NEW METHOD FOR POSITRON PRODUCTION AT SLAC*

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Summary

The focusing system for the positron beam at SLAC makes use of an adiabatically tapered solenoid, whose transverse acceptance remains high over a broad energy band. From the computed distribution in energy of the accepted positrons, one can estimate that approximately one-half of the total number of positrons lie within the energy band from 2 - 4 MeV. Due to the debunching effect over the drift space following the target, only the high energy part (from 4.5 MeV to 10 MeV, or more) of the accepted spectrum contributes to the useful current (within 1% energy bin). By decelerating the beam in a special short section very near the converter, it is possible to obtain a bunching of all the accepted positrons with energy above 2 MeV within approximately 5°, giving an improvement of a factor of two on the analyzed current.

Description

Focusing system

The focusing system for the positron beam[1] consists of a 7-meter long solenoid (up to an energy of 55 MeV) and of quadrupole triplets and doublets. The emittance matching between the plane of the target and the solenoid uses an adiabatically tapered solenoid. Recent computations made by M.B. James et al.,[2] showed that the transverse momentum accepted by the system remains high ($p_t \geq 0.6$ MeV/c) over a very broad energy band (2.5 MeV $\leq$ E $\leq$ 30 MeV). In addition they computed the distribution in energy of the accepted positrons from which one can estimate that approximately one-half of the total number of accepted positrons lie within the energy band from 2 - 4 MeV. To maximize the current of positrons it is then necessary to accelerate the broadest energy band possible, starting from the lowest energy possible. The limitation in that process is the debunching effect in the drift space following the target and in the first few cm of the acceleration, leading to a broadening of the energy spectrum.

Rf system

The distance between the target and the first accelerating section had been set rather large (93 cm), mainly to provide space for collimators. It was decided last year to insert in that space a special short section (Fig. 1). This section is a 5λ-long (52.5 cm), constant impedance structure; the group velocity is vg/c = 0.0204 The accelerating field is related to the input power by E (MeV/m) = 3.55 $\sqrt{P}$ (MW). The distance between the target and the first section is then reduced to 14.2 cm. The section is followed by a drift space of 26.5 cm, in which are located a collimator and a fast vacuum valve.

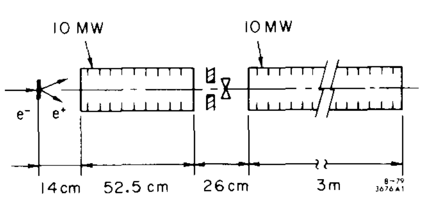

Fig. 1. Positron source accelerator structure.

Longitudinal Motion of Positrons

Debunching

In the drift space from the target to the section, the beam is debunched because of different energies, hence different velocities, and because of the spiralling path of positrons with different transverse momentum.

Let D be the length of the drift space. The phase slippage, with respect to a wave traveling at the velocity of light, is given by:

$$\Delta\phi = \frac{2\pi}{\lambda}\left(1 - \frac{1}{\beta_z}\right) D \quad , \qquad (1)$$

where β_z is the axial velocity in units of c, and λ is the rf wavelength. The axial momentum is given by:

$$p_z^2 = \gamma^2 - 1 - p_t^2 \quad ,$$

where p_t is the transverse momentum in units of mc and $\gamma = E/mc^2$. The axial velocity is $\beta_z = p_z/\gamma$. In the tapered field region the transverse momentum varies as:[3]

$$p_t(z) = p_{to}\left(\frac{B(z)}{B_o}\right)^{\frac{1}{2}} \quad , \qquad (2)$$

where p_{to} and B_o are the transverse momentum and the magnetic field on the target. The phase slippage is then:

$$\Delta\phi(\gamma_o \;\; p_{to}) = \int_o^D \frac{2\pi}{\lambda}\left(1 - \frac{\gamma_o}{(\gamma_o^2 - 1 - p_t^2(z))^{\frac{1}{2}}}\right) dz \quad , (3)$$

with $p_t(z)$ given by Eq. 2, which can be approximated by:

$$\Delta\phi = -\frac{\pi}{\lambda}\frac{1}{\gamma_o^2} p_{to}^2 \frac{1}{B_o}\int_o^D B(z)dz - \frac{\pi}{\lambda}\frac{1}{\gamma_o^2} D + \ldots \quad .$$

The total longitudinal phase-space can be calculated for a distance D and a range of energy and transverse momentum, for a given law of magnetic field $B_{(z)}$. Figure 2

