Proceedings of the 1981 Linear Accelerator Conference, Santa Fe, New Mexico, USA

COMPUTER SIMULATION OF LONGITUDINAL-TRANSVERSE SPACE CHARGE EFFECTS IN BUNCHED BEAMS*

I. Hofmann and I. Bozsik
Max-Planck-Institut für Plasmaphysik, D-8046 Garching, Germany

Summary

A newly developed 2 1/2 D particle-in-cell code with r-z geometry has been applied to derive criteria for longitudinal-transverse emittance transfer in ellipsoidal bunched beam with strong space charge. The main result is that emittance transfer occurs only if σ_ℓ/σ_t is near or above 1.5, in which case equidistribution is approached due to coherent instability. It is also shown that final bunch compression by an induction linac for Heavy Ion Fusion is as effective with a realistic distribution function leading to flat-top pulses at target as it is by simple envelope calculation.

I. Introduction

Early computer simulation of high current linac bunches has indicated the existence of longitudinal-transverse coupling and emittance transfer [1,2]. In recent analytic work space charge driven coherent instabilities have been made responsible for this coupling and thresholds have been presented for x-y geometry [3]. In view of the practical consequences of emittance transfer and "equipartitioning" emphasized in recent linac beam dynamics studies [4,5,6], we have used a newly developed code to check the applicability of these thresholds to r-z geometry (section II). Since this code has been developed primarily to study beam dynamics of long intense bunches for Heavy Ion Fusion Drivers we are also reporting here about first results of simulation of final bunch compression by an induction linear accelerator (section III).

The simulation program advances particles in x, y, z and utilizes a fast Poisson solver (by Schumann and Sweet [7]) on a mesh in r, z assuming a conducting radial cylindrical boundary with periodic boundary conditions in z. Periodic focusing forces have been replaced by equivalent constant forces. After each time-step charges are distributed on the four nearest grid points and Poisson's equation is solved. A 24 × 120 mesh and 8000 particles have been found sufficient for simulation of short ellipsoidal bunches which require about 150 msec CPU per time-step (~ 25% of this time for the Poisson solver) on a Cray 1. Simulation of long intense pulses demands for higher resolution and a larger number of particles, unless the very different scales for longitudinal and transverse motion in a real beam are brought closer together in the simulation. For a case with 130.000 simulation particles on a 32 × 80 mesh we have required ~ 1.4 sec CPU per time-step (3% for Poisson solver).

II. Coherent Instabilities and Emittance Transfer in Linac Bunches

1. Theoretical Model of the Instability

In recent theoretical work eigenfrequencies of "third-order" and "fourth-order" coherent modes have been calculated using the Vlasov equation for an initial Kapchinskij-Vladimirskij distribution with arbitrary emittance ratio $\varepsilon_x/\varepsilon_y$, tune ratio σ_x/σ_y and intensity.[3] For given $\varepsilon_x/\varepsilon_y$, thresholds for the onset of instability have been found to depend on σ_x/σ_y and the tune depressions σ_x/σ_{ox} and σ_y/σ_{oy}, as is shown in Fig.1. We note that the σ's are tunes in an equivalent continuous focusing; but subsequent simulation has shown, however, that in periodic focusing with the same tune ratios the same thresholds hold, provided that structure resonances are avoided (for instance by choosing $\sigma_0 < 60^0$).

It is interesting to compare these coherent instabilities with the nonlinear resonances of a single particle in the potential of the "third-order" mode, for instance. Assuming harmonic unperturbed equations of motion, the space charge potential of the "third-order even" mode $V^1 \sim x^3 + Axy^2$, yields for the y-motion a coupling term

$$y'' + \sigma_y^2\, y = \delta \cdot y \cos(\sigma_x s) \qquad (1)$$

whereas the "third-order odd" mode $V^1 \sim y^3 + Byx^2$ yields

$$y'' + \sigma_y^2\, y = \delta \cdot \cos(2\sigma_x s) \qquad (2)$$

Equ.(1), which describes a "gradient error" indicates the existence of a half-integer or parametric resonance, if

$$\frac{\sigma_x}{\sigma_y} = 2 \qquad (3)$$

The resonance condition for the "inhomogeneous" or integer resonance in Equ.(2) is

$$\frac{\sigma_x}{\sigma_y} = 1/2 \qquad 4)$$

The actual bandwidth of these resonances depends on the strength of the coupling term (here denoted by δ), which is however determined by the collective behaviour of all other particles. At this point the single-particle description of the resonances breaks down and one needs a collective description in terms of resonance between coherent eigenmodes (as was done to obtain the thresholds in Fig.1). Nonetheless it is instructive to recognize that the resonances of Equ's(3, 4) coincide with the peaks in the threshold plot of Fig.1. Hence we conclude the following:

(i) At low intensity ($\sigma/\sigma_0 \to 1$) the coherent instability occurs only if σ_x/σ_y is very close to the single-particle resonance values 1/2 or 2. Inside the unstable area an arbitrarily small deviation from uniform density (hence small nonlinear coupling term) is predicted to grow exponentially.

