# TIPLESS PERMANENT MAGNET QUADRUPOLE LENSES

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## Summary

Tipless quadrupole lenses of the rod type are proposed. Lenses with samarium-cobalt rods and < 2-cm aperture ensure a magnetic field gradient up to 6 kG/cm and allow regulation of the operating field over a wide range. The advantages of these lenses in comparison with conventional ones are shown.

## Introduction

The availability of rare-earth magnetic alloys opens new possibilities for designing magnetic optics elements with formerly unattainable parameters. For example, small high-gradient quadrupole lenses assembled from $Sm\text{-}Co_5$ cylinders for the PIGMI accelerator at Los Alamos.[1] Two new types of quadrupole lenses without pole tips were suggested in 1975-1977 in the USSR. They have some advantages in comparison with the lenses now used. In a sector-type lens (Fig. 1) suggested by J. D. Rabinovich[2] and investigated at the ITEP,[3] a gradient of 6.2 kG/cm in a 2.3-cm aperture was achieved with a lens of 5.2 cm o.d. and 5.0 cm length. The rod-type lens[3] designed at ITEP can ensure a gradient of $\sim$ 6 kG/cm, is simpler to manufacture, and, although its size is the same as that of its competitors, it can produce a lower gradient. Some properties of the sector-type lens and the possibility of manufacturing the rod-type lens were discussed by K. Halbach.[4] This paper deals with analysis of the rod-type lens and the results of measurements on the first prototype.

## Analysis of the Lens

The rods of this lens (Fig. 2) are mounted in cells in a non-magnetic holder (1) at a distance $R_o$ from the axis of the lens. The angle between the intensity of the magnetization vector $\vec{I}$ and the radius vector $\vec{R}_p(R_o,\psi_p)$ of rod p is twice as large as $\psi_p$. The radii of rods generally are unequal. The field of a multilayer rod-type quadrupole lens can be determined by superposition of the fields of all the rods. The operating field for the closest filling of the lens can be easily calculated when the neighboring rods in the same layer and in the adjacent layers touch each other.

If the numbers of rods and of layers increase to infinity, then the field of the lens approaches an ideal quadrupole:

$$\vec{B}(r,\phi) = Gr\vec{a} \quad ,$$

where

$G = 2\pi^2 I\left(\frac{1}{r_o} - \frac{2}{d}\right) =$ gradient,

$\vec{a} \equiv \vec{n}\cos 2\phi - \vec{\tau}\sin 2\phi$,

$r_o$ is the aperture radius,

d is the outer diameter, and

$\vec{n}$, $\vec{\tau}$ are unit vectors of the polar system $r,\phi$.

In a lens with a finite number of rods, N in a layer and a finite number of layers J, the base (quadrupole) harmonic of the field

$$\vec{B}_f = \frac{N}{\pi}\sin\frac{\pi}{N}\cos^2\frac{\pi}{N}\vec{B}$$

is less than $\vec{B}$ with the same o.d., $d = d_J$ (where $d_J$ is the o.d. of a J-layer lens), but the difference becomes negligible with increasing number of rods. The base harmonic is distorted by the influence of higher harmonics. The contribution of the higher harmonics, $\Delta\vec{B}$, can be defined in terms of the ratio of this contribution to the quadrupole harmonic modulus at the corresponding point of the field in the aperture limits $r = r^*$, $\phi = 0$:

$$\delta \equiv \frac{|\Delta\vec{B}(r^*,0)|}{|\Delta\vec{B}_f(r^*,0)|} \quad .$$

The values of $\delta$ and of the ratio $2\ r_o/d_J$ depend on the numbers of rods and layers, N and J, as shown in Fig. 3, where the points corresponding to the same value of J, calculated with $r^* = 0.75\ r_o$, are connected by smooth curves. Clearly, there is no need to use more than three layers to decrease the nonlinearity, and the most effective number of rods is between 10 and 16. For practical realization, it is simpler to keep the ratio $2\ r_o/d_J$ close to 0.5.

The discreteness of the distribution of the magnetization intensity leads to the appearance of a dispersion field. As N and J approach infinity, the dispersion field vanishes; at finite N and J, it decreases almost to zero with increasing distance from the surface of the lens. The dependence of the dispersion field on the ratio $2\ r/d_J$ for different values of N from 10 to 30 is shown in Fig. 4. The number of layers in the lenses with different N was chosen to satisfy the inequalities $d_{J-1} < 0.5 < d_J$. The graphs show that in a lens with N = 20 (J - 3), for example, at a distance $d_J/10$ from the surface, the field B is $< 10^{-2}\ B_r$ (i.e., $\leq$ 100 G).

