MOTION - A VERSATILE MULTIPARTICLE SIMULATION CODE

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The features of the newly developed multiparticle simulation code MOTION are outlined. It solves the general three dimensional equations of charged particle motion in arbitrary external fields, taking into account also the internal space charge forces. The transition from an unbunched to a bunched beam can be handled. Results are presented for focusing by a solenoid yielding a hollow beam profile. Further, the acceleration of a bunched beam by rf gaps is calculated and compared to the results of the MAPRO code.

Program Description

Initial Distributions

The program handles particles of any mass or charge. The initial particle distribution in the six-dimensional phase space can either be generated according to specified distribution functions and Courant-Snyder parameters, or it can be taken from an input file. In case the distribution is given at a fixed axial position, e.g. z=0, the code drifts the particles back such that their coordinates are then known at a fixed time and all z-coordinates are negative. Then, using time as the independent variable, the particles are traced forward again. External and internal fields are switched on individually for each particle when it crosses the measuring plane z=0. In case that bunching is to occur at a frequency $f=c/\lambda$, the axial extension of the initially unbunched beam is taken to be $\beta\lambda$. In case of no bunching elements also particle distributions with zero longitudinal extend can be traced.

External Fields

A major purpose of MOTION was to trace charged particles in external fields which cannot be described sufficiently well by analytical formula. Especially this is the case if higher order correction terms have to be taken into account, as might be necessary for high intensity, high duty cycle, high energy linacs. Examples are: bunching or accelerating by a rf gap in case the energy gain across the gap is not small compared to the particle energy; focusing by a solenoid in case that aberrations come into play. The fields in such elements can be calculated by means of computer codes like POISSON, SUPERFISH or CLAS, yielding the fields at a mesh which in turn are used as input data to MOTION. The field value at the actual particle position is obtained by interpolation. Alternatively, the external field can be described by an analytic formula, e.g. a series expansion.

Space Charge Fields

The space charge forces are calculated at prespecified time intervals. By this means the program user can choose the accuracy for computing the space charge force independent of that for solving the equations of motion. At present, the space charge forces are calculated by summing up the mutual Coulomb forces among the simulated macro particles. To avoid artificial close collision effects the macro particles are assumed to be uniformely charged spheres occupying the total bunch volume, and having altogether the total bunch charge and mass. The particle onto which the space charge forces are just calculated is assumed to be a point charge. Then the force onto this test particle by a neighboring macro particle simply drops linearly with distance once the test particle penetrates into the volume of the macro particle. The total number N of macro particles is chosen such that the results of MOTION do not depend any more on the choice of the macro diameter.

If the transition from an unbunched to a bunched beam is to be treated, the effect of the two neighbor bunches is taken into account. The neighbor bunch trailing the bunch in time is obtained from data stored when the bunch was a period back in time. The other neighbor bunch being ahead in time is estimated by assuming the space charge forces on it to be the same as those experienced by the bunch on the average during the last period, corresponding to a smooth approximation for the space charge force. The external fields for the neighbor bunches are taken properly into account.

Equations of Motion Solver

The equations of motion are solved numerically using cartesian space coordinates. Time is the independent variable, since then there are no special transformations necessary to obtain the space charge forces. The equations are solved in the following form:

$$\gamma_2 \vec{v}_2 = \gamma_1 \vec{v}_1 + \frac{q}{m}\int_1^2 (\vec{E} + \vec{v} \times \vec{B})\, dt$$

$$\gamma_2 = \gamma_1 + \frac{q}{mc^2}\int_1^2 \vec{v}\cdot\vec{E}\, dt; \quad \vec{x}_2 = \vec{x}_1 + \int_1^2 \vec{v}\, dt$$

Measuring Planes, Output, Restart

For data evaluation the particle coordinates are wanted as a function of the axial position. Such measuring planes can be specified as input data. The particle coordinates at these planes as well as at the specified time intervals can be stored in a mass storage system. Thus a restart of MOTION at the end of a previous run is possible to continue the calculation of a long beam line. At last, the information at all measuring planes is available e.g. for plots as a function of z.

