LINEAR ACCELERATORS WITH INTERTANK FOCUSING

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Introduction

In 1978 different concepts for an extension of the UNILAC energy range beyond the present limit of about 10 MeV/u were discussed at GSI. For energies up to 100 MeV/u linear accelerator structures as well as sector field cyclotrons were reconsidered. The straight-forward approach is the extension of the present Alvarez structure operating at 108 MHz. There are, however, good reasons to study linear accelerator structures that operate at higher frequencies, e.g. 216 or 351 MHz. In this high frequency range the well-known focusing scheme of magnetic quadrupole lenses, which are mounted inside the drift tubes, has the following disadvantages:

a. The capacitive loading of the rf-cavities due to drift tubes with a large diameter (20 cm or more) is rather high (216 MHz) or even not tolerable (351 MHz).
b. A large number of drift tubes (N) with magnetic quadrupole lenses is quite expensive (N ~ frequency).
c. Furthermore, an alignment system for the drift tubes, which is only required with drift tube quadrupoles, is quite expensive, too.

Permanent magnetic quadrupole lenses with a rather small outer diameter could reduce the capacitive loading. However, they do not fit the needs of a heavy ion linear accelerator, where a large aperture and variable field gradients for a broad range of ions are required. Therefore a different approach was studied, which does not make use of drift tube quadrupole focusing: short accelerator cavities without quadrupole focusing and intersections between cavities, which provide radial focusing with magnetic quadrupole lenses and, eventually, even phase focusing by means of rebuncher cavities.

Phase Focusing

In linear accelerator structures that make use of intertank quadrupole focusing instead of drift tube quadrupole focusing, rather short cavities and quite long intertank sections are used in order to provide the necessary radial focusing. It has to be proven that the phase motion is stable in accelerating structures with intertank sections which are long in terms of the rf cell length $\beta\lambda$, and that the phase acceptance is adequate.

For periodic structures the phase motion can be described in the same matrix formalism that is well-known for the radial motion. This linear approximation was used to study the stability criteria. In canonical coordinates $\delta\varphi$ and δW, the transport through an accelerating gap is given in the thin lens approximation by the matrix M_G with $k = -\zeta eU \sin\varphi_s$ and through a drift space of length L by M_D, with $\ell = -(2\pi/W_o\beta^3\gamma^3)*(L/\beta\lambda)$

$$M_G = \begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix} \qquad M_D = \begin{pmatrix} 1 & \ell \\ 0 & 1 \end{pmatrix}$$

The resulting transfer matrix M_S for the coordinates $\delta\varphi$ and δW through one section can be written as

$$M_S = \begin{pmatrix} \cos N\mu_1 + \frac{d}{2\beta_1}\sin N\mu \ , & -\beta_1(\sin N\mu_1 - \frac{d}{\beta_1}\cos^2 \frac{N}{2}\mu_1) \\ \frac{1}{\beta_1}\ (\sin N\mu_1 + \frac{d}{\beta_1}\sin^2 \frac{N}{2}\mu_1)\ , & \cos N\mu_1 + \frac{d}{2\beta_1}\sin N\mu_1 \end{pmatrix}$$

where
N is the number of cells per cavity
$\mu_1 \simeq \sqrt{-k\ell}$ is the phase advance per cell
d is the matrix element for the intertank space
$\beta_1 = (1/w)\sqrt{-l/k}$
$w = \sqrt{(1 + kl/4)} \simeq 1$

The stability criterion for the phase motion is

$$\frac{1}{2}\ \mathrm{TR}\ (M_S) = \left| \cos N\mu_1 + \frac{d}{2\beta_1}\ \sin N\mu_1 \right| < 1$$

For accelerating structures that have no intertank sections at all (d=0), the phase motion is stable for all phase angles with $\sin \varphi_s < 0$ and all values of N. The insertion of drift spaces between cavities will result in an unstable phase motion for some ranges of N, even with phase angles in the well-known stability region $-90^\circ < \varphi_s < 0^\circ$ (Fig. 1).

