A VARIABLE STRENGTH PERMANENT MAGNET DIPOLE

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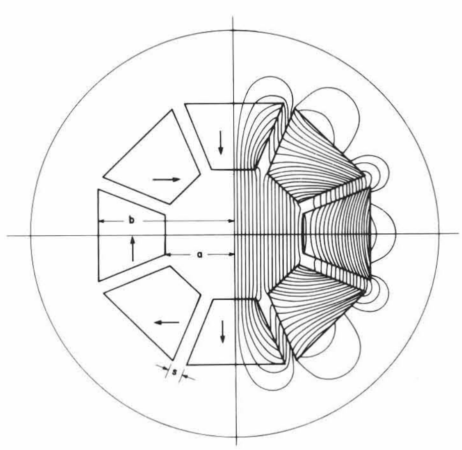
Abstract

The design strategy that has been successfully applied to segmented permanent magnet quadrupole lenses can equally well be used to produce dipole field configurations [1]. The use of homogenizing spaces between segments to produce beam optics quality fields over 80% of the magnet gap is described. The use of counter rotating dipole rings to vary the dipole bending strength and the effects of fringing fields are described.

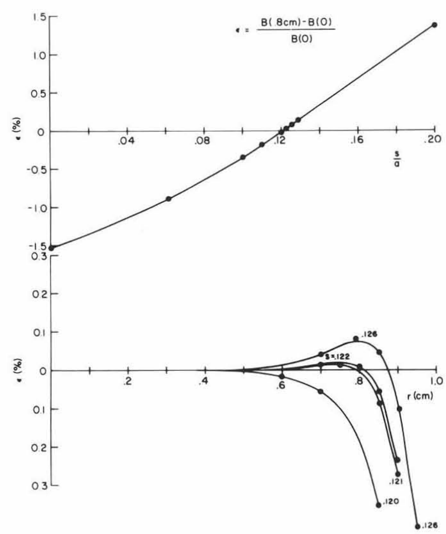
Dipole Description

A permanent magnet dipole is diagrammed in Figure 1. It takes the form of a ring assembled from eight trapezoidal samarium-cobalt pieces with magnetic vectors oriented so as to lead the return flux around the aperture almost entirely within the permanent magnet material. The central field is an increasing function of b/a and can be as much as 10 kilogauss. The dipole field uniformity can be maximized by incorporating homogenizing gaps (s in Figure 1) between the magnet segments as suggested by Halbach [2]. We will use a dipole with a = 1cm, b = 2cm and a field strength of 5 kilogauss as an example. Figure 2, top, shows the error at 80% of the aperture for values of s to 0.2cm. The bottom graph gives the radial field error for values of s near the optimum, showing a possible uniformity better than 10^{-3} for 85% of the aperture.



DIPOLE MAGNET CONFIGURATION AND FLUX PATH DIAGRAM

Figure 1



THE EFFECT OF THE HOMOGENIZING GAP, s, ON MIDPLANE FIELD UNIFORMITY, ε

Figure 2

Adjustable Strength Dipole Characteristics

The effective strength of a permanent magnet dipole for particle beam steering can be varied in a manner similar to that used for adjustable quadrupoles. This is done by counter-rotating successive rings so that there is a reduction, approximately proportional to the cosine of the rotation angle, of the net impulse received by the particles as the beam traverses the dipole. Figure 3 illustrates the configuration in which three identical dipoles are placed in series with the length of the center ring equal to twice the length of each end ring. By rotating the rings around the beam axis in the rotation senses shown, the effective bending strength of the dipole is reduced with no vertical displacement or impulse.

The vector diagram in Figure 4 shows the orientation of each dipole field, $\underline{B}_i$, and the resulting impulse, $\underline{J}_i$, on a particle beam resulting from each ring. The net impulse is reduced as the angle α is increased.

