

MATCHING THE RF QUADRUPOLE BEAM TO THE DRIFT TUBE SECTION IN THE FMIT ACCELERATOR\*

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Summary

The beam produced by the Fusion Materials Irradiation Test (FMIT) radio frequency quadrupole (RFQ) accelerating structure must be matched to the drift-tube linac (DTL) structure that follows. Because minimum beam spill is a primary concern, a matching criterion that considers the beam edges as well as the rms properties is needed.

We flared the RFQ's vanes and adjusted the strengths of the first four quadrupoles in the DTL to achieve optimum performance downstream. Numerical methods used to set the quad gradients, and some experience with various matching criteria are described. The match achieved is compared with matches obtained by other methods.

Introduction

The deuteron beam emerging from the FMIT RFQ must be matched to the following drift-tube section. To save space the match is accomplished by using the first four quadrupoles of the drift-tube section; no attempt is made to improve the longitudinal match. Beam spill is of overriding importance in FMIT because hands-on maintenance is desired. Hence the quality of the match is highly important. The RFQ output beam is not inherently matched to the drift-tube section for two reasons. First, the focusing period in the RFQ is  $\beta\lambda$ , while it is  $2\beta\lambda$  in the drift-tube sections. Second, the transverse beam dimensions in the RFQ cannot be expected to be those that produce the best beam in the accelerator's downstream section.

The FMIT DTL design is based on an assumed RFQ output beam having the nominal parameters: 2-MeV energy; 0.006 cm·mrad unnormalized transverse total emittance, in both x and y;  $\pm 25^\circ$  phase spread; and 0.2-MeV energy spread. The synchronous phase is tapered from  $-40^\circ$  to  $-30^\circ$  in the first 25 cells, then held constant. The accelerating gradient is constant at 1.4 MV/m.

Perhaps the most usual matching method would be to match the rms Courant-Snyder  $\alpha$ 's and  $\beta$ 's to the requirements of the DTL at a particular point. This was done, but the total beam size obtained was unsatisfactorily large. Using more information about beam performance over a whole section of the downstream DTL resulted in a much smoother match. Also, by including maximum beam size as part of the criterion, we were able, in simulation studies, to achieve a smaller maximum-beam size with little change in the rms properties. The procedure and some observations on experience with it will be presented below.

The Procedure

Of course, the first thing to do is to properly define the "output" beam from the RFQ. This

beam should not be the RFQ's entire output but only the part that represents the part of the beam that is "good." We exclude any particles with energy too far off the synchronous energy; that is, particles with no chance to transverse the DTL.

We took the basic assumption that any beam property chosen to make the match must experience a smooth transition, without wide excursions, from the entrance of the DTL to the point in the DTL where the matching may be considered complete. The criterion for matching was based on a group of beam properties in the cells downstream from the matching quads.

A least-squares optimization approach is used. Let there be  $m$  properties in a given cell that will be used to determine the match, and  $n$  downstream cells in which these  $m$  properties are to be optimized. We then have  $mn$  properties,  $P_{(\mu\nu)} = P_{ij}$ ,  $1 \leq i \leq mn$ , to optimize. Select a smooth function  $f_{\mu}(\nu)$ , ( $1 \leq \mu \leq m$ ,  $1 \leq \nu \leq n$ ), to which the  $\mu$ th beam property is to be fitted. Only the general form of this function is selected, (for example, linear). Its parameters will be determined by the  $n$  values of the  $\mu$ th property and the function so determined then will be used to determine the desired values of  $P_{(\mu\nu)}$ . Thus we first pass the beam through the matching section and through the  $n$  downstream cells to find the  $mn$  properties,  $P_{(\mu\nu)}$ , and the least-squares fit to the target functions  $f_{\mu}(\nu)$ . From this we derive a set of  $mn$  deviations of the properties from their desired values,  $\delta_i = P_{ij} - f_{\mu}(\nu)$ . Let  $\omega_j$  be the weight to be given to the  $\mu$ th property in the  $\nu$ th cell.

We define the vector  $d$  by

$$d^T = (\omega_1 \delta_1, \dots, \omega_j \delta_j, \dots, \omega_{mn} \delta_{mn}) \quad .$$

The beam is then passed through the system repeatedly, each time with a predetermined change in one of the first  $m$  quadrupoles in the matching section, to get the partial derivatives.

$$a'_{ij} = \frac{\Delta P_{ij}}{\Delta Q_j} \approx \frac{\partial P_{ij}}{\partial Q_j} \quad ; \quad 1 \leq j \leq m, \quad 1 \leq i \leq mn$$

and  $a_{ij} = \omega_j a'_{ij}$ , where  $\omega_j$  is the weight to be given the  $P_{ij}$  property. Now define the vector of quadrupole gradient changes by  $\Delta Q^T = (\Delta Q_1, \dots, \Delta Q_m)$  and the  $mn \times m$  matrix  $A = (a_{ij})$ . Then we can solve the matrix equation  $A \Delta Q = d$  for the required quadrupole changes in a least-squares sense.