Computing Time and Storage Requirements

MOTION was developed to be able to make accurate checks of beam line designs of linear accelerators. The method chosen requires large computing times and large storage systems. As an example, to trace 2500 macro particles fully threedimensional through an Alvarez linac MOTION needs at present about 40 minutes per linac cell on an IBM 3033, which is about a factor of 25 more than the MAPRO code (for 15 space charge calculations per linac cell compared to 2 in MAPRO, and for a relative error of less than $5\cdot10^{-5}$ in each time integration step).

First Applications of MOTION

Focusing by a Solenoid

By using MOTION to calculate the particle trajectories in a solenoid all aberrations can be included together with space charge effects. As an example the transport of a 20 keV proton beam through a solenoid has been calculated without space charge. At first, the magnetic field of the solenoid was calculated by POISSON, which yields the field values on a triangular mesh. From this the fields on a square mesh are generated which are input values to MOTION. As a check for the accuracy, the particle motion was calculated for several cases: the mesh size of the original mesh of the POISSON run was varied by a factor of two, the same was done for the secondary mesh; further the accuracy for solving the differential equations in MOTION was varied by a factor of 10. For all possible combinations of these parameters the change in the particle trajectories transported through the solenoid was negligible. The particle energy changed by less than 2×10^{-5} which is another check for the accuracy of the program, as in a magnetic field the particle energy must be a constant of the motion.

To demonstrate the accuracy of MOTION by physically meaningful quantities, two aberration constants for the solenoid were evaluated from a set of particle trajectories. Again this was done for several cases: for a beam initially parallel to the solenoid axis, for an initially divergent beam originating from a point source on the solenoid axis, and for three different solenoid focusing strengths. The particle trajectories for the strongest focusing strength used are shown in Fig. 1.

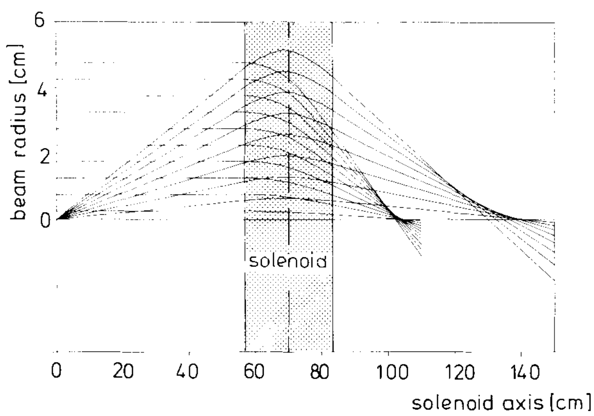

Fig. 1: Focusing by a solenoid

The effect of aberrations is obvious near the focuses, the constants related to them are plotted in Fig. 2 as a function of the particle radius at the solenoid center. Theoretically this must yield a constant value for particles moving close to the solenoid axis. This is confirmed by the evaluation for radii between about 1.5 cm and 3.5 cm. For smaller radii the error in evaluating the constants get large (substraction of 2 small numbers of nearly equal size, error in extrapolating the focus for the on-axis beam). For particles moving more than about 1/3 of the solenoid aperture away from the beam axis terms of fifth order start to come into play.

The focal lengths f_2 (=distance between focus and principal plane) for the three field values are given in Table 1, together with the thin lens focal lengths. The smallest field can be described by the thin lens approximation extremely well, whereas for the largest field the deviations are about 12 %.

The effect of aberrations can yield a hollow beam density profile close to the beam focus. To show this, the beam density was evaluated as a function of beam radius, 2.4 cm before the beam focus for an incoming parallel beam and for $B_z(r=0)$ = 0.166 T. The density of the initial beam was uniform in space, the beam

max $B_z(r=0)$ [T]	Ampere turns [A]	f_0 [m]	f_2 [m]	p [mm]
0.0415	9375	5.009	5.046	+1
0.083	18750	1.252	1.293	-2
0.166	37500	0.313	0.357	-9

Table 1: Thin and thick lens focal lengths f_0, f_2 for different magnet strengths. p = distance between principal plane and solenoid center, measured positive in the beam direction. $f_0 = 8\, mE_{kin}/e\int B^2(r=0)dz$.