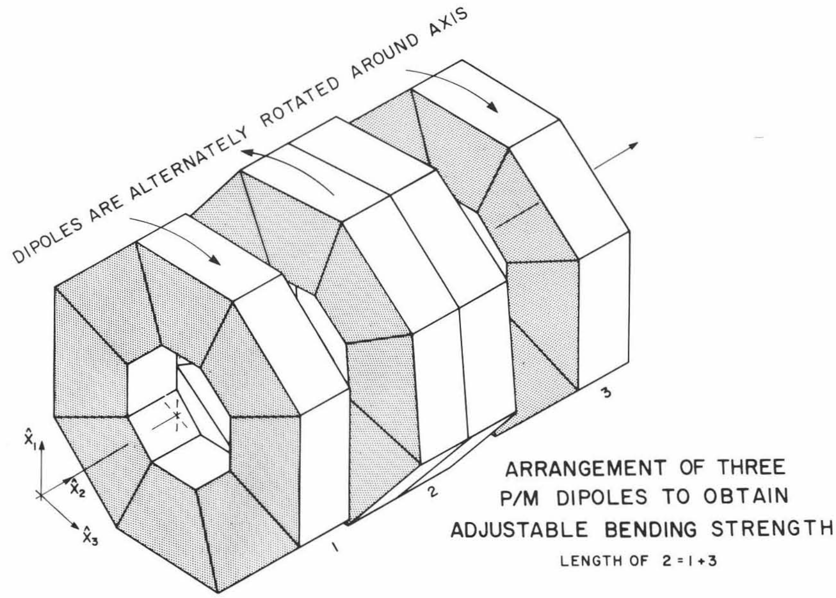


Figure 3

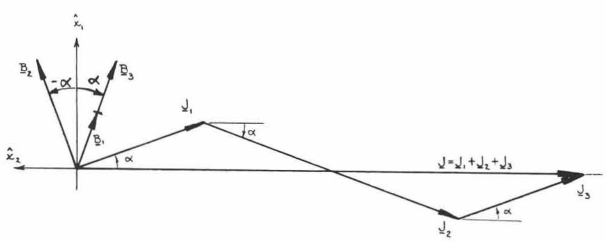


Figure 4

Beam Transport Through Three Dipoles in Series

Following the particle trajectory through each dipole, the entrance parameters of the downstream dipole are made equal to the exit parameters of the upstream one. Fringe field effects will be considered below. We will consider each ring separately.

For a proton passing through a dipole oriented along the x_1 axis, the momentum vector $\underline{p}$ is rotated through an angle ωt where ω is the cyclotron frequency and t is the time required to pass through the dipole. The new momentum vector is given by

$$\underline{p}_1 = \underline{p}_0 \bar{\underline{R}}_0 \qquad \bar{\underline{R}}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega t & -\sin\omega t \\ 0 & \sin\omega t & \cos\omega t \end{bmatrix}$$

The displacement of the particle is given by

$$\Delta\underline{r} = \frac{1}{m}\int_t \underline{p}\, dt = \underline{p}_0 \frac{1}{m} \int_t \bar{\underline{R}}_0\, dt = \underline{p}_0 \int_t \bar{\underline{Q}}_0\, dt$$

If the dipole is rotated around $\hat{x}_3$ by an amount α, a new rotation matrix results:

$$\bar{\underline{R}}_1 = \tilde{\underline{A}}\, \bar{\underline{R}}_0\, \bar{\underline{A}} \quad \text{where} \quad \bar{\underline{A}} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And $\tilde{\underline{A}}$ is the transpose of $\bar{\underline{A}}$. Now

$$\underline{p}_1 = \underline{p}_0 \bar{\underline{R}}_1 = \underline{p}_0 \tilde{\underline{A}}\, \bar{\underline{R}}_0 \bar{\underline{A}} \quad \text{and} \quad \Delta\underline{r}_1 = \underline{p}_0 \frac{1}{m} \tilde{\underline{A}} \left(\int_t \bar{\underline{R}}_0\, dt\right) \bar{\underline{A}} = \underline{p}_0 \bar{\underline{Q}}_1$$

The second dipole is twice as long (so that the particle stays in the field for a time t') and is rotated about $\hat{x}_3$ in the opposite sense:

$$\underline{p}_2 = \underline{p}_1 \bar{\underline{R}}_2 = \underline{p}_1 \bar{\underline{A}}\, \bar{\underline{R}}_0'\, \tilde{\underline{A}} \quad \text{where} \quad \bar{\underline{R}}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega t' & -\sin\omega t' \\ 0 & \sin\omega t' & \cos\omega t' \end{bmatrix}$$

$$\Delta\underline{r}_2 = \underline{p}_1 \frac{1}{m} \bar{\underline{A}} \left(\int_{t'} \bar{\underline{R}}_0'\, dt\right) \tilde{\underline{A}} = \underline{p}_1 \bar{\underline{Q}}_2$$

Finally the third dipole is identical to the first so that

$$\underline{p}_3 = \underline{p}_2 \bar{\underline{R}}_1 \quad \text{and} \quad \Delta r_3 = \underline{p}_2 \bar{\underline{Q}}_1$$