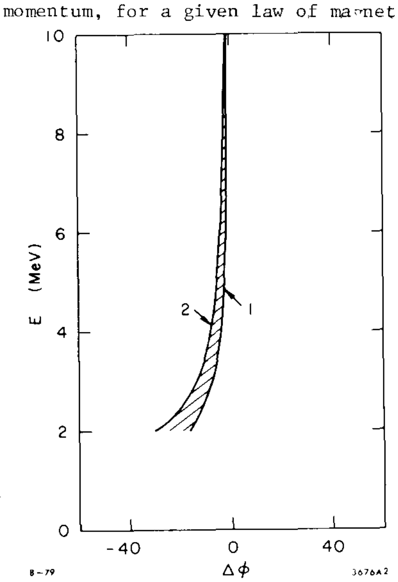

Fig. 2 Longitudinal phase-space after a 14.2 cm drift: Curve 1 for p_{to} = 0; Curve 2 for p_{to} = 0.6 MeV/c.

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shows the results in the case of D = 14.2 cm, for energies from 2 - 10 MeV and a maximum transverse momentum on the target of 0.6 MeV/c. The total bunch length is about 30°. The bunch length for energies from 4 - 10 MeV is only 8°. The contribution to the phase dispersion from the spiralling path is approximately equal to that of the velocity dependent effect for a given energy. Clearly, one needs a rebunching process to accelerate all the accepted positrons within a reasonable energy spectrum.

Principle of bunching

1.) Let us first consider the case of positrons moving on the axis ($p_{to} = 0$) and assume that the beam enters an accelerator which is long enough to consider the asymptotic behavior of the positrons. The motion of the positrons can be described by orbits, Fig. 3, in the

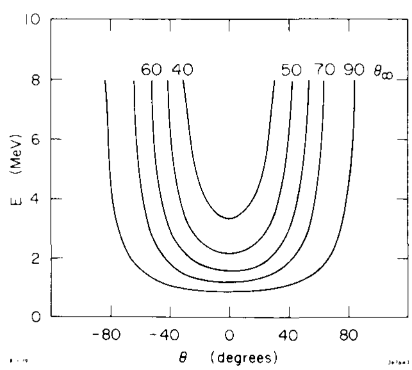

Fig. 3 Orbits in longitudinal phase for particles in a linac.

phase-space E, ϕ. The equation for the orbits[4] is:

$$\cos\phi - \cos\phi_\infty = \frac{2\pi}{\alpha}\left[\gamma - (\gamma^2 - 1)^{\frac{1}{2}}\right] \quad , \quad (4)$$

where $\alpha = eE'\lambda/mc^2$ is the energy gain per wavelength in the units of mc^2, and ϕ_∞ is the asymptotic phase. The reference of phase is such that the electric field is 0 for $\phi = 0$.

On the other hand, there exists a relation between phase and energy as the beam enters the accelerator, which can be written, using Eq. 1 :

$$\phi_o = \Phi_o + \frac{2\pi}{\lambda} D \left(1 - \frac{\gamma_o}{(\gamma_o^2 - 1)^{\frac{1}{2}}}\right) \quad , \quad (5)$$

Φ_o being a parameter which represents the input phase of the field for a $\beta = 1$ positron.

By using Eqs. 4 and 5 one can compute optimum values for the parameters α, D and Φ_o, in order to obtain a minimum dispersion in asymptotic phase for a given range of γ_o. The asymptotic phase ϕ_∞ must be chosen near -90° in order to obtain the maximum acceleration. The orbits (γ, ϕ) resulting from Eq. 4 are symmetric with respect to $\phi = 0$. Positive values of ϕ_o (decelerating field) give a positive slope $d\gamma_o/d\phi_o$. Equation 5 gives a positive slope $d\gamma_o/d\Phi_o$. The best fit between relations 4 and 5 will then be obtained when the beam enters the accelerator in a decelerating field. The optimum value of Φ_o will be near 90° for $\phi_\infty = -90°$. This can be easily understood by looking at Fig. 2 and comparing it with the general shape of the phase-space orbits in Fig. 3.

