor of $\sqrt{2}$ at injection, for the range of linac parameters under discussion. At a given $\sigma_0{}^t$, the sensitivity to matching becomes more pronounced as the tune is depressed.

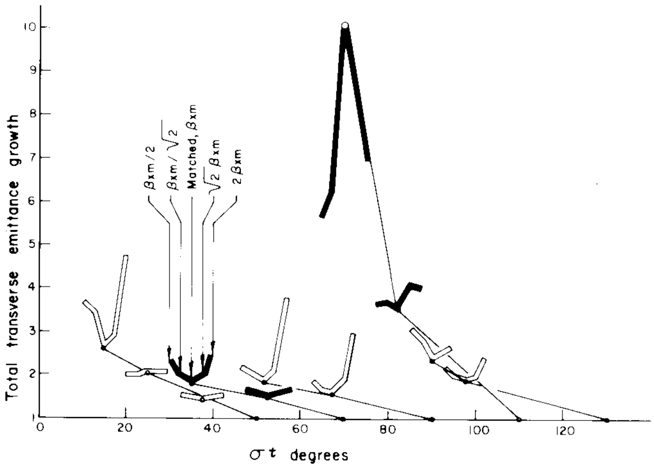

Fig. 8. Sensitivity of transverse emittance growth to input matching. Linac has constant accelerating gradient E_0. At each tune, the smaller dimension of the input beam is varied by changing the input matching-ellipse parameter β by $\pm\sqrt{2}$ and ± 2. The y and z inputs are matched. Growth is shown after 20 cells.

As $\sigma_0{}^t$ increases, the sensitivity for a given tune depression increases, an effect of the alternating gradient. The smaller absolute size of the beam (in one dimension) also becomes more important in terms of the required measurement resolution. For K-V beams, a "mismatching" instability mode has been identified[4] for $\sigma_0{}^t > 90^o$; its analog for these distributions may be a factor here.

In the vicinity of the unstable mode, the behavior becomes somewhat unpredictable. For the parameters in Fig. 8, the betatron oscillations generally subjected the beam to a lower average μ_t (higher σ^t) over the 20 cells, sometimes resulting in less growth. The $\sigma^t/\sigma_0{}^t$ = $70^o/110^o$ case is particularly dramatic in this respect. The changes in transverse emittance growth from mismatching are generally rather uniform with respect to the shape of the distribution function, as shown in Fig. 9, or sometimes show more growth for higher percentages.[17]

Influence of Off-Axis Trajectories

For a single gap without space-charge, Crandall[10] showed that the increase in total emittance is proportional to $(a^2 + d^2)$ if $|d| \leqslant a$, and $2ad$ if $|d| \geqslant a$, where a is the half-width of the beam and d is the displacement of the beam center from the axis. The increase in rms emittance is proportional to $(1 + d^2/\overline{a^2})$, where $\overline{a^2}$ denotes the rms half-width of the beam. It is seen that the rms emittance grows relatively faster than the total emittance. The growth over n gaps will depend on what happens to the relative sizes of a and d. Figure 10 shows the emittance growths for (x-offset/average input-beam radius) = 1.0 for the cases of Fig. 1. Again the sensitivity increases for larger tune depressions and for higher $\sigma_0{}^t$, with a diminishing of the unstable mode's effect. The longitudinal emittance growth also is increased by the transverse oscillation. Figure 11 shows the typical redistribution that occurs in the transverse-phase space. This feature,

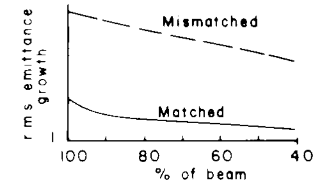

Fig. 9. Typical redistribution of transverse emittance growth for mismatched beams.

