CAVITY LOADING ASSOCIATED WITH HIGH-CURRENT RF LINACS

by

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ABSTRACT

The limitations on high-current rf linacs due to cavity loading are studied. A linear analysis as well as self-consistent particle simulations of a multipulsed 10 kA beam indicate that only a negligible small fraction of energy is radiated into nonfundamental cavity modes.

Cavity loading due to beam driven radiation is a well-known problem in radiofrequency linear accelerators. In fact, one of the major criticisms of the high-current rf linac concept was that beam driven radiation into unwanted modes would be catastrophic since the radiation energy source scales as $\vec{E} \cdot \vec{J}$, or as the charge of a micropulse squared. In particular, enhanced spectral content in unwanted modes not only serves as a sink for beam energy but it can seriously magnify transverse beam emittance. Even worse, the coupling to the non-axisymmetric $\ell = 1$ mode would deflect the beam into the drift tube wall. This last issue of the beam breakup mode[1] is addressed in another report. In the present section we have undertaken a quantitative study of high-current driven cavity radiation to indicate both the magnitude and scaling of the axisymmetric beam loading.

A simple right circular, cylindrical cavity is employed to represent the essential physics, if not the details of more complicated, realistic cavity shapes. A major advantage of this is that the eigenmodes for such a system are readily calculable. Even though the ideal structure is perturbed by drift tube apertures of radius r_d this is a relatively small effect so long as the cavity radius R satisfies $R \gg r_d$. As a further simplification, we will consider only single cavities, driven at the fundamental TM frequency. While coupling of cavities is often done to enhance synchronism, we feel that this more complex configuration can best be illuminated initially by treating the single cavity loading exhaustively.

The study is limited to intense electron beams, which are at least modestly relativistic upon injection into the cavity. In fact, when treating the beam dynamics with simulation, the electrons are constrained to be neither relativistic nor to follow one-dimensional trajectories. Transport of multikiloampere beams into rf cavities, however, does require that they be relativistic enough to avoid space-charge limitations. For 10 kA micropulses this corresponds to about 3 MV.[2] In the simulations, the transport itself was aided by assumption of a straight solenoidal magnetic guide field. Inclusion of more realistic focusing fields can be treated but there is no reason for doing so at this juncture.

The model we use to calculate the response of the cavity modes to the beam is well known.[3] It is not a self-consistent calculation in that the modification of the beam distribution by the cavity fields is not taken into account. However, we find the results to be in good agreement with self-consistent electromagnetic, relativistic, two-dimensional, particle-in-cell simulation results.

In terms of the vector potential $\vec{A}$ and the scalar potential ϕ, Ampere's law becomes

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J} - \frac{1}{c^2}\frac{2}{\partial t}\nabla\phi \quad . \qquad (1)$$

We have chosen the Coulomb gauge: $\nabla \cdot \vec{A} = 0$. The vector potential can be expressed in terms of the cavity eigenmodes $\vec{a}_\lambda$,

$$\vec{A} = \sum_\lambda q_\lambda \vec{a}_\lambda \quad , \qquad (2)$$

which satisfy $\nabla^2\vec{a}_\lambda + (\omega_\lambda^2/c^2)\vec{a}_\lambda = 0$. Because $\phi = 0$ on the cavity surface and $\nabla \cdot \vec{a}_\lambda = 0$ in the cavity volume, this term vanishes, and the time evolution of a cavity mode is given by

$$\ddot{q}_\lambda + \omega_\lambda^2 q_\lambda = \frac{j_\lambda}{\varepsilon_0} \quad , \qquad (3)$$

where

$$j_\lambda = \int_\Omega \vec{a}_\lambda \cdot \vec{J}\, d^3x \quad . \qquad (4)$$