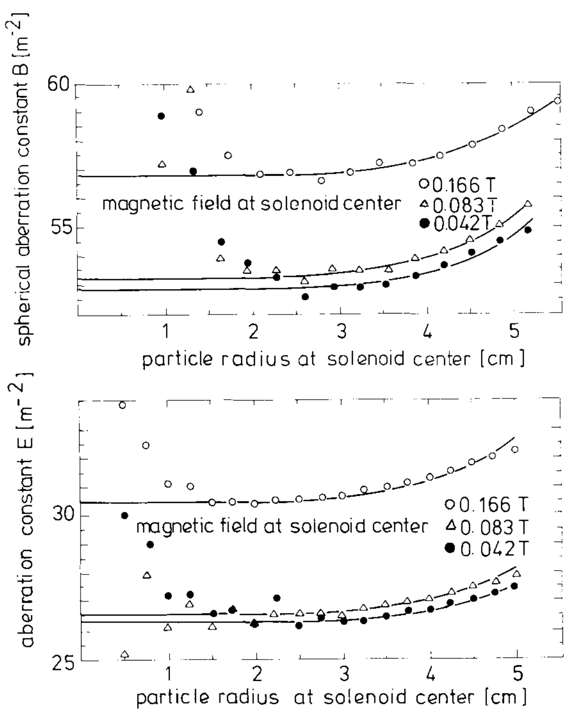

Fig. 2: Aberration constants of a solenoid for 20 keV protons. $E = \Delta r_E/r_E^3$ for an incoming parallel beam. $B = \Delta r_B/r_B^3$ for a divergent beam originating from a point source on the beam axis $2f_0$ before the solenoid center. r = ray radius at the solenoid center in single thin lens approximation. Δr = ray radius at the plane of paraxial focus. Solenoid geometry: aperture radius = 10 cm, total length = 25 cm, width of iron yoke at the aperture = 1 cm, width of the coil = 22 cm, outer coil radius = 24 cm = inner iron radius, outer iron radius = 25 cm.

radius was limited to 2.75 cm in front of the solenoid (= 27.5 % of the solenoid aperture). The density was evaluated by assuming it to be proportional to the inverse of the distance between neighboring particles, the result is shown in Fig. 3. This profile changes rapidly when approaching the beam focus, as then the particle trajectories cross.

Acceleration by an Alvarez Linear Accelerator

The theory of Lapostolle-Schnizer on the acceleration by a rf gap [1] takes into account terms up to the order of (energy gain / 2 kinetic energy), which amounts to 0.11 at the first gap of the proposed proton linear accelerator SNQ [2]. A bunched beam (protons, 450 keV injection energy, 100 mA beam current, 108 MHz Alvarez) was traced through the first 10 gaps of this linac by MOTION and the results obtained were compared with those of the MAPRO [3] code. The fields in the beam aperture region were calculated from the potentials given by CLAS [3]. One of