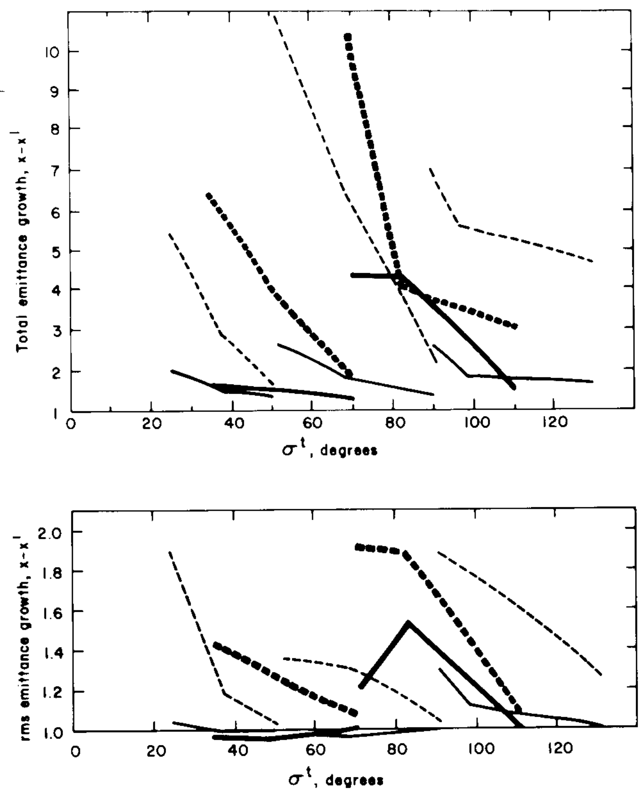

Fig. 10. Sensitivity of transverse emittance growth, after 20 cells for 100% of beam, to horizontal offsets equal to the average input beam radius. Solid -no offset; Dashed - offset.

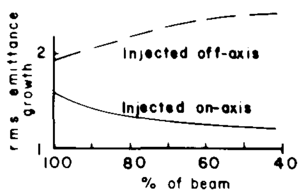

Fig. 11. Typical[17] redistribution of transverse emittance growth for mis-steered beams.

and the contrasting signature of the mismatched beam, Fig. 9, can be valuable aids in machine tuning for detecting the presence of a centroid oscillation or mismatch.

Detailed Matching

It has been hypothesized[9,12] that detailed matching procedures, which prepare the input distribution in ways more specific than just the average properties, might result in reduced emittance growth. This idea finds a general form in the theory of equilibrium distributions,[1,9] and also can be expressed in terms of introducing certain desirable correlations in the particle distribution.[20] A simple example concerns the smearing of the transverse emittance caused by the finite phase spread of the bunch, illustrated in Ref. 12. It would seem reasonable that a first-order improvement, which might be practical using an rf focusing system, would result from adjusting the transverse focusing depending on the phase of the particle. Starting with a Gaussian 6-D uniform distribution, the particles were sorted into five bins according to their initial phase. The average phase advance and matched ellipse parameters for the first two cells of the $\sigma_0{}^t = 70^o$ case were computed for each bin. A new input distribution was then prepared in which each particle was matched to the transverse parameters appropriate to the bin containing its phase. Over the 20 cells a 5-7% reduction in emittance growth resulted for both the transverse and longitudinal planes. Other cases, or other recipes, have not yet been explored. More general systems, which "adiabatically" introduce the desired correlations, are being studied. In particular, the development of the RFQ low-beta structure to accomplish capture, bunching, and preacceleration is a complete embodiment of this concept. Preparations are being made to do combined RFQ/drift-tube-linac studies in this context.

Conclusion

Some new tools useful for generating and observing matched linacs in simulation studies have been forged, and used to study some aspects of emittance growth. Some general guidance on acceptable tune ranges and the effects of some types of errors is apparent. However, the parameter space is large, and further study is needed. For example, there is interest in how far the particular parameters, such as voltages, fields, frequency, and particle type, can be condensed into relatively general parameters such as phase advance or tune depression. The concept of detailed matching appears worthy of further consideration. A better theoretical foundation would be most helpful.

Acknowledgements

We would like to thank K. Crandall, W. Lysenko, and R. Gluckstern for many helpful discussions, and C. Watson for her diligent and enthusiastic help in making the computer runs.