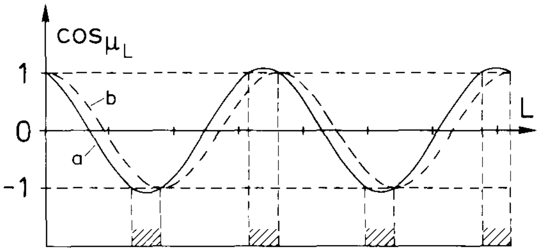

Fig. 1: Instability regions of a linear accelerator structure with intertank sections (a). The phase motion for a continuous accelerating structure is always stable with $\sin \varphi_s < 0$ (b).

In Fig. 2 the limitation of the cavity length is shown as a function of the normalized particle velocity β. It is well-known for the radial motion, and it will be shown for the phase motion, that the optimum acceptance is achieved for the phase advance per section $\mu_L = \pi/2$. Therefore the cavity lengths for $\mu_L = \pi/2$ are also plotted versus β. In the velocity range from 0.1 to 0.2 (5 MeV/u to 20 MeV/u), which has to be passed immediately behind the UNILAC, the optimum cavity length lies in the range from 1 m to 3 m.

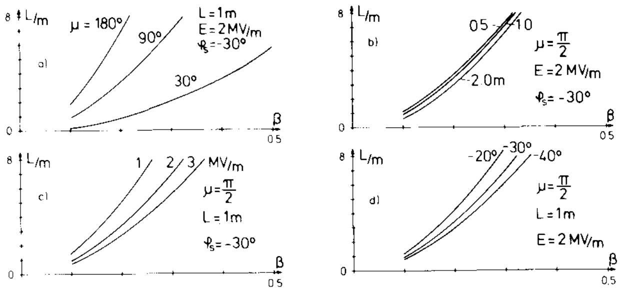

Fig. 2: Limitation of the cavity length L at $u_L = \pi$ versus normalized velocity ß and optimum cavity lengths at $\mu_L = \pi/2$ for some values of the mean accelerating field E_S, the phase angle φ_S and the intertank distance L

The linear treatment for the phase motion has only approximate answers for the following questions:

a. What is the phase acceptance in the non-linear theory?

b. How many sections will be required to develop the effects of unstable phase motion (Fig. 3)?

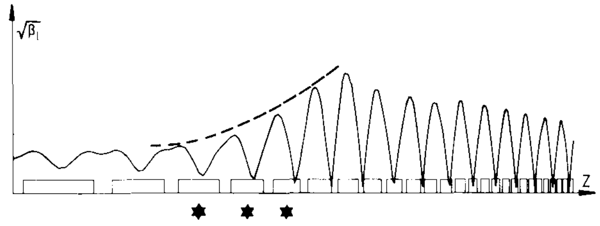

Fig. 3: Phase amplitudes for a particle bunch that is accelerated from ß = 0.1 through a linac structure with 3.4 m long cavities and 1 m long intersections. The three sections marked with an asterisk are operated in the instability region (Fig.1) causing an exponential growth of the phase amplitudes.

In order to study the non-linear phase motion, first iso-Hamiltonians of the phase motion in periodic structures were derived numerically (Fig. 4). These iso-Hamiltonians can be transformed in a second step through the real semi-periodic accelerating structure in order to display the development of instabilities (Fig. 5). From these studies the following conclusions were derived:

a. The optimum acceptance is achieved at $\mu_L = \pi/2$. At an intertank distance of 1 m the acceptance is comparable in size to the acceptance for the continuous accelerating structure.

b. A phase advance much above $\mu_L = \pi/2$ is not advisable. Even at $\mu_L = 2\pi/3$ the acceptance is remarkably reduced.

c. In accelerating structures that are operated in the instability region the phase acceptance shrinks drastically.

