

FOCUSING CONSIDERATIONS IN THE RACE TRACK MICROTRON AND DOUBLE-SIDED MICROTRON\*

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Summary

We have developed general designs for quadrupole multiplet systems for use in both the Race Track Microtron (RTM) and Double-Sided Microtron (DSM). The quadrupole multiplet consists of five quadrupoles forming two mirror symmetric optical systems; furthermore, both systems are telescopic in the bending plane and exhibit dispersion zeroes in the centers of the second and fourth quadrupoles. The systems perform achromatic transformations and either rotate the incoming beam phase ellipse by  $90^\circ$  or introduce focussing into the system. Alternate telescopic systems utilizing three quadrupoles can be used at higher energies. Compact systems can be constructed using permanent magnets. We present examples of these systems as applied to the proposed Argonne National Laboratory 2-GeV RTM-DSM configuration.

I. Introduction

The crucial requirement for both the RTM and DSM is longitudinal phase stability. Nevertheless, effective transverse focussing schemes are required for the multiple passes through these devices. The beams should be dispersion free in the linac(s). The planned ANL RTM-DSM complex is shown in Fig. 1.<sup>1</sup> The DSM traveling wave linacs each boost the electron energy by 25 MeV per traversal while 6 MeV is gained per turn in the RTM. Referring to Fig. 1, 5 MeV electrons are injected into linac 1 and alternately accelerated and focussed in a system of six 3.0 m long rf sections and seven periodic quadrupole triplets situated between the rf sections and at the ends of the structure. The 30 MeV electrons are bent  $20^\circ$  left by a septum magnet (which is the first dipole of the chicane system for returning high energy electrons); after passing through a  $170^\circ$  isochronous bending system, the electrons are injected into the RTM where the energy is boosted to 180 MeV after 25 turns. This beam is then reinserted into the 180 MeV short straight section (SSS) of the DSM and is then accelerated up to a maximum energy of 2005 MeV in 36.5 turns.

In this document, we specifically discuss the active focussing systems that are required in the return paths of the RTM and short straight sections of the DSM. We do not discuss correction devices for maintaining phase stability. In Sec. II, we examine the geometry and requirements of each system. The theory is presented in Sec. III, and several examples are developed in Sec. IV. Concluding remarks are added in Sec. V.

II. Geometry and Focussing Requirements

Figures 2(a) and 2(b) exhibit plan views of one orbit through the RTM and one-half turn through the DSM, respectively. The system between the entry (i)