As an illustration of this bunching process, Fig. 4 shows the phase space resulting from the drift space of 14.2 cm for energies from 2 MeV to 10 MeV (solid curve), together with the orbits ϕ_∞ = -85°, -90° and -95°, for an accelerating field of 12 MeV/m (dotted curves) and Φ_o = 86° (decelerating field). It is seen that the

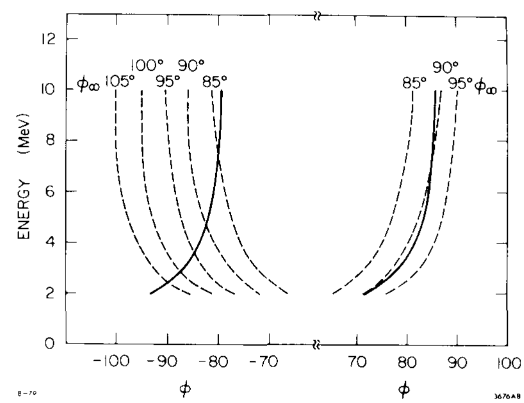

Fig. 4 Longitudinal phase space at entrance to acceleration section

fitting with the -90° orbit is achieved within less than ± 2° and one can expect a dispersion of asymptotic phase (i.e., a bunch length) of less than 4°. On the other hand, Fig. 4 also shows the same phase-space for the beam together with the orbits -75°, -80°, -90° and -100° in the case Φ_o = -70° (accelerating field). It is evident that the dispersion in asymptotic phase will be greater than 25°. The integration of the equations of motion over a distance of 3 m gives a total bunch length of 31°; the output energy varies from 37 MeV to 44 MeV.

It appears that this bunching process is very efficient for an initially bunched beam, which contains particles within a large energy band extending down to a very low energy, and which has been debunched under the action of different velocities over a drift space.

2.) Positrons With Transverse Momentum - If it is assumed that there exists no other transverse force than the focusing effect of the dc magnetic field, then the transverse momentum remains constant in a constant focusing field, or varies according to Eq. 2 if the magnetic field varies slowly. The equations of longitudinal motion can be written in that case:

$$\begin{cases} d\gamma = -\alpha \sin\phi \, dz \\ d\phi = 2\pi\left(1 - \dfrac{\gamma}{(\gamma^2 - 1 - p_t^2(z))^{\frac{1}{2}}}\right) dz \end{cases} \quad (z \text{ in units of } \lambda)$$

For constant p_t, these equations can be combined and integrated, giving:

$$\cos\phi_o - \cos\phi_\infty = \frac{2\pi}{\alpha}\left[\gamma_o - (\gamma_o^2 - 1 - p_t^2)^{\frac{1}{2}}\right] \quad , \quad (6)$$

which defines new orbits in the phase space E, ϕ. Let us consider two positrons of the same energy γ_o, one moving on the axis and one leaving the target with a transverse momentum p_{to}. Let ϕ_{o1} be the entrance phase for first one and $\phi_1 = \phi_{o1} - \delta\phi$ the phase for the second one, where

$$\delta\phi = \Delta\phi\ (\gamma_o, 0) - \Delta\phi\ (\gamma_o, p_{to}),$$

where $\Delta\phi$ is given by Eq. 3. In the approximation of small angle of emission, one can write

$$\delta\phi = \frac{\pi}{\lambda} \frac{p_{to}^2}{\gamma_o^2} \frac{1}{B_o} \int_o^D B\, dz \quad ,$$

which can also be written

$$\delta\phi = \frac{\pi}{\lambda} \frac{p_t^2}{\gamma_o^2} \cdot K \quad ,$$

where p_t is the transverse momentum in the section and

$$K = \frac{1}{B_2} \int_0^D B_{(z)} dz$$

B_2 being the magnetic field in the section, which is supposed to be constant. Equation 6 can be approximated by:

$$\cos \phi_o - \cos \phi_\infty = \frac{2\pi}{\alpha} \left[\gamma_o - (\gamma_o^2 - 1)^{\frac{1}{2}} + \frac{1}{2} \frac{p_t^2}{(\gamma_o^2 - 1)^{\frac{1}{2}}} \right]. \quad (6b)$$

Using Eqs. 4 and 6b together with the expression for $\delta\phi$, the dispersion in asymptotic phase is given by:

$$\delta\phi_\infty \sin \phi_\infty = \left[\frac{\pi}{\lambda} \frac{1}{\gamma_o^2} K \sin \phi_{o1} - \frac{\pi}{\alpha} \frac{1}{(\gamma_o^2 - 1)^{\frac{1}{2}}} \right] p_t^2 \quad . \quad (7)$$

It is seen that one can choose the accelerating field and input phase, for a given energy, to make the asymptotic phase independent of the transverse momentum.