As a starting point, we consider the cavity radiation driven by a point charge which enters the cavity at $t = 0$ and moves rigidly along the z-axis at velocity c. For such a charge the current density is

$$J_z(z,t) = \rho c\, \delta(z - ct)\delta(r)/2\pi r \quad . \qquad (5)$$

Suppose we have a train of point charges separated by a time $2\pi/\omega_x$, the current density is then

$$J_z(z,t) = \rho c \sum_{n=1}^{N} \delta[z - ct + 2\pi(n - 1)\dot{c}/\omega_x]$$

$$\times\ \delta(r)/2\pi r \quad . \qquad (6)$$

If we let $\omega_x \to \infty$ and $N \to \infty$ while keeping $T = N \cdot 2\pi/\omega_x$ constant, we get the result for a constant current pulse of width T. Thus, for a train of micropulses of finite pulse width the energy in an initially undriven mode after N pulses is then

$$U_\lambda = \frac{2N^2\rho^2}{\varepsilon_0\pi d(1 + \delta_{0p})}\ \frac{1}{x_{0n}^2 J_1^2(x_{0n})}\left[\frac{\sin(N\pi\omega_\lambda/\omega_{010})}{N\sin(\pi\omega_\lambda/\omega_{010})}\right]^2$$

$$\times \left[\frac{\sin(\omega_\lambda T/2)}{(\omega_\lambda T/2)}\right]^2 [1 - (-1)^p\cos(\omega_\lambda d/c)] \quad . \qquad (7)$$

Finally, the energy radiated into the fundamental mode is

$$U_{010} = \frac{1}{2}\varepsilon_0 E_{z010}^2 \pi R^2 d\, J_1^2(x_{01}) - \frac{2|\rho| NRE_{z010}}{x_{01}}$$

$$\times \sin(\omega_{010}d/2c)\left[\frac{\sin(\omega_{010}T/2)}{(\omega_{010}T/2)}\right] + \frac{2N^2\rho^2}{\pi\varepsilon_0 d}$$

$$\times \frac{1}{x_{01}^2 J_1^2(x_{01})}\left[\frac{\sin(\omega_{010}T/2)}{(\omega_{010}T/2)}\right]\sin^2(\omega_{010}d/2c) \qquad (8)$$

Our first concern was to check the analytical results against particle-in-cell simulations. In Fig. 1 the amount of energy radiated into an initially empty cavity is plotted as a function of the number of micropulses passing through the cavity. The parameters are $\rho = 1.74 \times 10^5$ Coulomb/micropulse, $R = 2.3$ m, and $d = 2.5625$ m. The comparison between the analytic result and the "slug" beam is very good. Where a fully self-consistent beam is injected into the cavity, a combination of space-charge and induced fields reflects the beam after about 7 pulses, i.e., the energy in the cavity modes becomes about equal to the kinetic energy of the micropulse. The details of this simulation will be discussed in more detail later, but we feel that the analytic results will yield essentially the correct beam loading for an actual beam.

With some confidence in our result, the energy in unwanted modes was calculated for a 10 kA beam for various micropulse widths after 100 pulses. Again, the cavity dimensions are $R = 2.3$ m and $d = 2.5625$ m. Basically, we find that the energy going into unwanted modes is negligible compared to the energy going into the fundamental. This will be true even for $\ell \neq 0$ modes as long as the cavity is "detuned", i.e., $\omega_\lambda/\omega_{010} \neq$ odd integer. A summary is given in Table I.

The simple model derived in the previous section is very useful for calculating the beam loading on the cavity. But it neglected space charge, finite transverse dimensions of the beam, and beam distortion by the cavity fields, however. In short, it did not attempt to evaluate the effect of cavity fields on the beam. To study the self-consistent dynamics, we have employed the two-dimensional particle-in-cell