(ii) With increasing intensity the unstable bands become broader and extend towards large values of σ_x/σ_y, hence the collective behavior dominates entirely over the single-particle behavior.

A similar analysis can be made for the "fourth-order" mode, which has a peak at $\sigma_x/\sigma_y = 1$ (for "even" symmetry). In order to connect the limits in σ_x/σ_y to the energy anisotropy in a frame moving with the bunch, we use the relationship

$$\frac{E_x}{E_y} = \frac{\sigma_x}{\sigma_y} \cdot \frac{\varepsilon_x}{\varepsilon_y} \qquad (5)$$

and predict that anisotropy limited by

$$\frac{E_x}{E_y} \lesssim \frac{1}{2}\frac{\varepsilon_x}{\varepsilon_y} \qquad (\varepsilon_x >> \varepsilon_y) \qquad (6)$$

does not lead to emittance transfer. The question of how far σ_x/σ_y can be above the limit 1/2 without getting significant instability and emittance transfer is beyond the linearized theory and requires computer simulation.

2. Simulation of a Bunch in r-z Geometry

The purpose of this simulation has been to check whether the predictions on instability from x-y geometry also hold for the equivalent quantities in r-z geometry with longitudinal-transverse coupling (quantities in paranthesis in Fig.1). Hence, we assume $\varepsilon_\ell > \varepsilon_t$ and consider ellipsoidal bunches, which are matched to equivalent continuous focusing forces by means of the r.m.s. envelope equations. The initial charge distribution is chosen uniform with the phase space distribution located on a hyper-ellipsoidal surface. External force constants $\sigma_{0,\ell}^2$ and $\sigma_{0,t}^2$ are chosen in such a way that the desired tune depressions σ/σ_0 are achieved in longitudinal and transverse directions.

Results obtained for $\varepsilon_\ell/\varepsilon_t = 4$ are shown in Table 1. The locations of the different cases are indicated in Fig.1; a contour plot for case D is shown in Fig.2.

Case	A	B	C	D	E	F	G
σ_ℓ/σ_t	0.25	0.5	1	2	3	3.1	3.1
$\sigma_\ell/\sigma_{0\ell}$	0.12	0.50	0.45	0.62	0.80	0.36	0.89
σ_t/σ_{0t}	0.20	0.49	0.30	0.30	0.40	0.11	0.50
$\left.\frac{\varepsilon_{final}}{\varepsilon_{init.}}\right\|_t$	1.0	1.0	1.15	2.0	2.6	2.0	1.0
$\left.\frac{\varepsilon_{final}}{\varepsilon_{init.}}\right\|_\ell$	1.0	1.0	0.9	0.6	0.6	0.7	1.0
$\left.\frac{E_\ell}{E_t}\right\|_{init.}$	1.0	2.0	4.0	8.0	12	12.4	12.4
$\left.\frac{E_\ell}{E_t}\right\|_{final}$	1.0	2.0	3.0	1.3	1.9	1.6	~ 12

Table 1. Emittance change and energy anisotropy of r-z simulation of bunches with 8000 particles (ε is defined as r.m.s. emittance, E as r.m.s. velocity squared in moving frame).

We note that results obtained for $\varepsilon_\ell/\varepsilon_t = 12$ have been surprisingly similar to those for $\varepsilon_\ell/\varepsilon_t = 4$. Case F has also been run with 32.000 particles, which gave the same r.m.s. emittance growth (within < 1% error). Typical growth times have been 1 - 2 periods of transverse betatron oscillations.

The main conclusion is that emittance transfer in the linearly unstable region $0.5 \lesssim \sigma_\ell/\sigma_t \lesssim 1$ is negligible, whereas significant transfer requires that $\sigma_\ell/\sigma_t \gtrsim 1.5$. This suggests that linac design could tolerate energy anisotropy as long as

$$\frac{E_\ell}{E_t} \lesssim 1 \ldots 1.5 \times \frac{\varepsilon_\ell}{\varepsilon_t} \quad \text{or} \quad \frac{\sigma_\ell}{\sigma_t} \lesssim 1 \ldots 1.5 \qquad (7)$$

III. Simulation of Long Pulses in Induction Linacs

The application of accelerators as drivers for inertial confinement fusion requires extremely high power density at the target. The actual current limit is set by space charge effects in transverse and longitudinal directions. While it has been shown previously that there is no limitation in the transverse direction [8,3] provided that $\sigma_0 \leq 60^0$, longitudinal limitations are not as well explored yet. Bunch compression [9] and longitudinal waves [10] have been investigated previously with purely longitudinal simulation codes. The purpose of using a r-z simulation code is to study

(i) the stability of longitudinal-transverse coupled collective oscillations[11]

(ii) the influence of a proper r-z self-field calculation on final bunch compression with realistic assumptions (line charge density leading to nonlinear force, dependence of g-factor on r etc.).