It is evident from Eq. 7 that ϕ_{o1} must be positive, i.e., the beam must be decelerated. The accelerating field E' and input phase are simply related to the energy E_o by:

$$E' \sin \phi_{o1} \approx \frac{E_o}{K}$$

In the case of SLAC, K is of the order of 0.45 m. For E_o = 2 MeV, this gives $E' \sin \phi_{o1} \approx$ 4.4 MeV/m. The equations of motion have been integrated over a distance of 3 m for E_o = 2 MeV, E' = 10 MeV/m, p_{to} = 0 to 0.6 MeV/c, and ϕ_{o1} = 25° (ϕ_o in Eq. (5) ≃ 40°). The resulting phase dispersion is less than 1°, for an input dispersion of 15°.

This bunching process could be very useful in the case of a high transverse acceptance over a narrow energy band centered on a low energy (use of a quarter wave transformer).

It is concluded that by first decelerating the positron beam, one can take advantage of using very low energy and high acceptance (in this case the angular acceptance at 2 MeV is 18°) together with large energy band, leading to a maximum current of positrons within a narrow energy spectrum. The limitation may be the initial energy dispersion in the case of a low final energy.

Results of Calculations in the Case of SLAC

The first section is a very short one, followed by a drift space (Fig. 1). With an accelerating field of the order of 10 MeV/m, it is evident that one cannot take full advantage of the bunching process described above, mainly because the minimum energy at the end of the first section must be high enough to avoid a debunching effect over the second drift space. The longitudinal motion was computed for several values of the parameter ϕ_o (see Eq. 5) and the accelerating field. Figure 5 shows the resulting phase space at the entrance of the second section in the cases ϕ_o = 70° and 30° (beam initially decelerated) and ϕ_o = −70° (beam accelerated), and a field of 10 MeV/m. For each case, the two solid curves with cross-hatching between them represent the phase space for positrons with initial transverse momenta between 0 and 0.6 MeV/c. The effect of the second drift space is clearly seen in the case ϕ_o = 70°: the minimum energy is about 1 MeV and the phase-space is completely distorted. In the case ϕ_o = 30° the minimum energy is 2.5 MeV and the distortion of the phase space is much less important. The orbits for ϕ_∞ = −40° and ϕ_∞ = −50° are shown on the same scale for an accelerating field of 10 MeV/m (the second section is long enough to consider at a first approximation the asymptotic behavior). The fitting between these orbits and the phase-space of the

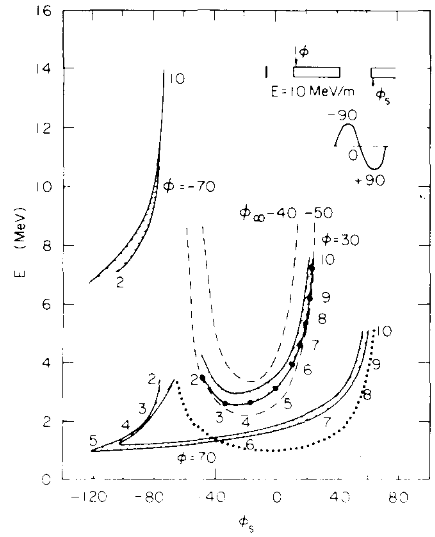

Fig. 5 Longitudinal phase-space at entrance to second section, for three phases entering short section: Φ_o = 70°, Φ_o = +70° and the optimum Φ_o = +30°.

beam is reasonably good and one can expect a total bunch length of less than 10°. In addition, it can be seen that the bunching of low energy positrons with transverse momentum from 0 to 0.6 MeV/c has been achieved in the short section.

In the case ϕ_o = −70° (beam accelerated), no bunching effect has occurred; the total bunch length is 46° and no further bunching can occur in the second section. The phase space at the end of the second section has been computed for various values of the accelerating field, input phase and different phase space configurations resulting from the first section. Figure 6 shows, on the

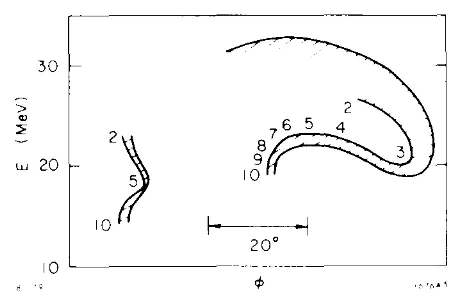

Fig. 6 Phase space at output of second section;
Left: New section in use.
Right: Without new section.