After solution for  $\Delta Q$ , new values of the first  $m$  quads are obtained and the entire process is repeated. The iteration continues until suitable convergence is obtained.

The above completes the procedure for finding the input DTL quadrupole settings. However, more

\*Work supported by the US Department of Energy.

was done to accomplish the match with the RFQ. We found that some of the matching section quadrupole strengths were higher than desired when the RFQ maintains a constant dimensionless focusing strength,  $B$ , in its accelerator section.<sup>1</sup> We knew that an increase in beam size between the RFQ and the DTL was required, because of the doubling in the transverse periodicity. To accommodate some of this before the matching quads, we flared the accelerating section of the RFQ slightly and found that the required gradients in the DTL matching quads decreased. Such flaring has become part of the present procedure. Figure 1 shows a plot of  $B$  versus the DTL matching quad strengths.

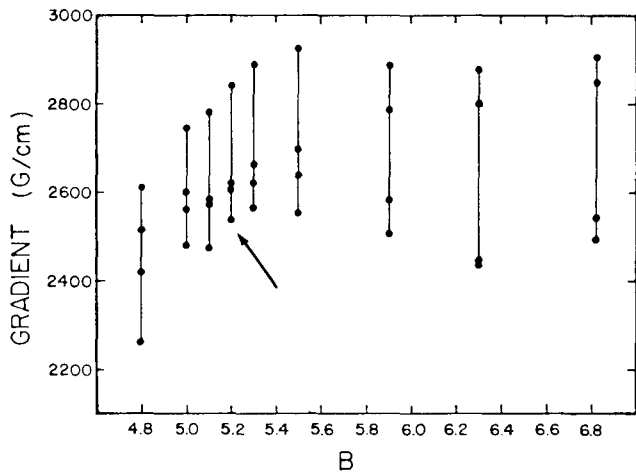


Fig. 1. Plot of range of matching quadrupole gradients versus the dimensionless focusing strength,  $B$ .  $B = 5.2$  was chosen.

Choice of Fitting Functions and Weights

The chosen set of matching properties consisted of rms waist, rms bust, maximum waist, and maximum bust in 27 cells downstream of the four matching quads. This corresponds to one property for each transverse coordinate; by adjusting the weights assigned to the various properties, the influence of any one property can be emphasized.

Several functions were tried to fit the beam properties. The first was a straight line. A straight line violates the smooth change criterion at the end of the set of cells whose properties determine the match and, although this discontinuity should be small, it appeared to be sufficient to affect the beam quality downstream. An exponential taper failed because of numerical difficulties evaluating exponentials with large negative arguments. The final choice, which produced a smooth beam, was a parabola  $f_{\mu} = a + bz + cz^2$ .

Any chosen beam-property function can be made to "push" on the beam by the proper choice of its constants during the iterations. Thus, if we want to make the beam smaller we multiply the beam-size function constants from each pass (by a number <1) before using them for the next pass; we used this method in making the match. The multiplying number

was chosen as  $t = 1/(1 + k/I)$ , where  $I$  is the number of the iteration and  $k$  a constant entered into the code as input data. ( $k = 0.1$  is good for quick movement in early stages,  $k = 0.001$  is sometimes used if the beam is thought to be close to the minimum size.)

The choice of  $n$ , the number of cells in which criteria are to be met, was also examined over a range of  $n = 7$  to  $n = 47$ . In general, the more cells used the better the match, as judged by the beam profile; however, the improvement obtained by using more than 27 cells was not great. Because the code running time is a linear function of the value of  $n$ , we chose  $n = 27$ .

Weighting was done so that

$$(\omega) \left( \frac{\overline{W}_{\max} + \overline{B}_{\max}}{\overline{W}_{\text{ave}} + \overline{B}_{\text{ave}}} \right)$$

was between 0.75 and 1.1. ( $W =$  waist,  $B =$  bust). Waists and busts were additionally weighted so that each had an equal effect on the problem.

Experience with the Technique

We compared several possible ways of making the match between the RFQ and the DTL. These cases are described below and summarized in Table I. For brevity in the description and in the table we make use of the following definitions.

RFQ Input to the DTL refers to a standard set of 5272 particles generated by passing an initial set of 6000 particles through a numerical simulation of the FMIT RFQ. Particles emerging from the RFQ with energies  $>0.09$  MeV below the synchronous energy were excluded. This limit was specifically chosen so that no particles would be lost during the matching. This makes the following comparisons more straightforward. (In our actual FMIT design, we have used a somewhat more elaborate criterion.)

Uniform Input refers to a set of random particles generated by PARMILA uniformly in phase space to have the same rms properties as the RFQ beam.

Cases IA and IB: In Case IA the RFQ input was used and the match was determined using rms waist, rms bust, maximum waist, and maximum bust in 27 downstream cells. Case IB was the same except that a weight of zero was assigned to the maximum values.

Cases IIA and IIB: These two cases parallel Cases IA and IB. A uniform input was used instead of RFQ input.