Regarding the first issue simulation of a rather cold long beam with thermal velocity ratio $\langle v_{long.}\rangle/\langle v_{transv.}\rangle \sim 1/2$ has not given evidence of instability, so far. This case has been run with up to 260.000 particles to reduce noise effects. Further work is planned on this issue.

Regarding bunch compression we have first considered an ideal bunch with parabolic line charge density and designed a compression scheme using the longitudinal envelope equation [12]. A 10 GeV Bi^{+2} pulse initially 200 nsec long with averaged particle current of 120 A is tilted in longitudinal phase space by applying a ramped voltage of $\lesssim \pm$ 550 keV/m over ~ 330 m which suffices to generate a 10-fold pulse compression after the bunch has travelled 820 m (Fig.3).

The simulation with 32.000 particles was first started with an initial longitudinal Neuffer distribution (consistent with a linear force) and a transverse K-V distribution, which gave the same compression as the envelope prediction, as expected. A more realistic longitudinal distribution with a Maxwellian in v_z and a flat line charge density profile (shown in Fig.4, where the same r.m.s. length and emittance was used as in the parabolic case) has given almost as good a compression (9.4 fold). The strong nonlinear space charge force at focus (> 1 MeV/m, has caused some aberration (as is recognized from the four-armed z-v_z plot in Fig.5 (instead of a tilting ellipse for the ideal Neuffer distribution), without disturbing much the final pulse profile.

The conclusion is that bunching against a nonlinear space charge force due to a realistic distribution function is as effective as a linear force bunching. Furthermore, we have shown that a fast current rise (within < 20% pulse duration time) to a flat-top profile is feasible, which is important for target design.

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*Work supported by the Bundesminsterium für Forschung und Technologie

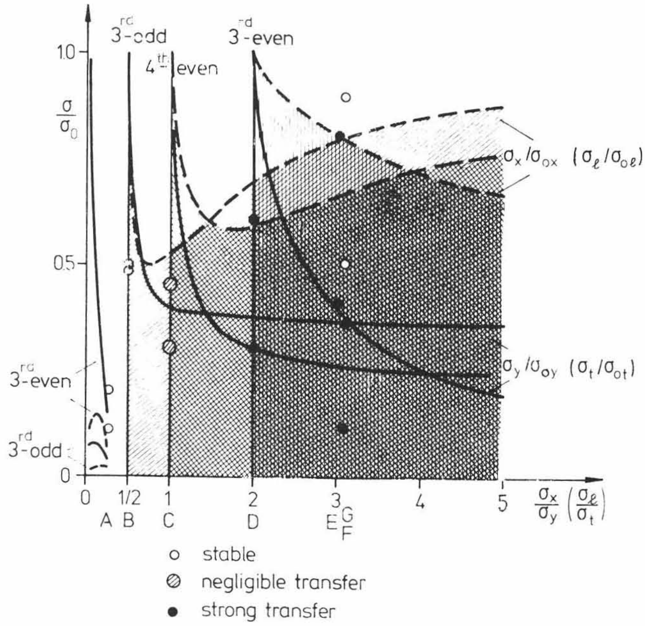

Fig.1 Thresholds for lowest order x-y coupling modes (only non-oscillatory) with $\varepsilon_x/\varepsilon_y = 4$. A given mode is linearly unstable, if both tune depressions (in x, y or ℓ, t) are within shaded area; circles indicate location of r-z simulations.