left, the result in the case of the phase-space shown in Fig. 5 for Φ_o = 30°. The input phase for the second section is ϕ'_o = 50° (phase of the field for a β = 1 particle). The average output phase is −45°, giving a total phase slippage for the beam of about 95°. The bunch length is 6° and the energy dispersion 10.5 MeV. Shorter bunches can be obtained at the expense of final energy and energy dispersion. The transition between the solenoid and the quadrupole must take place at an energy of at least 55 MeV in order to keep an admittance of 0.15 π MeV/c-cm[1]. With the phase-space of Fig. 6, adding a 3 m-long section with the same field gives an energy from 44 MeV to 54 MeV. Studies have been made of two ways of increasing the energy without perturbing the bunching process, for the same accelerating field:

- Detuning of three cavities of the long section at 1.2 m from the rf input (giving a phase shift of 20° to 45°) increases the output energy by about 5 MeV, giving a bunch length of 7°.
- Lowering the temperature of the section by 4°C, to increase the phase velocity, gives approximately the same result.

These techniques are possible because the bunching process takes place mainly in the first meter of the structure, after which the phase of the field remains around -45°.

Remarks on the Calculations

1. It is known [5] that during the bunching of electrons, the space harmonics of the field can lead to a broadening of the bunch width. The equation of motion has been integrated, taking into account the space harmonics n = 1 and n = -1, whose amplitudes for a SLAC structure are $E_1 = -0.077$, $E_{-1} = 0.454$ for an amplitude of the fundamental $E_o = 1$. Even in the case of a strongly decelerated positron, the changes in phase and energy were found to be not significant within the accuracy needed.

2. Influence of the E_r and B_ϕ components of the field

(a) Transverse motion - During their motion with respect to the wave, in a region where $\partial E/\partial z < 0$, the positrons will experience a defocusing force due to the E_r and B_ϕ components of the field. The radial equation of motion can be written [6]

$$\frac{d}{dz}\left(\beta\gamma\frac{dr}{dz}\right) = r\left[\frac{\pi}{\lambda}\frac{E_o}{\beta^2\gamma^2}\cos\phi - \frac{B^2_{ext}}{4\beta\gamma}\right] , \quad (8)$$

where r = distance from the axis, E_o = accelerating field in units of 0.511 MeV/cm, and B_{ext} is the dc magnetic field in units of $mc^2/e/cm$ (= 1704 Gauss). The first term, which is due to the rf fields, can have a magnitude about 1/4 the magnitude of the second term for distances of a few centimeters, when a low energy positron passes through zero phase.

(b) Longitudinal Motion - When the bunching of positrons of the same energy and different transverse momentum was studied, the effect of the axial force resulting from the interaction of radial velocity and azimuthal magnetic field was neglected. The complete equation of motion can be written in the case of the fundamental alone [7] :

$$d\gamma = -\alpha\left[\sin\phi - \frac{2\pi}{\lambda}\frac{r\, p_r}{\beta\gamma}\cos\phi\right] dz \quad . \quad (9)$$

This equation has been integrated in the case mentioned before (E_o = 2 MeV, E' = 10 MeV/m, ϕ_o = 25°, p_{to} = 0 to 0.6 MeV/c) for various values of the initial azimuth angle. It was found that the maximum output phase variation is about 2°.

Comparison with the Present Mode of Operation

In the present mode of operation, the short section is not used. The rf phases of the linac are not changed from electron to positron acceleration. The positron beam is then decelerated in the 3-m long section. The phase-space at the end of the section was computed for different values of the input phase and field which give a total phase slippage for the beam of about 180°. Figure 6 shows the resulting phase-space for a field of 10 MeV/m and an input phase of Φ_o = 80°. Positrons with energies from 4.5 MeV to 10 MeV are bunched within approximately 10°. Positrons from 2 to 4 MeV are far away in phase and will not contribute to the useful beam (within 1% energy bin).

Results of Experiment

The short section was used in June 1979 for an experiment with positron beams of final energy 1.8 GeV and 10 GeV. The analyzed current within 1% was increased in both cases by at least a factor of two with respect to the usual current. The optimum shift for the rf downstream of the target, versus electron operation, was found to be about 80°, giving a phase slippage for the beam of about 100°. These results are in good agreement with the calculated values. The short section will be put into normal operation next October, with 75° pulsed phase shifters for the kylstrons upstream the converter.

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