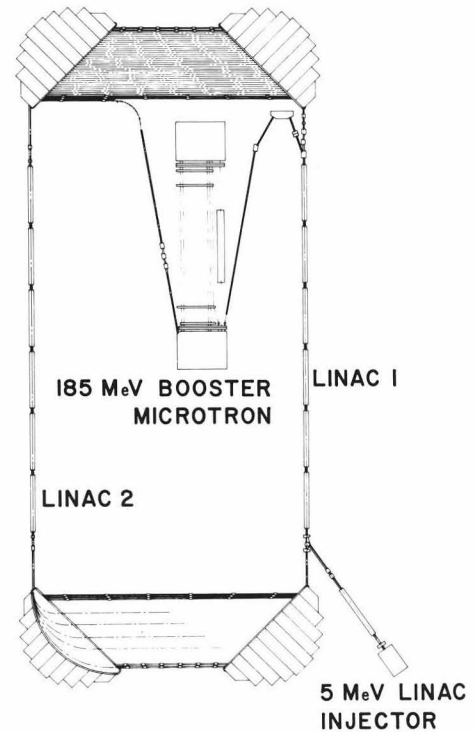


Fig. 1 Plan View of Proposed Argonne Double-Sided Microtron Project

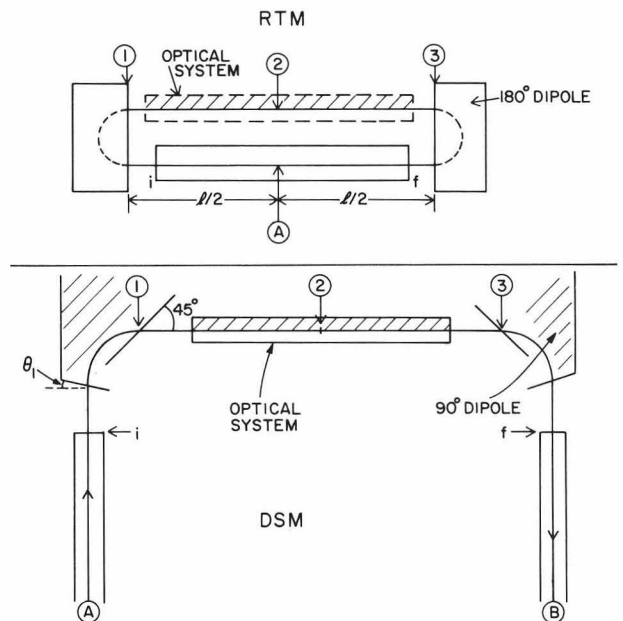


Fig. 2 Plan Views of One Orbit through the RTM and One-Half Turn through the DSM. Points A and B Represent the Centers of the DSM Linacs

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and exit (f) points must perform an achromatic transformation. We require nominal beam waists at the linac centers  $\beta = \beta_A$ ,  $\alpha_A = 0$ , and at the midpoint  $2\beta = \beta_2$ ,  $\alpha_2 = 0$ ; the system must be variable so as to affect other conditions, however. It must also be compact -- the spacing between adjacent orbits is  $\lambda/\pi$  for the RTM and  $\lambda/(\pi-2)$  for the DSM where  $\lambda$  is the rf wavelength ( $\lambda = 0.125$  m).

The choice of dipole entrance angle  $\Theta_1$  in Fig. 2(b) has been studied intensively. Entrance and exit angles of  $-45^\circ$  produce  $90^\circ$  bends, a parallel to parallel transformation in the bend plane, and severe vertical defocussing. The minimum energy is near 200 MeV, however. Attempts to lessen this defocussing using external negative fields, i.e., reverse field stripes, are planned.<sup>2</sup> However, they give rise to a path length problem. Presently, we plan to use  $\Theta_1 \geq 30^\circ$  with a small quadrupole between the  $90^\circ$  dipoles and linacs. This decision was partly historical because the early ANL design accelerated directly up from 5-2005 MeV in the DSM. The positive  $\Theta_1$  value reduces the vertical defocussing but causes a horizontal crossover downstream of the dipole. Stability studies have indicated an acceptable value of  $\beta_A = 1.5$  m for both the RTM and DSM.

### III. Beam Transfer Theory

A mirror symmetric system will be located between, e.g., points (1) and (3) in Fig. 2. Then the  $2 \times 2$  transfer matrices from points  $i$  to  $f$  are given by

$$M_{fi} = \begin{pmatrix} r & s \\ t & r \end{pmatrix} \quad (1)$$

in both transverse planes. The horizontal plane behavior in the interior section (1 to 3) is constrained to be telescopic and to have unity diagonal elements because of the achromatic requirement and

$$M_{31}(x) = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \quad (2)$$

The matrix (2) can be written in the form

$$M_{31}(x) = \begin{bmatrix} \sqrt{\frac{\beta_3}{\beta_1}} f(\psi, \alpha_1) & \sqrt{\beta_1 \beta_3} \sin \psi \\ 0 & \frac{\beta_1}{\beta_3} f(\psi, -\alpha_3) \end{bmatrix} \quad (3)$$

where the betatron phase shift is given by  $\tan \psi = (\alpha_1 - \alpha_3)/(1 + \alpha_1 \alpha_3)$  and  $f(\psi, \alpha) = \cos \psi + \alpha \sin \psi$ . If  $\beta_1 = \beta_3$  and  $\alpha_3 = -\alpha_1$ , we have  $\sin \psi = 2\alpha_1/(1 + \alpha_1^2)$  and  $a = 2\beta_1 \alpha_1/(1 + \alpha_1^2)$ . We can rewrite Eq. 2 as

$$M_{31}(x) = \begin{pmatrix} 1 & a/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a/2 \\ 0 & 1 \end{pmatrix} \quad (4)$$