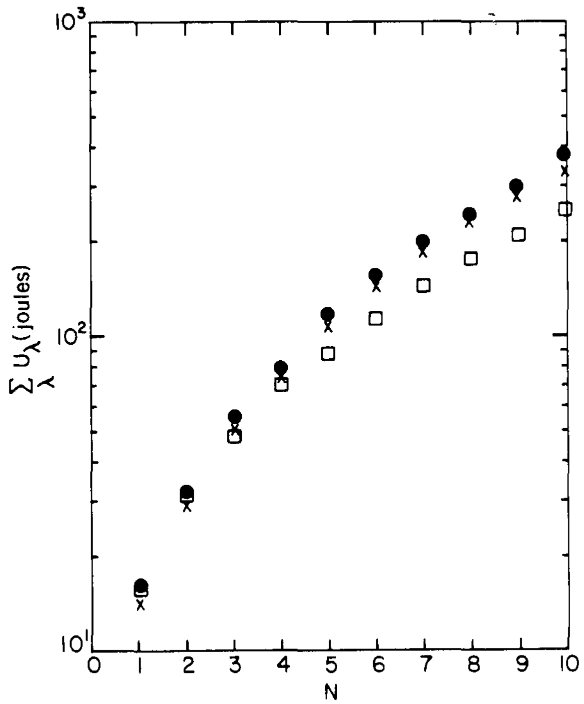

Fig. 1. Energy radiated into cavity modes $\Sigma_\lambda U_\lambda$ as a function of pulse number N. The open boxes represent the self-consistent simulation results, the closed circles the "slug" simulation results, and the x's the analytic theory.

code, CCUBE. This fully electromagnetic, relativistic simulation code has been used previously in a wide variety of intense non-neutral beam and accelerator studies. The present calculations were performed in cylindrical (r,z) coordinates, with azimuthal symmetry (that is, $\ell = 0$).

A pill-box, right-circular cavity was used for these simulations. The cavity, when driven, was operated on the TM_{010} mode. In fact, this cavity fundamental was the dominant mode excited in undriven cavities when a series of beam pulses was injected. The cavity length, d, was taken to be slightly larger than its radius, R, with d/R ranging from 1.07 to 1.16. Since the calculations were scaled to the TM_{010} mode, no absolute

TABLE I

SUMMARY OF ENERGY IN UNWANTED MODES AFTER 100 PULSES HAVE PASSED THROUGH THE CAVITY

I_0 (kA)	T(nsec)	$\Sigma_\lambda U_\lambda$; $\lambda \neq 010$	U_{010} (klystron)
10	1.25	266 J	11.0 kJ
10	1.50	323 J	15.8 kJ
10	2.50	223 J	43.0 kJ
10	3.3	257 J	71.5 kJ

dimensions are attached to them. In fact, however, we are quite interested in PHERMEX, which operates at 50 MHz, or similar high-current linacs. For a PHERMEX-like cavity, R = 2.3 m and d = 2.6 m.

Because of the complicated dynamics of the full self-consistent loading, a series of calculations was performed to explicitly isolate various aspects of the problem. In the first, a sequence of fixed current profiles was propagated through the single cavity. These current "slugs" radiated electromagnetic fields into the cavity but were not in turn acted upon by the fields. These simulations were closest to the assumptions of the loading model derived above. The cavities were not driven so there was no confusion between the radiated field distributions and a pre-loaded field. The second type of simulation also contained no cavity pre-excitation but simulation macro-particles were used to construct the injected pulses. Because these pulses were free to respond to the self-excited cavity, space-charge effects and kinetic energy depletion due to $\vec{J} \cdot \vec{E}$ were self-consistently calculated. To facilitate pulse propagation across the cavity, a uniform solenoidal field B_z, such that $\Omega_0 = \omega_p$, $\Omega_0 = |e|B_z/mc$, was included. Although imposition of a non-fringing field of this magnitude around a 2.3 m radius cavity is possible, it is admittedly not practical. For these calculations, it was unnecessary to complicate the beam dynamics with focusing effects, however. Finally, the realistic accelerator problem was treated. We limited the studies to early cavity loading, because cavity loading is most severe before the beam has become too "stiff" ($\gamma \gg 1$). The cavity was driven in the TM_{010} mode to between 9.0 MV/m at 50 MHz. Maximum peak currents injected were 18.5 kA and minimum, 0.6 kA. The former corresponded to an average current of almost 1.4 kA.

The base line calculations were slug simulations that can most easily be compared with the analytic model. There were several salient features of the model amenable to simple tests. The model, for instance, predicted that the total energy radiated into the cavity should vary as the square of the total charge per pulse. This was repeatedly verified for pulses of various radius and longitudinal extent. The spectral distribution of cavity energy moreover was identical for pulses of the same physical shape but different density. For pulses injected at the same frequency as the cavity fundamental, the model moreover predicted that the fraction of the total cavity energy outside the fundamental would decrease as the number of pulses increased. For a 9.2 kA peak current injection, Fig. 2 shows the fraction of the energy not in TM_{010} as a function of pulse number in the simulation compared with the theory. As a complement to this, Fig. 1 shows the total radiated energy for this case as a function of pulse number. From these, it is evident that the pulses are very effectively driving the cavity TM_{010} mode.