### Adjusting the Gradient

Because the layers are independent of each other in a rod-type lens, it is easy to regulate the gradient. This can be done by allowing for the azimuthal displacement of one group of layers relative to another. If $\vec{B}_1 = G_1 r\vec{a}$ and $\vec{B}_2 = G_2 r\vec{a}$ are the fundamental harmonics of the fields of an inner and an outer group respectively, then, with the angle $\phi_o$ of relative turning of the base planes, the fundamental harmonic $\vec{B}$ of the resulting field is

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = Gr\left[\vec{n}\cos 2(\phi-\theta) - \vec{\tau}\sin 2(\phi-\theta)\right], \qquad (1)$$

where

$$G = \sqrt{G_1^2 + G_2^2 + 2G_1G_2\cos 2\phi_o}\ ,$$

$$\theta = \tfrac{1}{2}\arc\cos\frac{G_1 + G_2\cos 2\phi_o}{G}\ . \qquad (2)$$

Equation (1) indicates that the quadrupole field $\vec{B}$ is shifted in phase relative to the field of an immovable (inner) group. The gradient of the resulting field, as seen from Eq. (2), depends on the angle $\phi_o$. As this angle changes from zero to $\pi/2$, the gradient changes from a maximum value of $G_{max} = G_1 + G_2$, to minimum, $G_{min} = |G_1 - G_2|$. As a rule, the appearance of the angle $\theta$ in the phase of the resulting harmonic $\vec{B}$ is undesirable, and it must be compensated for by turning the lens as a whole, through the angle $\theta$. The turning arrangement is simplest when $G_1 = G_2$; in this case $\theta = \phi_o/2$ and $G = 2G_1|\cos\phi_o|$, i.e., to fix the coordinate axes of the resulting field, it is enough to rotate the movable groups of layers in opposite directions by equal angles.

It is important in manufacturing the lens, to select identical rods. The higher harmonics of the rods' fields can be compensated for in the lens when their magnetic moments are equal. If the rods are not identical, the effect of demagnetization of the rod material can be compensated for outside the lens. This can be accomplished by lapping the rods to decrease the radii $R_i$ in order to obtain satisfactory equality of the products $I_iR_i^2$. In this case the field of the lens with equalized rods corresponds to minimum magnetic moment in the primary group selected for rod lapping.

It is also important to tune the lens, i.e., to install the rods in accordance with the law of vector $\vec{I}$ rotation in a quadrupole lens. A convenient way to accomplish this is to measure the direction of the field generated at the axis of the lens by the rod being tuned. The effectiveness of such measurement is ensured by use of a Hall pickup. Every rod field at the axis of the quadrupole lens forms an angle $\psi_p$ with the base plane. For tuning, the plane of the Hall pickup should be put at the angle $\pi/2 - \psi_p$ and the rod should be rotated around its axis to the position where the Hall emf is maximum. For final tuning it is convenient to use the zero reading of the Hall pickup by turning its plane $90^o$.

### The Prototype Lens

The prototype of a rod-type quadrupole lens manufactured at ITEP consists of two layers of Sm-$Co_5$ alloy rods (Fig. 5). Each layer is mounted in a separate cylindrical drum made of brass. The rods have brazed centers (Fig. 5) and are fixed in the drum cells by a stop screw, which presses them to the bottom of the cells. The length of the magnetic rod, $\ell_r$, is 2 cm, and its diameter is 0.6 cm.

Each drum with its tuned rods, comprises a separate lens. The inner drum contains 12 rods and the outer one, 20. The diameter of the hole in the inner drum is 2 cm; the outer diameter of the compound lens is 4.8 cm and of the inner lens, 3.4 cm. The gradient at the axes of the inner and outer lenses (average for the lens without a yoke) are 2.6 and $\sim$ 1 kG/cm, respectively. These are not high values considering the overall sizes of the lenses. They result from the relatively low magnetization of the rods (in the inner lens, for example, $I \simeq 550$ G), from the low packing density, and from the fringing effects, since rods had a diameter $2r_o \gtrsim \ell_r$ in these lenses. This relationship between diameter and length has a strong influence on the harmonic structure of the operating field, if the dependence $G(z)$ in such a short lens does not have a flat part. For example, the measurements showed that the inner lens with rods equalized according to their magnetic moments (with accuracy $\pm$ 2%) has an operating field with the amplitudes of the 6th, 10th, and 14th harmonics equal to 2.7%, 0.7%, and 0.6% that of the quadrupole field, with $r^* = 0.75$ cm. The other harmonics have lower amplitudes.

### Designs For A Linac

The rod-type construction allows tighter packing of a lens with rods than was used in the prototype, and the length of the lens in the focusing channel of an ion linear accelerator would normally be several times that of the experimental model. Therefore, much higher gradient and linearity of the field could be obtained for the same size lens. Calculations show that a gradient of up to 7 kG/cm could be obtained for a bore diameter of 2 cm and a lens o.d. of 8 cm if a Sm-$Co_5$ alloy with higher magnetization ($4\pi I \simeq 9$ kG attainable at present) is used.[5]

The proposed rod-type quadrupole lens, which encompasses the whole range of gradients needed in the focusing channel of a linear accelerator, reduces the variety of required designs. It also simplifies the manufacturing technology and the tuning procedure in comparison with a sector-type lens or a lens with pole tips. Its linearity and dispersion are as good as those of the sector-type quadrupole lens and are much better than those of a salient pole lens. One of the major merits of the rod-type lens as an example of a tipless lens, is that it allows smooth regulation of the gradient over a wide range. This, and the other advantages mentioned above, simplify the design and manufacture of focusing channels with a large number of lenses.

