RESISTANCE DRIVEN BUNCHING MODE OF AN ACCELERATED ION PULSE*

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Summary

Amplification of a longitudinal perturbation of an ion pulse in a linear induction accelerator is calculated. The simplified accelerator model consists only of an applied field (E_a), distributed gap impedance per meter (R) and beam-pipe capacity per meter (C). The beam is treated as a cold, one-dimensional fluid. It is found that normal mode frequencies are nearly real, with only a very small damping rate proportional to R. This result is valid for a general current profile and is not restricted to small R. However, the mode structure exhibits spatial amplification from pulse head to tail by the factor $\exp(RCLv_o/2)$, where L is pulse length and v_o is drift velocity. This factor is very large for typical HIF parameters. An initially small disturbance, when expanded in terms of the normal modes, is found to oscillate with maximum amplitude proportional to the amplification factor. Unlike the analogous problem in a circular machine, linear growth is limited in amplitude by the finite pulse length. But the fact that frequencies are real cannot be taken to indicate the absense of large amplitude distortion.

Introduction

An induction linac driver for HIF involves the transport of an intense ion pulse through several kilometers of accelerator structure; this is long enough that the pulse is strongly coupled to itself via its interaction with the vacuum pipe and acceleration gaps.[1] A longitudinal bunching instability is intrinsic for this system, and unless suppressed by appropriate design it will generate an unacceptably large momentum spread.

The field in the gaps may be considered to act continuously and is programmed to axially confine as well as accelerate the beam. Spontaneous bunching is opposed by the local increase in space charge, and mode growth is not expected from this interaction alone.[2] The resistive character of the gap impedance is destabilizing, and application of the growth rate formula derived for circular machines gives an exponentiation length of only hundreds of meters for typical parameters.[3] That treatment, however, is not applicable to a finite length pulse in a straight system since the growing wave moves backwards in the pulse and is expected to convert to a decaying forward wave at the pulse tail. Qualitatively, we expect a distorted mode structure, with amplitude which is large in the tail and small in the head. This may be viewed as the result of a balance between resistive drag on the mode peaks and repulsion by the excess space charge in the tail. Momentum spread on the order of 1% may be sufficient for stability, but even this amount may be too large to meet final focusing requirements. In the following we assume that the pulse is cold and that the unperturbed field of the beam is exactly cancelled by the programmed field of the gaps. The one dimension fluid model for the pulse is the same as that of Channell, Sessler and Wurtele (C.S.W.), but the assumed gap impedance is less general than theirs.[4] This simplification allows for a more complete calculation of mode structure and definite restrictions on system parameters are obtained.

System Model

To avoid confusion about currents all quantities are defined in the laboratory frame. The pulse line density is n(t,z) and it drifts in the +z direction with velocity v(t,z). Since thermal effects are neglected the (non-relativistic) equations of motion are

$$\partial n/\partial t + \partial nv/\partial z = 0, \qquad (1)$$

$$\partial v/\partial t + v\partial v/\partial z = qE/M, \qquad (2)$$

and the electric field consists of applied and beam induced components:

$$E + E_a - Rqnv - (q/C)\partial n/\partial z \quad . \qquad (3)$$

As mentioned, R and C are the continuous representation of the effect of gaps and pipe. The capacity is taken to be that of the co-axial beam of radius a and pipe of radius b:

$$C = 4\pi\epsilon_o/[2\log(b/a)+.5] \qquad (4)$$

For simplicity the unperturbed velocity v_o is held constant and E_a is just the field required to cancel the field induced by the unperturbed current profile, i.e. the net unperturbed field vanishes.

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The assumed parameters are (C.S.W.): $R=200\,\Omega$ /m, b/a=1.5, M=200 Mp, q=2e, L=20 m, and total ion number $N=10^{15}$. Velocity is taken to be the final value $v_o = \beta_o c = .3c$, so we have the typical HIF parameters: T=9.06 Gev, W=1.45 MJ, I=1.44 kA. In order to reach the required power on the pellet (~100TW), the pulse must be subdivided or compressed by a factor of x 15; this would be done subsequent to acceleration and is not considered here.

It is important to note that there is some latitude in the selection of parameters. The value $C=.849\times10^{-10}$ F/m is fixed within a factor of two by geometry, and v_o is similiarly restricted by the target gain requirement. Considerable reduction of L is possible, but at increased cost since space charge density is thereby increased. In principal R can be reduced, however the driver efficiency is also reduced. To see this we note that in the simplist model of the gaps, efficiency is 100% if the impedance is matched to the current and accelerating gradient, i.e. if

$$R = E_o/I \equiv R_m \quad . \tag{5}$$

For the typical value $E = 10^6$V/m we have $R = 694\,\Omega$ /m at I=1.44 kA, and higher values at lower I. The assumed value $R=200\,\Omega$/m (which is used only at the final current) is therefore a moderate mismatch. More generally the efficiency (for the simple gap model) is

$$\eta = 4(R/R_m)(1+R/R_m)^{-2} \quad , \tag{6}$$

with value $\eta = .695$ for the assumed parameters.[5] Since non-zero R drives the bunching mode it may be desirable to consider a mismatch as great as $R_m/R=10$, which yields $\eta = .331$.