The magnitude of this field is increasing linearly with the number of pulses, so that after 10 pulses, the peak TM_{010} field has attained a

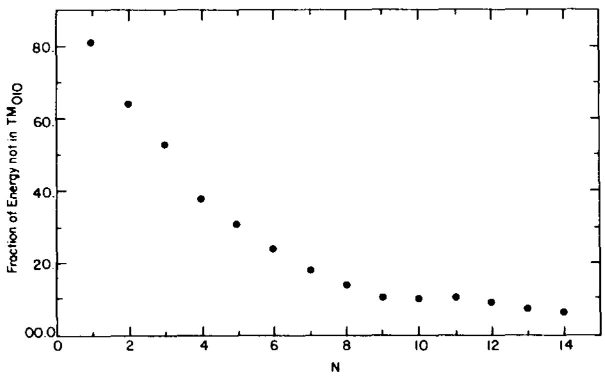

Fig. 2. Relative fraction of field energy in cavity fundamental ω_{010} as a function of pulse number N for slug simulation ($\gamma = \infty$); I_{max} = 9.2 kA, I_{ave} = 0.6 kA, T = 14.1 ω_p.

magnitude of 2.8 MV/m. While this field is in the decelerating phase, it is a significant fraction of the accelerating gradient. This suggests the fascinating prospect of building an rf auto-accelerator. The purpose ofsuch a configuration would be to overcome power limitations of existing radiofrequency sources. Creating an initial beam by pulse power certainly has limits, but these appear to be in the range of 10's of terrawatts as opposed to 10's of gigawatts for conventional power supplies. Thus, if a multi-terrawatt electron beam can be induced to radiate its energy into a given cavity mode, it seems quite feasible to operate rf linacs in as high a gradient as the cavities can withstand (either Kilpatrick or field emission).

As mentioned above, we have verified that the total energy in the cavity increases as the square of the number of pulses but that the relative fraction outside the fundamental decreases. The excitation of higher order modes, therefore, becomes less important as the number of pulses increases. Even if the magnitude of fields in these unwanted modes were to remain high, we are confident that loading of the cavities with a frequency dependent absorber could reduce the levels to acceptable values. A more fundamental limitation is depletion of the pulse energy after the gradient reaches sufficient magnitude.

If the kinetic mean energy of an injected pulse is $(\gamma - 1)mc^2$, this pulse will lose all its energy once the loaded field has reached a magnitude E_z, such that

$$\int_0^d E_z \cos \omega t \, dz \geq (\gamma - 1)mc^2 \qquad (9)$$

where $t = (z - z_0)/V_0$. This places an upper bound on E_z, $E_z = \omega(\gamma - 1)mc^2/(ec \sin \omega d/2c)$.

Once this gradient is attained, we find that a moving virtual cathode forms on the pulse. By this we mean that a fraction of the beam is reflected, while the rest propagates through the cavity. In a low-current beam for which $\Delta\gamma/\gamma \ll 1$, it is plausible to treat single particle trajectories in which all particles are either transmitted or reflected. For these high-current pulses, however, the collective behavior of a virtual cathode is observed whenever the pulse kinetic energy drops below a finite, non-zero value. The reflected portion of the pulse is moving in the opposite direction, and so is in an acceleration phase. Since it extracts energy from the cavity, the net energy radiated is reduced from the nonreflecting case. Fig. 3 shows growth of rms E_z field at the cavity mid-point for a slug simulation compared with a fully interacting particle one. The field has reached pulse reflecting levels by pulse number seven. Interestingly, even after partial pulse reflection, the TM_{010} field continues to grow. The net efficiency of field generation is clearly reduced, but as Fig. 4 shows, the relative magnitude of the TM_{010} mode increases vis-a-vis any of the non-fundamental and presumably deleterious modes. Actual operation in this fashion may prove undesirable because of breakdown problems associated with residual charge left in the cavity, but it is not ruled out on the basis of the spatial field distribution.