Acknowledgement

The authors would like to thank Prof. I. M. Kapchinskii for formulating the problem and for other help, Dr. V. V. Sergeev for consultation and assistance, and Drs. V. S. Rasstegaev and E. D. Shalyaev, who manufactured the magnetic rods in their laboratory.

References

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3 V.S. Skachkov, "Permanent Magnet Multipole Lenses Without Pole Tips", ITEP-76, Moscow (1979).

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Discussion

Bentley, NEN: Can you give us an idea of how much the material costs and what the cost of the lines you showed us might be?

Lazarev: Yes, of course. The cost of this Samaerium cobalt material is high but with such lenses there is no consumption of energy. For the most part, the cost of lenses within drift tubes is in their manufacturing and not the cost of a piece of iron, so the difference is not so much as it may seem.

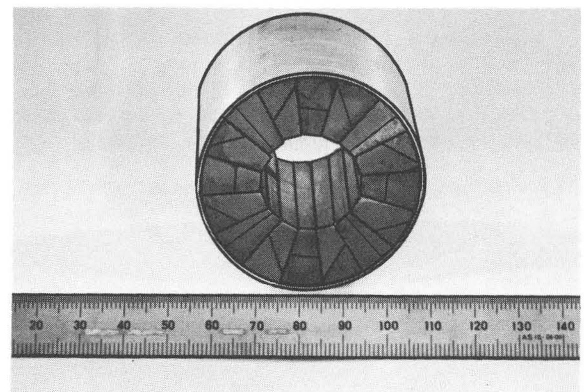

Fig. 1 Sectoral samarium-cobalt quadrupole lens with a gradient of 6.2 kG/cm in an aperture of 2.3-cm diameter

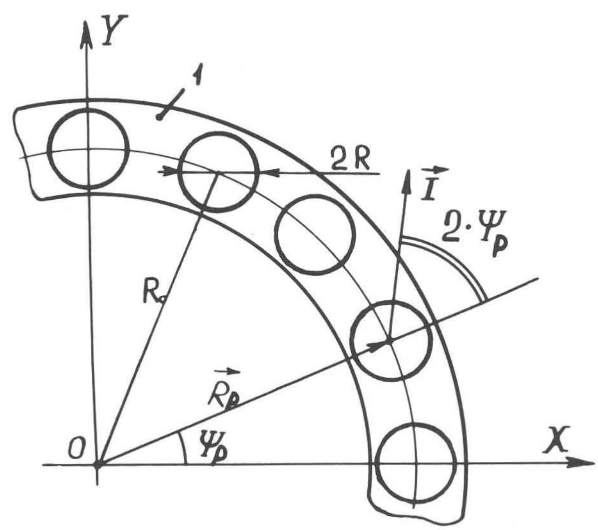


Fig. 2 Cross-section of a rod-type quadrupole lens

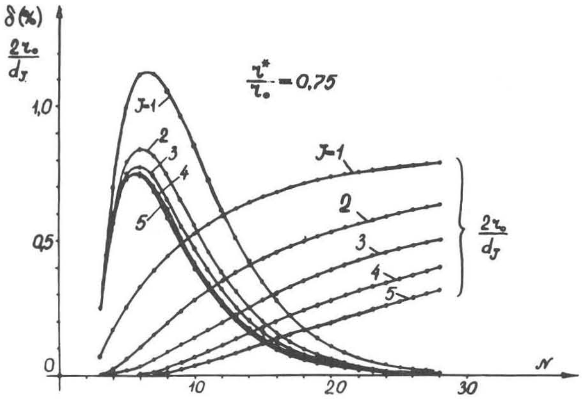


Fig. 3 Dependence of the field nonlinearity δ and the ratio 2 $r_0/d_J$ on number of rods N in each layer of a quadrupole lens with J layers


Fig. 5 Rod-type, two-layer, samarium-cobalt quadrupole lens

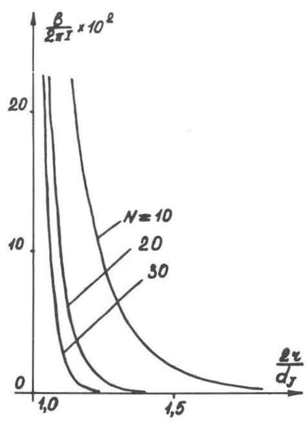


Fig. 4 Dependence of the dispersion field on distance from the lens surface and on the number of rods in each layer