The integrations are easily done to yield

$$\int_t \bar{\underline{R}}_0\, dt = \frac{1}{\omega} \begin{bmatrix} \omega t & 0 & 0 \\ 0 & \sin\omega t & 1-\cos\omega t \\ 0 & \cos\omega t - 1 & \sin\omega t \end{bmatrix}$$

$$\int_{t'} \bar{\underline{R}}_0'\, dt = \frac{1}{\omega} \begin{bmatrix} \omega t & 0 & 0 \\ 0 & \sin\omega t' & 1-\cos\omega t' \\ 0 & \cos\omega t' - 1 & \sin\omega t' \end{bmatrix}$$

The rotations are applied to give

$$\bar{\underline{R}}_1 = \begin{bmatrix} \cos^2\alpha + \cos\omega t \sin^2\alpha & \cos\alpha\sin\alpha(\cos\omega t - 1) & -\sin\omega t \sin\alpha \\ \cos\alpha\sin\alpha(\cos\omega t - 1) & \sin^2\alpha + \cos^2\alpha\cos\omega t & -\sin\omega t\cos\alpha \\ \sin\omega t \sin\alpha & \sin\omega t \cos\alpha & \cos\omega t \end{bmatrix}$$

$$\bar{\underline{R}}_2 = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha\cos\omega t' & \sin\alpha\cos\alpha(1 - \cos\omega t') & \sin\alpha\sin\omega t' \\ \sin\alpha\cos\alpha(1 - \cos\omega t') & \sin^2\alpha + \cos^2\alpha\cos\omega t' & -\cos\alpha\sin\omega t' \\ -\sin\alpha\sin\omega t' & \cos\alpha\sin\omega t' & \cos\omega t' \end{bmatrix}$$

$$\bar{\underline{Q}}_1 = \frac{1}{m\omega} \begin{bmatrix} \omega t\cos^2\alpha + \sin\omega t\sin^2\alpha & \sin\alpha\cos\alpha(\sin\omega t - \omega t) & \sin\alpha(1 - \cos\omega t) \\ \sin\alpha\cos\alpha(\omega t - \sin\omega t) & \omega t\sin^2\alpha + \sin\omega t\cos^2\alpha & \cos\alpha(1 - \cos\omega t) \\ \sin\alpha(\cos\omega t - 1) & \cos\alpha(\cos\omega t - 1) & \sin\omega t \end{bmatrix}$$

$$\bar{\underline{Q}}_2 = \frac{1}{m\omega} \begin{bmatrix} \omega t'\cos^2\alpha + \sin\omega t'\sin^2\alpha & \sin\alpha\cos\alpha(\omega t' - \sin\omega t') & \sin\alpha(\cos\omega t' - 1) \\ \sin\alpha\cos\alpha(\omega t' - \sin\omega t') & \omega t'\sin^2\alpha + \sin\omega t'\cos^2\alpha & \cos\alpha(1 - \cos\omega t') \\ \sin\alpha(1 - \cos\omega t') & \cos\alpha(\cos\omega t' - 1) & \sin\omega t' \end{bmatrix}$$

The exact phase space equations of motion are

$$\underline{p}_1 = \underline{p}_0 \bar{\underline{R}}_1 \qquad \underline{r}_1 = \underline{r}_0 + \underline{p}_0 \bar{\underline{Q}}_1$$

$$\underline{p}_2 = \underline{p}_1 \bar{\underline{R}}_2 \qquad \underline{r}_2 = \underline{r}_1 + \underline{p}_1 \bar{\underline{Q}}_2$$

$$\underline{p}_3 = \underline{p}_2 \bar{\underline{R}}_1 \qquad \underline{r}_3 = \underline{r}_2 + \underline{p}_2 \bar{\underline{Q}}_1$$

Fringe Field Effects

The forces due to fringing fields on a particle traversing a dipole aligned along the $\hat{x}_1$ axis can be represented by:

$$\underline{\dot{p}} = \frac{e}{m}\,\underline{p}\,\underline{\bar{f}} \qquad \underline{\bar{f}} = \begin{bmatrix} 0 & f_3 & -f_2 \\ -f_3 & 0 & f_1 \\ f_2 & -f_1 & 0 \end{bmatrix}$$