The spectral distribution of radiated energy is of considerable interest. Unfortunately, exact details of the spectra depend sensitively on details of the cavity structure. As an example, a series of calculations were performed in which the pulse structure and current were identical, with only the width of the cavity varied. Although the ratio of width to radius of

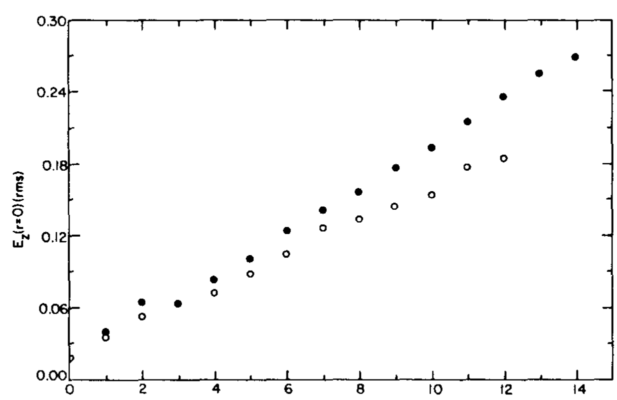

Fig. 3. Comparison of rms magnitude of E_z in slug ($\gamma_0 = \infty$, closed circles) and particle (γ_0 = 10.0, open circles) simulations as a function of pulse number N; I_{max} = 9.2 kA, I_{ave} = 0.6 kA, Ω_0 = 1.0 ω_p. Because of partial pulse injection initially on particle calculations, pulse number is shifted by one relative to actual time for plotting.

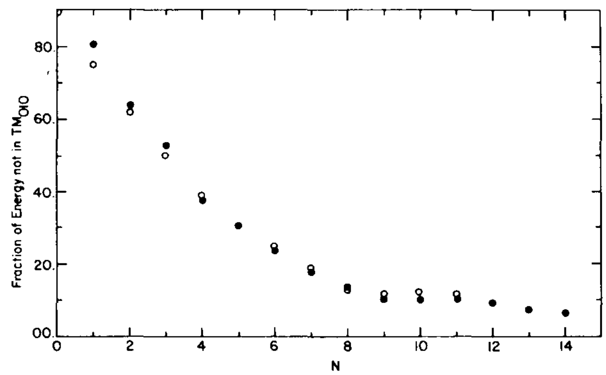

Fig. 4. Comparison of relative fraction of field energy not in TM_{010} for same calculations as Fig. 4.

the cavity, d/R, varied from only 1.105 to 1.159, there was significantly different spectral structure away from the fundamental, $\omega_{010} = 0.052\ \omega_p$. In all cases, however, the TM_{010} spectral component after 10 pulses was overwhelmingly dominant, and within graphical accuracy, at the same level. It should also be noted that while the overall agreement between simulation and analysis is excellent, numerical deviations may have played some role in choosing between particular resonances.

To summarize these results, we find that the frequency of pulse injection strongly determines the dominant cavity mode excited by the beam loading. As predicted by the analytic model, the cavity energy increases as the square of the number of pulses, while the fraction of this energy not in the fundamental monotonically decreases. The slug calculations, which correspond to $\gamma \to \infty$, are found to yield the same cavity loading dynamics as fully mobile particle calculations of the accelerator mode. The main features of the radiated spectra are in quantitative agreement with the analysis, but secondary features may be model dependent in the simulations.

Finally, self-excitation of the cavities by intense injected pulses does not appear to have fundamental problems. A practical limitation may prove to be virtual cathode formation when too much energy is extracted from the pulse, but even this condition did not degrade the relative energy flow significantly. The efficiency of field accretion was reduced, however. Further work on this concept is apparently needed to assess its ultimate utility.

ACKNOWLEDGMENT

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Discussion

One could increase the amount of stored energy available by using higher order modes.

A second comment is that the rate at which power goes into the cavity does not depend on the Q, but on the rf power source.

In point of fact, the only thing demonstrated about the relative decrease in the nonfundamental mode energy was that it was just a consequence of pulse length. Clearly, if pulses are injected at the fundamental frequency, the Fourier transform indicates that this mode will dominate. What wasn't obvious was that this result wouldn't change when finite beam injection gave very nonlinear effects in the beam dynamics, with virtual cathode formation, violent fluctuations in the beam, and so on.