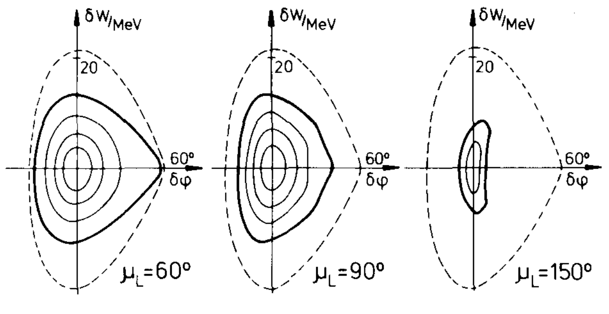

Fig. 4: Iso-Hamiltonians at ß = 0.1 for three different values of phase advance μ_L in linac structures with 1 m intertank distance and, for comparison, the separatrix (dashed) for a continuous accelerator structure.

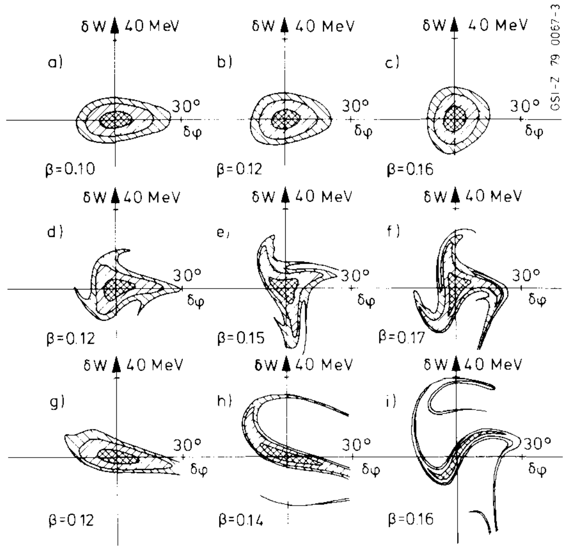

Fig. 5: Transformation of the iso-Hamiltonians (a) through structures with $\mu_L = 90^\circ$ (b,c), $\mu_L = 120^\circ$ (d,e,f) and through a structure with 3.5 m cavities (g,h,i) all with 1 m intertank distance.

If for technical or economical reasons linac sections longer than about 1 m are desired, rebunchers can be introduced in order to restore phase stability. Two schemes for the insertion of rebunchers were considered: either one rebuncher cavity in the center of the intertank sections (I), or two rebunchers before and behind the section, which can be integrated in the neighboring accelerator cavities (II). The rebuncher focusing strengths required for perfect matching are respectively:

$$b_R = -\frac{d}{\beta_1^2 + \frac{d^2}{4}} \qquad \text{(I)}$$

$$b_R = -\frac{1}{2d}\left(1 - \sqrt{1 - \left(\frac{2d}{\beta_1}\right)^2}\right) \qquad \text{(II)}$$

Figure 6 shows the amplitude functions β_l, for both configurations. The rebuncher amplitudes for $\beta = 0.1$ and $\beta = 0.25$ are given in Table I.

	double	single
$\beta = 0.1$	undefined	0.75
$\beta = 0.25$	1.25	0.96

Table I Rebuncher amplitudes (in MV) for two schemes of insertion

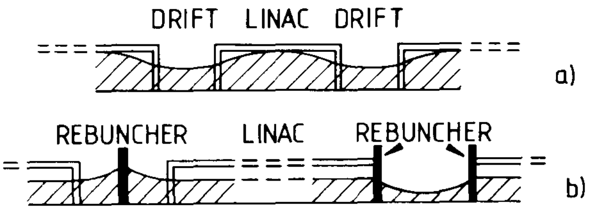

Fig. 6: Amplitude functions β_1 for linear accelerator structures with intertank sections (a). The intertank sections can be matched to the accelarator by means of rebunchers.