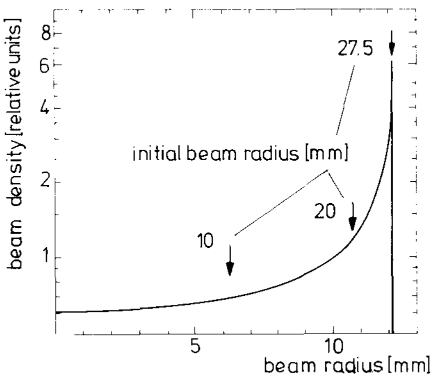

Fig. 3: Hollow beam density profile before the solenoid focus

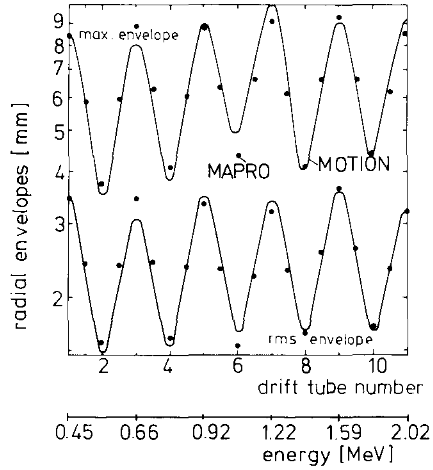

Fig. 4: Transverse bunch envelopes along the SNQ linac

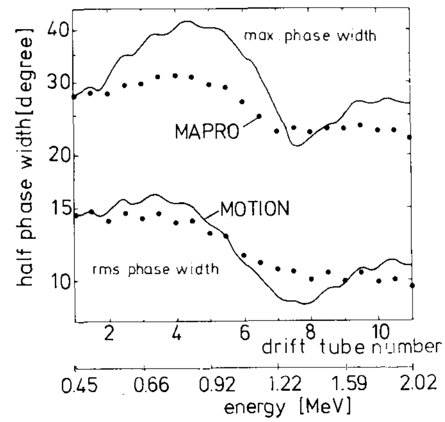

Fig. 5: Phase width along the SNQ linac

various checks of MOTION was to compare for zero current and zero rf, that is quadrupole fields alone. The agreement was better than 10^{-3}. Also the difference between MOTION runs with 10 compared to 15 space charge calculations per linac cell was about 10^{-3} in rms quantities. The differences between MAPRO and MOTION results at the end of the first rf gap, zero current, is given in Table 2. It is of the order of 10 %. The differences in bunch envelopes and rms emittances are shown in Fig. 4, 5, 6. The conclusion is that MAPRO certainly is accurate enough for traditional rf linac designs. In future designs in which particle loss problems play an important role the MOTION code will be an useful tool.

plane	$\Delta\ \varepsilon$	$\Delta\ \beta$	$\Delta\ \gamma$	$\delta\ \alpha$	Δ R
xx'	- 0.28	5.4	- 4.2	- 16	8.5
yy'	0.44	12	- 16	- 19	13
ϕW	- 0.12	- 9.9	10	- 9.5	7.2

Table 2: Relative (Δ) and absolute (δ) deviations between MAPRO and MOTION for the first rf gap of the SNQ Alvarez linac, zero beam current. ε = rms emittance, β, γ, α = Courant-Snyder parameter, ΔR = mismatch factor = relative residual rms envelope oscillation in a phase space coordinate system in which the rms ellipse of the MOTION results is a circle [4]. ΔR is invariant against transformations having unity determinant.

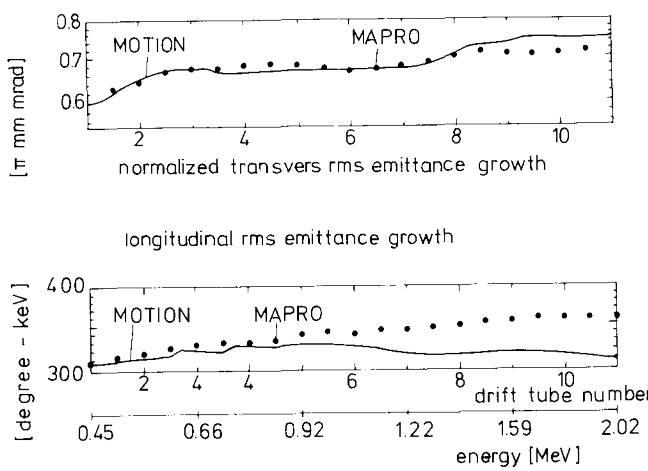

Fig. 6: Emittance growth along the SNQ linac

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Acknowledgements

The stimulating discussions with K. Bongardt and M. Pabst are appreciated.