It is very convenient to make use of the variable

$$x = z - v_o t \quad , \tag{7}$$

which measures distance with respect to the pulse tail (x=0), with the head at x=L. We use x and t as the independent variables of the calculation - this has the appearance of a Galilean transformation, but current is still defined in the laboratory frame. Eqn (3) yields for given profile $n_o(x)$

$$E_a(x) = Rqn_o v_o + (q/C)dn_o/dx. \tag{8}$$

Perturbed System

We consider a longitudinal perturbation $n=n_o+\delta n$, $v=v_o+\delta v$, $E=\delta E$. Then using the variables (x,t), eqs (1-3) yield

$$\partial\delta n/\partial t = -\partial n_o \delta v/\partial x \quad , \tag{9}$$

$$\partial\delta v/\partial t = q\delta E/M \quad , \tag{10}$$

$$\delta E = -Rq(v_o\delta n + n_o\delta v) - (q/C)\partial\delta n/\partial x \quad . \tag{11}$$

The analysis is simplified by using the Lagrangian dispacement variable $\xi(t,x)$:

$$\delta v = (\partial\xi/\partial t)_x \quad , \tag{12}$$

and this is the point of departure from previous work. Eqs (9-11) become

$$\delta n = -\partial n_o \xi/\partial x, \tag{13}$$

$$\partial^2\xi/\partial t^2 = q\,\delta E/M \,, \tag{14}$$

$$\delta E = Rq(v_o\partial n_o\xi/\partial x - \partial n_o\xi/\partial t) + (q/C)\partial^2 n_o\xi/\partial x^2 \,. \tag{15}$$

Eliminating δn and δE, and grouping dimensional factors, we have

$$\frac{1}{v_o^2}\frac{\partial^2 n_o\xi}{\partial t^2} = \frac{\epsilon^2 n_o}{\bar{n}}\left[\frac{1}{r}\left(\frac{\partial}{\partial x} - \frac{1}{v_o}\frac{\partial}{\partial t}\right)n_o\xi + \frac{\partial^2 n_o\xi}{\partial x^2}\right]. \tag{16}$$

Here $\bar{n} = N/L$ is the mean density and we define

$$\epsilon^2 = (q^2\bar{n}/Mv_o^2\, C) = 2.24 \times 10^{-5} \,, \tag{17}$$

$$r = (RCv_o)^{-1} = .655 \text{ m} \quad . \tag{18}$$

The variable x is compared with lengths L and r. Time $(v_o t)$ is compared with the system length (~4.5km), and a natural frequency $(\epsilon/2r) = (277\text{m})^{-1}$. If values of v_o other than .3c are considered, then the product Rv_o/L should be held fixed to realize constant fractional mismatch of impedance.

The boundary conditions are $n_o\xi=0$ at the pulse ends. This makes the perturbed potential energy finite as $n_o \to 0$. In the special case of flat top $n_o(x)$, $n_o\xi$ must go smoothly to zero at the pulse ends even though n_o has a step. If n_o goes smoothly to zero at the ends then ξ may be finite there.

Mode Structure

Equation (16) is of an inconvenient form for analysis because of the first derivative in x. This is removed by defining

$$\psi(t,x) = n_o\xi\exp(x\lambda/L), \tag{19}$$

where

$$\lambda = L/2r = LRCv_o/2 = 15.3. \tag{20}$$

We are removing a mode distortion factor (which is large for the assumed parameters). Eq. (16) gives

$$\frac{1}{v_o^2}\frac{\partial^2\psi}{\partial t^2} = \frac{\epsilon^2 n_o}{\bar{n}}\left(\frac{\partial^2\psi}{\partial x^2} - \frac{\psi}{4r^2} - \frac{1}{rv_o}\frac{\partial\psi}{\partial t}\right) . \tag{21}$$

Note that the time derivative in Eq. (21) is small of order ϵ and therefore the last term on the right may usually be neglected; this is done in most similiar derivations and is equivalent to dropping the term proportional to δv in Eq. (11). If that term is dropped then Eq. (21) is of self-adjoint form and mode frequency (which is real and positive) can be evaluated for general $n_o(x)$ using a variational technique.