It can be seen that for the double rebuncher system (II), the maximum intertank space is restricted while for the single rebuncher system the only limitation is the linearity range of the rebuncher voltage, which does not appear in the linear approximation. If rebuncher focusing is provided, it is also possible to operate the cavities at a phase angle of $\varphi_s = 0^\circ$. This means that neither phase focusing nor radial defocusing occur in linear approximation; the cavity can be treated as a drift space. Phase stability has to be maintained by appropriate settings of the rebunchers, giving about 90° phase advance per period . Numerical calculations have shown, however, that the acceptance decreases rapidly with increasing length of the cavity. Sufficient phase acceptance at a length of about 3.5 m cannot be achieved. For the use of rebuncher matching it may be concluded:

a. With rebuncher matching there is no limitation for the lengths of the accelerator sections.
b. In the velocity range from $\beta = 0.1$ to $\beta = 0.2$ with the parameters considered here, rather high rebuncher voltages are required for perfect matching (Table I).
c. Although phase matching can be achieved for longer accelerator sections, radial focusing and radial acceptances will be reduced with increasing cavity length as shown below.

Radial Focusing

In linear accelerator structures that have no drift tube focusing, magnetic quadrupole lenses are installed in the intertank sections. The maximum radial acceptance is again achieved for a betatron phase advance $\mu_R = \pi/2$. In Fig. 7 the radial amplitude functions are plotted for two different structures, one with short cavities, which are matched in length to 1 m intertank sections, and the other with long cavities and rebuncher matching. The focusing strengths, quadrupole parameters and radial acceptances for an aperture of 2R = 4 cm are listed in Table II.

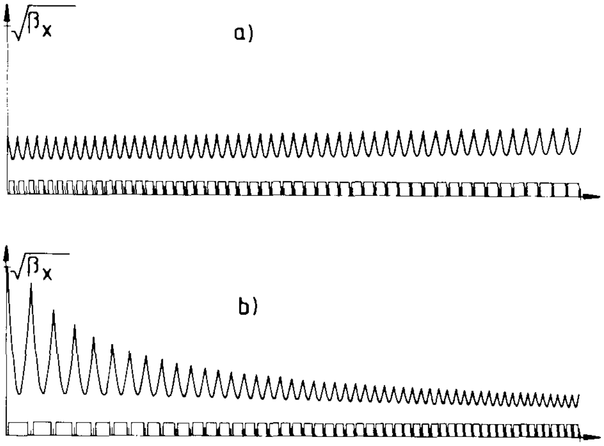

Fig. 7: Radial amplitude functions for structures with $\mu_L = 90^\circ$ (a) and with 3.5 m cavities (b), all with 1 m intertank distance. The intertank quadrupole triplets are tuned to the radial phase advance $\mu_R = 90^\circ$.

β	short cavities B' (T/m)	short cavities A_n (mm*mrad)	long cavities (3.5 m) B' (T/m)	long cavities (3.5 m) A_n (mm*mrad)
0.1	15.0	9.39	14.0	1.05
0.25	25.3	12.67	25.3	12.67

Table II Focusing parameters for two accelerator structures with intertank focusing.

Alternate Phase Focusing

A structure with 3.5 m long cavities and 1 m intertank distance has a very poor radial acceptance due to the strong radial defocusing in the first part of the linac. There are two ways to reduce radial defocusing and to improve radial acceptance. One can either shift the equilibrium phase angle φ_s towards 0°, or one can introduce sections with positive equilibrium phase angle and hence, with inherent radial focusing (APF structures). Both schemes improve radial acceptance at the expense of phase acceptance.

For APF structures, alternate sections with $\pi/4$ phase advance per period produce minimum excursions of the amplitude function and thus the best phase acceptance. Nevertheless, the phase acceptance is reduced by a factor of 4 compared to the continuously focusing structure in the linear approximation and even more, by a factor of 10 taking into account the strong non-linear phase motion. Thus the perfect APF structure designed for heavy ion beam parameters at $\beta = 0.1$ has an adequate radial acceptance but only a poor phase acceptance. A compromise with moderate acceptances for both the phase and the radial motion can be found, if one reduces the length of the radially focusing sections with positive phase angles in the APF structure (Fig. 8).