We find that  $a/2$  is the distance to the horizontal waist as measured from points 1 or 3. The unity matrix in Eq. (4) represents a mirror symmetric focussing system acting in the  $x$  (bend) plane from a distance  $a/2$  downstream of point 1 to  $a/2$  upstream of point 3. The chosen value of  $a$  ( $\equiv 2\beta_1 \alpha_1/(1 + \alpha_1^2)$ ) results in a  $90^\circ$  beam ellipse rotation in going from point  $i$  to  $f$  (see Fig. 2(b)), i.e.,  $\beta_i(x) = \beta_f(x)$  and  $\alpha_f(x) = -\alpha_i(x)$ . This can be altered by changing the value of  $a$ ; then  $\tan \psi = a/(\beta_1 - a\alpha_1)$ ,  $\alpha_3 = \alpha_1 - a\gamma_1$  and we obtain

$$\beta_3 = \beta_1 \left[ \frac{1 + \tan^2 \psi}{(1 + \alpha_1 \tan \psi)^2} \right] \quad (5)$$

This change still maintains the achromatic transformation of Eq. (2).

### IV. Optical Systems

Two configurations have been adopted to deal with the RTM and the  $\Theta_1 \neq -45^\circ$  optics of the DSM:

(1) a five quadrupole design, and (2) a three quadrupole system. Both use a quadrupole singlet at the center symmetry point 2. For the RTM, the  $x$  transfer matrix for one turn  $M_{AA}(x)$  is a drift (ignoring acceleration) of length  $(\ell + a)$ . The  $\beta_A$  function will grow rapidly unless  $a \sim -\ell$ ; thus, one must have  $\sin \psi < 0$  (from Eq. (3)). This can be accomplished with the five quadrupole design. In the DSM, a similar system is used for energies below 1300 MeV; the triplet will suffice at higher energies.

Figure 3 shows the thin-lens analogues for the two systems along with the behavior of the dispersion  $\eta$  ray. The lens positioning relative to the horizontal waist location  $a/2 = \beta_1 \alpha_1/(1 + \alpha_1^2)$  is indicated. For the RTM  $\alpha_1 < 0$ , thus  $a/2 < 0$  is required. The multiplet is adjusted to form beam

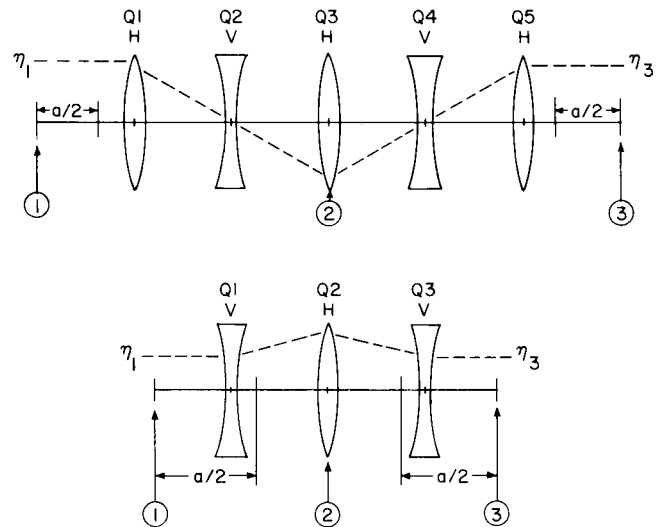


Fig. 3 Thin-lens Equivalents for Quadrupole Multiplet and Triplet Focussing Systems as Discussed in the Text

waists and  $\eta' = 0$  at point 2, and  $\eta = 0$  in Q2 and Q4. The tune can be varied by changing Q2, Q4 without affecting the dispersion ray. In the DSM, the value of  $a/2$  increases with energy; Q1 and Q5 must remain inside of these locations and their strengths increase quadratically.

For 1300 to 1800 MeV, the lower scheme in Fig. 3 is adopted. The quadrupole strengths are varied to obtain  $\eta_2' = 0$ , and an x waist  $\alpha_2(x) = 0$ ; the optimum location of Q1 (and Q3) minimizes  $|\alpha_2(y)|$ . Above 1800 MeV,  $a/2$  exceeds the distance between points 1 and 2 which is given by  $s_{12} = 5.585 - \lambda(E-30)/[50(\pi-2)]$  for even energies and by  $s_{12} = 5.53 - \lambda(E-55)/[50(\pi-2)]$  for odd energies. We treat this case by reversing the quadrupole polarities to H,V,H.