Here we estimate mode frequency for general $n_o(x)$. Let $\psi = g(x)\exp(-i\omega v_o t)$; then eqn (21) yields

$$-\left(\frac{\omega}{\epsilon}\right)^2 g = \frac{n_o}{\bar{n}}\left[\frac{d^2 g}{dx^2} - \frac{g}{4r^2} + \left(\frac{i\omega}{\epsilon}\right)\frac{\epsilon g}{r}\right] \quad . \qquad (22)$$

Multiplying all terms by$(Lg^*\bar{n}/n_o)$ and intergrating over x we obtain the quadratic in ω

$$A\left(\frac{L\omega}{\epsilon}\right)^2 + \frac{iBL\epsilon}{r}\left(\frac{L\omega}{\epsilon}\right)\left(\frac{L^2 B}{4r^2} + C\right) = 0, \qquad (23)$$

where A, B, and C are the positive quantities

$$A = \int \frac{dx}{L}\frac{\bar{n}}{n_o}|g|^2, \quad B = \int \frac{dx}{L}|g|^2, \quad C = \int \frac{dx}{L}\left|\frac{Ldg}{dx}\right|^2 . \qquad (24)$$

The eigenfrequencies are

$$\omega = -i\left(\frac{\epsilon^2}{2r}\right)\frac{B}{A} \pm \left[\left(\frac{\epsilon}{2r}\right)^2\left(\frac{B}{A} - \frac{\epsilon^2 B^2}{A^2}\right) + \left(\frac{\epsilon}{L}\right)^2\frac{C}{A}\right]^{\frac{1}{2}} \qquad (25)$$

For a flat top profile the normalized eigenfunctions are

$$g_m(x) = \sqrt{2L}\ \sin(m\pi x/L), \qquad (26)$$

with m any positive integer. In this case we have

$$A = B = 1/2\ ,\ C = m^2\pi^2/2. \qquad (27)$$

For general $n_o(x)$ we expect A/B to be of order unity and C/A to be of order $(\pi m)^2$, so we have

$$\omega \approx -i\left(\frac{\epsilon^2}{2r}\right) \pm \left[\left(\frac{\epsilon}{2r}\right)^2\left(1-\epsilon^2\right) + \left(\frac{\epsilon}{L}\right)^2\pi^2 m^2\right]^{\frac{1}{2}} . \qquad (28)$$

For the nominal parameters scale lengths are

$$2r/\epsilon^2 = 58.6\ \text{km},\ L/\epsilon = 4.23\ \text{km}, \qquad (29)$$

$$2r/\epsilon = .277\ \text{km}\quad ,$$

and their relative ordering is maintained for lower v_o if v_oR is fixed. We therefore expect frequencies clustered near $(277\text{m})^{-1}$ at low mode number $(m<10)$ and increasing proportional to m for $m > 10$. The damping length (58.6km) is of negligible consequence.

Discussion

The most important feature of the mode structure is the distortion factor [eq. (19)]. An initial perturbation must be expanded in the normal modes g_m:

$$n_o\xi = \exp(-x\lambda/L)\Sigma\ A_m g_m(x)\cos(\omega_m v_o t), \qquad (30)$$

with coeficients

$$A_m = \int dx\ g_m(x) n_o \xi(x,t=0)\exp(x\lambda/L). \qquad (31)$$

For mode numbers close to that of the initial disturbance A_m contains the factor e^{λ}. In particular, the flat top profile yields, for a perturbation $n_o\xi(t=o)=A\sin(\ell\pi x/L)$,

$$\frac{A\ell}{A} = \frac{e^{\lambda}-1}{\lambda}\left[1 + \left(\frac{\lambda}{2\pi\ell}\right)^2\right] \quad . \qquad (32)$$

This amplification factor is nearly $3\text{x}10^5$ for low mode number (nominal parameters). The result is that the small disturbances due to field errors in the gaps produce very large disturbances in the pulse tail after a drift length of ~300m. This disturbance is oscillatory with a spectrum of frequencies and eventually converts to a large momentum spread or loss of particles from the pulse ends.

Several possible cures are apparent. First, it is clear that sufficient velocity spread will damp the disturbance. A simple estimate based on a continuous pulse with a Lorentzian velocity distribution gives the requirement (hwhm)

$$\frac{\Delta v}{v_o} \geq \epsilon\sqrt{\frac{\lambda}{\pi}} = .0104 \quad , \qquad (33)$$

which is on the borderline of acceptability for final focussing requirements. If spread is used for mode suppression and is initially to low, it seems likely that the system will "overshot" by a large factor when it generates spread velocity by mode distortion.

A second class of cures consist of the use of feedback or high frequency filtering of δE in the accelerating modules - this is not considered here.

Finally, one may design the system such that the factor $\lambda = RCLv_o/2$ is only of order unity. A possibility would be $L\to 10\text{m}$, $R\to 25\Omega/\text{m}$, $I\to 2.88$ kA. Then $\lambda = .956$, but the efficiency drops to $\eta = .251$ and there is an increased cost for transport.

References

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