Case III: RFQ input was used to find quadrupole settings that would result in matched values of  $\alpha_x, \beta_x, \alpha_y, \beta_y$  at the end of Cell 4 of the DTL. With intense beams, it is not trivial to determine the ellipse parameters precisely. We started with  $\alpha$ 's = 0 and calculated  $\beta$  values from the envelope equations; then adjusted the  $\beta$ 's by trial and error.

Case IV: This is the parallel case to Case III. Uniform input was used to find the match to the same  $\alpha$ 's and  $\beta$ 's as Case III.

Table I

DESCRIPTION OF MATCHING CASES

Case	Input	Max Weight	Match To
IA	RFQ	1	27 cells
IB	RFQ	0	27 cells
IIA	Uniform	1	27 cells
IIB	Uniform	0	27 cells
III	RFQ	-	$\alpha$ 's, $\beta$ 's
IV	Uniform	-	$\alpha$ 's, $\beta$ 's
VA	Uniform + extra	1	27 cells
VB	Uniform + extra	0	27 cells

Cases VA and VB: These two cases were used to examine the effect of weighting on a few particles placed well outside the bunch transversely. A uniform input was used, with matching to the rms and maximum busts and waists in 27 downstream cells.

All cases represent possible methods of matching. Case IV is perhaps a standard technique.

We used two criteria to judge the quality of the match. First, the beam must be "smooth," without large changes in size in or after the matching section. Smoothness can be judged by looking at profiles and can be quantized by calculating the standard deviation ( $\sigma$ ) of the maximum and rms radii of the beam as it passes through all the cells of the DTL. The second criterion used was beam size. This is quantized by calculating the average maximum radius,  $r_{max}$ , and the average rms radius,  $r_{rms}$ .

For direct comparison of Cases I-IV, we passed the RFQ input beam through the entire DTL and calculated those quantities. In Cases VA and VB, where we were interested only in seeing the effect of weighting the maximums, we used a uniform input supplemented by halo particles.

Table II summarizes the results obtained for the eight cases.

Table II

COMPARISONS OF MATCHING CASES

CASE	IA	IB	IIA	IIB
$r_{max}$	1.2069	1.2047	1.5507	1.5745
$\sigma(r_{max})$	0.1034	0.1024	0.2587	0.2774
$r_{rms}$	0.00472	0.00472	0.00510	0.00515
$\sigma(r_{rms})$	0.000598	0.000600	0.000769	0.000829

CASE	III	IV	VA*	VB**
$r_{max}$	1.3982	1.5681	1.5543	1.1195
$\sigma(r_{max})$	0.2304	0.2978	0.3393	0.2671
$r_{rms}$	0.00481	0.00501	0.01226	0.01177
$\sigma(r_{rms})$	0.000692	0.000752	0.001610	0.00150

\*Cases VA and VB can be compared only with each other.

\*\*Particles lost in DTL.

Cases IA and IB are clearly the best of the six comparable input cases. There is negligible difference between Case IA and IB. Our experience suggests that this is due to the deliberate exclusion of particles with energies  $>0.09$  MeV below synchronous energy. When particles of slightly lower energy are kept, the moderate use of a maximum weight seems to give a smaller  $r_{max}$  than the use of the rms values only.

Comparison of Cases IIA and IIB shows that when uniform input is matched to 27 downstream cells, there is some slight advantage to using the maximum values as well as the rms values. Obviously a uniform approximation to the RFQ input gives neither as small nor as smooth a beam as true RFQ input. This is true even when the match is made to the  $\alpha$ 's and  $\beta$ 's needed at Cell 4, as seen by comparing Cases III and IV; therefore, the distribution shape can be important in achieving the best match.

Comparison of either IA or IB to Case III shows that matching to a single set of  $\alpha$ 's and  $\beta$ 's results in a beam substantially larger in overall size and somewhat larger in rms size. It is also not nearly as smooth.

The effect of using or not using the maximum values when particles are well outside the expected input is examined in Cases VA and VB. From Table II, it appears that using only the rms values (VB) produces a smaller and smoother beam. However, in this case, the "halo" we introduced was lost in the DTL; whereas, when the maxima were used in determining the match, these halo particles were transported through. It has been noted at LAMPF that rms properties alone are insufficient to achieve the lowest beam loss; because this technique handles halo particles, it may prove useful, provided adequate measurements on the DTL can be made.

Conclusions

We have described the least-squares method we used to match the FMIT RFQ beam to the DTL. We have compared this method with other methods that use approximate particle distributions, or less information, and we have shown that our method results in a smaller, smoother beam when halo particles are ignored. We have shown that the method is capable of providing a match that may allow halo particles to traverse the DTL.

The most convenient feature of our method is that it is unnecessary to determine, a priori, either the priorities of the RFQ input beam or the necessary properties of a good DTL beam to make the match because the determination of these properties is implicit in the method.

References

1. K. R. Crandall, R. H. Stokes, T. P. Wangler, "RF Quadrupole Beam-Dynamics Design Studies", Proc. 10th Linear Accelerator Conf., Montauk, New York, September 10-14, 1979, Brookhaven National Laboratory report BNL-51134, p. 205 (1980).