The three independent matrix elements are, to second order,*

$$f_1 = -\frac{(x_1^2 + x_2^2)}{8}\frac{d^2B}{dx_3^2} \qquad f_2 = \frac{x_1 x_2}{4}\frac{d^2B}{dx_3^2} \qquad f_3 = x_1 \frac{dB}{dx_3}$$

To get the impulse due to the fringe fields integrate over the time required to go from the uniform field part of the magnet to infinity:

$$\underline{J} = \frac{e}{m}\int_0^\infty \underline{p}\,\underline{\bar{f}}\,dt$$

Assuming the fringe region to be sufficiently short so that $\underline{p}$ is not deflected significantly from along the x_3 axis gives

$$\underline{J} = \frac{e}{m}\,\underline{p}\int_0^\infty \underline{\bar{f}}\,dt = \frac{e}{mv}\,\underline{p}\int_0^\infty \underline{\bar{f}}\,dx_3$$

$$\underline{J} = \frac{e}{mv}\,\underline{p}\begin{bmatrix} 0 & -x_1 B & 0 \\ x_1 B & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{J} = \frac{e x_1 B}{mv}\,[\,p_2, -p_1, 0\,]$$

For 10MeV protons 1 cm above the axis of a 5 Kg dipole and moving at an angle of 6° from the dipole axis, this gives a deflection of about 1 milliradian. For a particle entering the dipole, the limits of integration change:

$$\underline{J} = \frac{e}{m}\int_{-\infty}^0 \underline{p}\,\underline{\bar{f}}\,dt = \frac{-e x_1 B}{mv}\,[\,p_2, -p_1, 0]$$

For a dipole oriented at an angle α to the x_1 axis,

$$\underline{J} = \frac{eB}{m}(x_1\cos\alpha + x_2\sin\alpha)\,\underline{p}\,\tilde{\underline{A}}\,\underline{\epsilon}\,\underline{A} \qquad \underline{\epsilon} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix $\underline{\epsilon}$ has the property $\tilde{\underline{A}}\,\underline{\epsilon}\,\underline{A} = \underline{\epsilon}$. Therefore

$$\underline{J} = \frac{eB}{m}(x_1\cos\alpha + x_2\sin\alpha)\,[\,p_2, -p_1, 0]$$

For a rotation in the opposite sense

$$\underline{J} = \frac{eB}{m}(x_1\cos\alpha - x_2\sin\alpha)\,[\,p_2, -p_1, 0\,]$$

Thus, if the particle leaves a dipole rotated by $+\alpha$ and enters a similar dipole oriented at $-\alpha$, the total impulse is

$$\underline{J} = \underline{J}_+ + \underline{J}_- = \left(\frac{eB}{m}(x_1\cos\alpha + x_2\sin\alpha) - \frac{eB}{m}(x_1\cos\alpha - x_2\sin\alpha)\right)[\,p_2, -p_1, 0]$$

$$\underline{J} = \frac{2eB}{m}x_2\sin\alpha\,[\,p_2, -p_1, 0\,]$$

*Adapted from an expression for the field values in the permanent magnet dipole geometry developed by R. Gluckstern for NEN

Conclusion and Design Example

An adjustable permanent magnet dipole can be constructed from three counter-rotating sections, the middle section being twice as long axially as the first and last sections to make t' = 2t. To first order, assuming both the deflection in the first section (ωt) and rotation (α) angles are small, a beam entering the dipole on axis with a momentum of [0, 0, p] exits on axis with a momentum of

$$\underline{p} \simeq [0,\, 4p\cos\alpha\,\sin\omega t,\, p]$$

the characteristic time (t) can be calculated using the formula

$$t = \frac{mL}{p}$$

where L is the axial length of the first section. For a beam entering off axis, the fringe fields introduce some aberrations. The following table illustrates the magnitude of this effect.

BEAM PARAMETERS	MAGNET PARAMETERS
$\underline{r}_o$ = [1 cm, 1cm, 0]	B = 0.5T
$\underline{p}_o$ = [0, 0, $\sqrt{2mK}$]	α = 0.1 radian
K = 10 MeV protons	Total length = 36 cm

Beam Deflection = 23°

Fringe field aberrations = 4.3 milliradian

ACKNOWLEDGMENT

We wish to acknowledge the contributions at New England Nuclear of Ronald Holsinger* and Robert Lown** to the initial development of a permanent magnet dipole. The work reported here is derived from the fundamental magnetic analysis of Dr. Robert Gluckstern*** and the original contributions of Klaus Halbach [2,3].

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