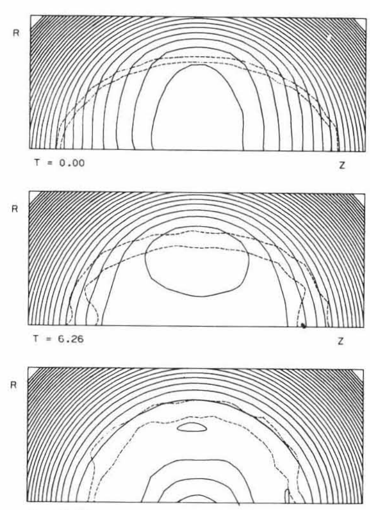

Fig.2 Equi-potential lines (total potential) for unstable case D. Dashed lines are density contours at 1/3 and 2/3 of maximum density. Initial energy anisotropy is evolving rapidly into almost equidistribution with a strongly nonlinear intermediate potential distribution (time scale: $\sigma_{ot} = 1\ \mathrm{sec}^{-1}$)

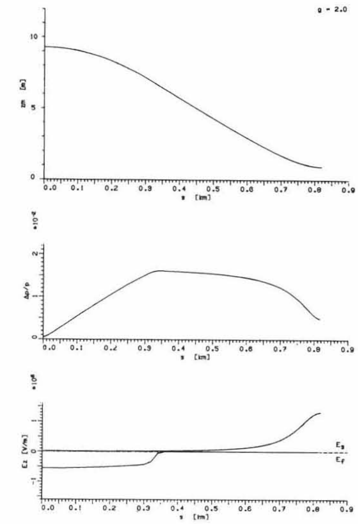

Fig.3 Long bunch compression scheme showing half-length (zm), momentum spread (± $\Delta p/p$), applied force (± E_f) and space charge force (± E_s) as function of distance.

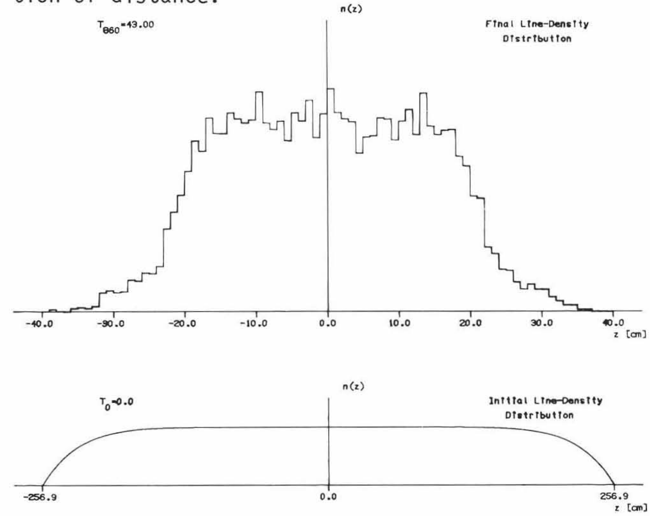

Fig.4 Line density of realistic bunch (initially r.m.s. equivalent to ideal case in Fig.3), compressed with linear applied force as in Fig.3. Flat-top profile is conserved at focus, only bunch ends are leaking out.

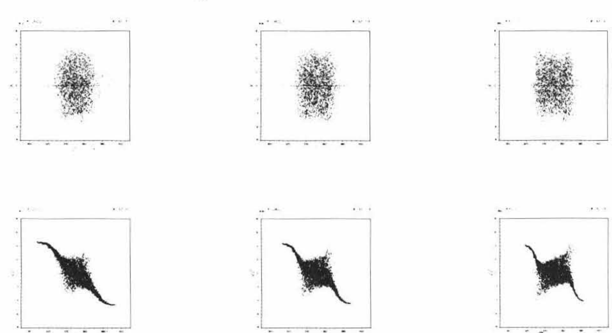

Fig.5 Projections of 6D phase space into z-x and z-v_z planes near focus, showing 8000 out of 32000 simulation particles. Aberration "wings" due to unbalanced excessive self-force near bunch ends, where line density gradient is large. Frames apply to 60 m, 30.0 m and 0 m away from focus.

Discussion

It was asked if we saw transverse emittance growth when compressing a long pulse in the compression studies. We did not have enough betatron oscillation periods in this study to observe what happens in the transverse plane. The time scales of betatron to synchrotron or longitudinal oscillation is very different, and to save computer time, we did not run out to several betatron wavelengths.

With the bunched beams and an initial rms emittance ratio of four, we did observe growth of the smaller rms emittance by a factor of 2 to 3 (90% emittance growth grew even more), and the initially larger rms emittance decreased a little.

To the questions of whether the bunched beam modes are a classic Weuball instability: the Weibull is electromagnetic. What we have here can be illustrated in terms of a plasma in a magnetic field--if the temperature perpendicular to the magnetic field is different from the longitudinal temperature, the situation is unstable. When the medium is infinite, a minor temperature difference or anisotropy will set up long-wavelength modes and isotropization will occur. In a bunch, however, the finite geometry means you can't have wavelengths longer than the bunch, and so there must be a threshold on the amount of anisotropy required before instability occurs.