For a heavy ion accelerator behind the UNILAC, a frequency jump from 108 MHz to 351 MHz was proposed. Therefore any reduction of phase acceptance is unwelcome and as a consequence has to be ruled out.

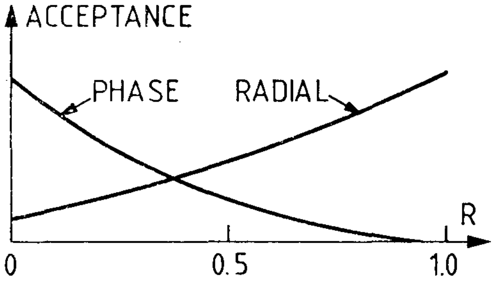

Fig. 8: Phase and radial acceptance for APF structures versus the ratio R of phase defocusing to phase focusing sections at $\beta = 0.1$

Parameter Discussion for a 100 MeV/u Linear Heavy Ion Accelerator

For an extension of the UNILAC energy range beyond the present limit of 10 MeV/u, a 351-MHz Alvarez structure with intertank focusing was studied. In order to provide adequate phase and radial focusing short sections starting with 1 m length are required in the first part of the accelerator. Figure 9 presents a scheme for an 100-MeV/u accelerator with intertank focusing. The machine parameters are listed in Table III. Even for the acceleration to an energy of 30 MeV/u, about 36 cavities are required, which have, with intertank sections, a total length of 120 m. The great accelerator length is due to the assumption of the acceleration of U^{41+} at a mean accelerating field of 2 MV/m. With a second stripper at 5.9 MeV/u and with an increased field of 4 MV/m, which is still low compared to PIGMI prototype results, the total length could be reduced to about 35 m for 30 MeV/u and 120 m for 100 MeV/u. For comparison with an 108-MHz Alvarez accelerator with drift tube focusing the following arguments have to be evaluated:

a. Accelerating structure and quadrupole focusing are inexpensive at 351 MHz.
b. Rf power amplifiers operating at 351 MHz are not yet commercially available.
c. Accelerator cavities of 1 m length are too small units for the 351-MHz Rf power amplifiers. Therefore methods of coupling several cavities should be studied.

Since it was finally proposed to extend the UNILAC energy range only to maximum energies of about 20 MeV/u, the 108-MHz Alvarez acclerator was preferred as the most appropriate solution in order to avoid the lengthy development of new power amplifiers.

β	0.1		0.25		0.42
W (MeV/u)	5		30		100
ΔU (MV)		151		403	
Number of cavities		36		66	
L (m)		122		299	
$A_{x,y}$ (mm*mrad)	13 (normalized)				
A_φ	27 MeV * 60°				

Table III Parameter list for a 351-MHz Alvarez accelerator. At an average field of E = 2 MV/m U^{41+} ions are accelerated from 5 to 100 MeV/u.

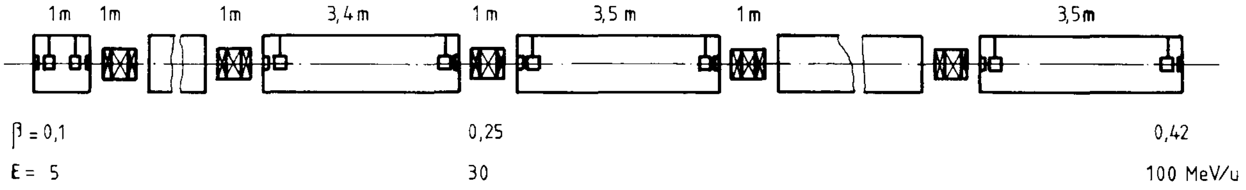

Fig. 9: Heavy ion linear accelerator with intertank quadrupole focusing for the acceleration of U^{41+} from 5 to 100 MeV/u