A program has been written to study the DSM optics over the entire range 200-2000 MeV. Quadrupoles are treated as thin lenses; dipoles use the standard transformations and Enge short-tail fields<sup>4</sup> for reduced vertical focussing at the edges. A one dimensional minimizer is used to optimize quadrupole locations; information on each orbit is available. We show in Fig. 4 the quadrupole pole-tip fields required for a bore radius of 1 cm and length of 10 cm;  $\Theta_1$  was taken as  $30^\circ$ . The discontinuities occur when  $a/2$  occurs in the centers of Q1 and Q2, respectively.

We have calculated beam envelopes using the program TRANSPORT.<sup>5</sup> In Fig. 5, we show DSM sample results for 430 MeV/c optics. The initial conditions were  $\beta_A = 1.50$  m,  $\Theta_1 = 30^\circ$ , and a geometric emittance of  $5/430 \pi$  mm-mr. Beam waists occur at

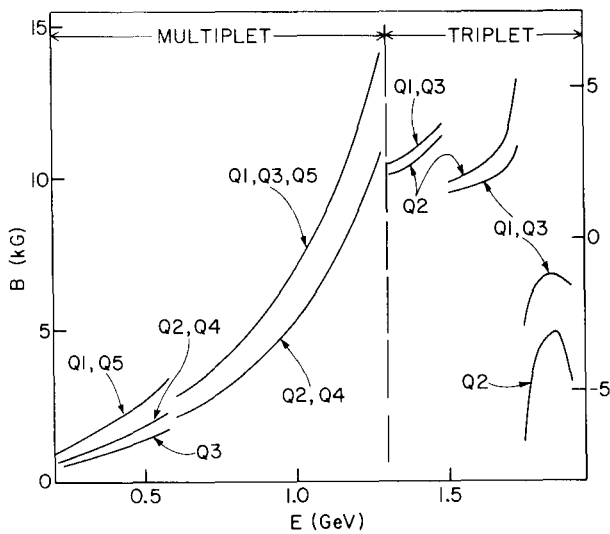


Fig. 4 Energy Dependence of DSM Quadrupole Pole-Tip Fields Assuming a Length of 10 cm and Bore Radius of 1 cm

the midpoint; the dispersions (not shown) behave similarly to those shown in Fig. 3(a). Envelopes have also been generated for the low-energy orbits in the RTM. They exhibit the same trends as those observed in Fig. 5. Similarly, high-energy orbits have been evaluated for the triplet geometry.

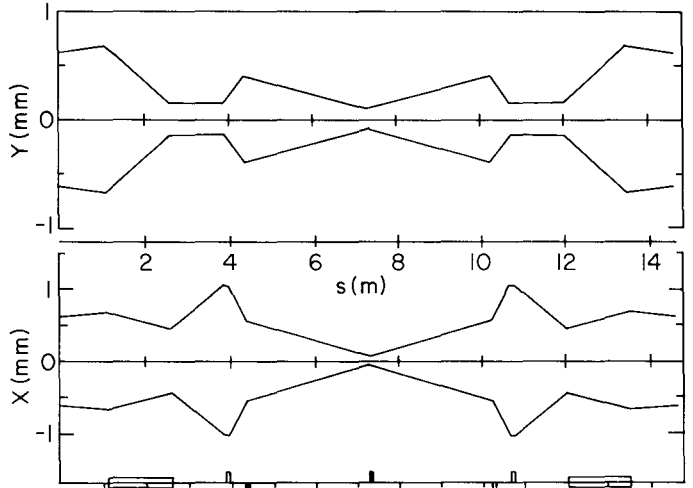


Fig. 5 Sample Transport Beam Envelopes for 430 MeV/c Electrons in a DSM SSS

#### V. Conclusions

More work remains to be done on these systems. The general stability problem must be addressed in regard to choices for  $\Theta_1$ , and the option of different values for  $\beta_A$ . The cumulative effect of perturbations in the DSM short-straight sections needs to be understood. The beam breakup threshold remains to be evaluated; the maintenance of beam waists in the linacs minimizes these effects.<sup>6</sup>

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