References

1 R. Gluckstern, R. Chasman, K. Crandall, Proc. 1970 Linac Conference, Batavia, pp. 823-849.

2 R. Chasman, Proc. 1968 Proton Linac Conf., BNL, p. 372, IEEE Trans. Nucl. Sci., NS-17, 202 (1969).

3 P. Lapostolle, IEEE Trans. Nucl. Sci., NS-18, 1101 (1971), various CERN reports, summarized in CERN-ISR/78-13.

4 L.J. Laslett, L. Smith, IEEE Trans. Nucl. Sci., NS-25, 3080 (1979).

5 I. Hofmann, Proc., op. cit. p. 3083.

6 S. Penner, A. Galejs, Proc., op. cit. p. 3086.

7 I. Haber, Proc., op. cit. p. 3090.

8 M. Reiser, Particle Accelerators, 1978, Vol. 8, pp. 167-182.

9 W.P. Lysenko, LASL Report LA-7010-MS, Oct. 1977, and Proc. 1979 PAC op. cit. p. 3508.

10 K.R. Crandall, "Transverse Emittance Increase in an rF Gap", LASL Priv. Comm. MP-9, Jan. 12, 1978, and "Transverse Emittance Growth Due to a Sequence of Rf Gaps", with R.S. Mills, LASL Priv. Comm. MP-9, Feb. 28, 1978.

11 R.A. Jameson, R.S. Mills, "Factors Affecting High-Current, Bright Linac Beams", April 8, 1977, LASL Priv. Comm.

12 J.W. Staples, R.A. Jameson, op. cit. p. 3698.

13 K.R. Crandall, R.H. Stokes and T.P. Wangler, "Rf Quadrupole Beam Dynamics Design Studies", to be presented at 1979 Linear Accelerator Conference.

14 O.R. Sander, R.A. Jameson, R.D. Patton, op. cit. p. 3417.

15 K.R. Crandall, "Summary of 805-MHz Linac Length Corrections", LASL Priv. Comm., March 26, 1975.

16 R.A. Jameson, W.E. Jule, "Longitudinal Tuning of the LAMPF 201.25 MHz Linac Without Space-Charge", LA-6863, March 1978, LASL.

17 R.A. Jameson, "Emittance Growth in the New CERN Linac - Transverse Plane, Comparison Between Experimental Results and Computer Simulation", LASL Priv. Comm. AT-DO-377(U), Jan. 15, 1979.

18 O.R. Sander, G.N. Minerbo, R.A. Jameson, D.D. Chamberlin, "Beam Tomography in Two and Four Dimensions", Proc. this conference.

19 P. Channell, LASL, Priv. Comm.

20 A. Chabert et al., op. cit. p. 3612.

Discussion

Ohnuma, FNAL: If somebody is building a high intensity linac right now, from your data or from your experience, what would be your recommendation regarding the most important thing they have to do to suppress the emittance growth?

Jameson: I have not tried to write down what all the rules are, because I don't think we are at the end of the study yet. I think if you study these pictures, it would suggest that one should probably have a tune around 70° for no current. These results so far, would suggest that in an accelerating system we can stand a somewhat higher tune than Lloyd Smith and someone found for transport systems. They suggested that you should limit the zero intensity tune to 60° and not de-tune it with current down to less than 24°, which is the .4 constant of nature again. It looks like in an accelerator, you can probably stand a somewhat higher phase advance, but 70° seems to be a good number to minimize the growth.

Penner, NBS: I think you're cutting it a little fine there, Bob. I don't think there is any difference between what you would recommend at 70° or so, and the 60° that Smith and Laslett recommend. I think their point is, and I think it is true in our numerical simulations and yours and everybody elses, that you must not be at 90° or above or there are these barometric resonances. They will always be there because you always have part of the beam that is at lower current than the main part. So the real rule is below 90°.

Curtis, FNAL: Just a small point. You said when you sliced up the beam you got a somewhat smaller emittance growth. Did you end up with the same density of particles or did you throw some of them away?

Jameson: No, I didn't throw any of them away. The paper that John Staples and I had in March showed that if you slice up the beam and plot the phase space of each slice, you will find that at the end of the beam which has the least rf focusing you will see the thing rotate so that the projection gets turned into kind of a cross and basically that was what I was doing. I was just matching to that, and it went through; no particles were lost.