ELECTRODE SHAPES FOR SPHERICAL PIERCE FLOW

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Summary

The problem of obtaining the electrode shapes to produce a conically converging proton beam that has constant current density over each spherical surface of convergence is treated in spherical coordinates. A cone is taken from the Langmuir and Blodgett[1] solution for the region within, and at the edge of, the conically converging beam. A solution for the LaPlace equation, required for the region outside the beam, is in terms of a power series in r and the Legendre polynomials of cos ϕ

$$V(r,\phi) = A_n P_n r^n \quad .$$

The Langmuir and Blodgett potentials required at the edge of the beam are fitted by a power series in r,

$$V(r) = \sum a_n r^n \quad .$$

At the beam edge the two solutions are matched by making $A_n P_n = a_n$.

Introduction

A mathematical method has been used to obtain the shapes of electrodes required to produce a conical spherically converging beam that has a uniform current density over each spherical surface of convergence. This is generally referred to as Pierce flow.

The theory for converging flow between concentric spheres was determined by Langmuir and Blodgett. The potentials required as a function of radius, those required along the edge of the beam, can be determined from Table I of the reference.

The problem is treated in spherical coordinates. A solution for the LaPlace equation, required for the region outside the beam, is known. The variables are separable and the solution is in terms of a power series in r and the Legendre polynominals of cos ϕ

$$V_{\phi,r} = A_0 P_0 r^0 + A_1 P_1 r^1 + A_2 P_2 r^2$$
$$+ A_3 P_3 r^3 + A_4 P_4 r^4 + \ldots .$$

where,

$$P_0 = 1, \quad P_1 = \cos\phi, \quad P_2 = 3/2 \cos^2\phi - 1/2 \quad ,$$

$$P_3 = 5/2 \cos^3\phi - 3/2 \cos\phi \quad ,$$

$$P_4 = 7/4 \cdot 5/2 \cos^4\phi - 5/4\ 3/1 \cos^2\phi + 3/4 \cdot 1/2 \quad .$$

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Each term of the series is a solution of the differential equation. In addition, any sum of such solutions is also a solution. Consequently, a truncated series is a valid solution.

Five terms of $V_r = \sum_{n=0}^{n=4} a_n r^n$ were found to provide a good fit to the table of potentials given by Langmuir and Blodgett for converging flow between concentric spheres (see Table I). Any larger number of terms is also allowable.

Following their procedure the equations are reduced to dimensionless form by using r as

$$r = \frac{R\text{cm}}{R_0\text{cm}} \quad \text{and V as } V = \frac{V\text{volts}}{V_{max}\text{volts}} \quad V_{max} \quad .$$

The values of r for the case in which the diameter of the beam is reduced to half its initial value are such that $r \leqslant 1$ so r^n decreases as r decreases and n increases.

In the first try, the coefficients a_n, of

$$V = \sum_{n=0}^{n=4} a_n r^n$$

were chosen to fit the reference table at five places. Subsequently, the coefficients were chosen by a least squares fit (see final paragraph, page 3) to all the values given in the table.

The coefficients of the two series

$$V = \sum_{n=0}^{n=4} a_n r^n$$

for the region within, and at the edge of, the beam and

$$V = \sum_{n=0}^{n=4} A_n P_n r^n$$

for the region outside the beam are matched at the edge of the beam; that is,

$$a_n r^n = A_n P_n r^n \quad \text{or} \quad A_n = a_n / P_n \quad \text{beam edge} \quad .$$

Note that P_n is a function of ϕ, and the match is made for ϕ corresponding to the angle between the axis of the beam and the conical beam edge.

Table I gives the Langmuir and Blodgett values for V as a function of r, and those obtained by the least-squares fit to the power series

$$V = \sum_{n=0}^{n=4} a_n r^n \quad .$$

Table I

LEAST SQUARES FIT

$$V = \frac{v_{volts}}{v_o volts} V_{max}$$

$r = \frac{Rcm}{R_o cm}$	Langmuir and Blodgett	fit	Differences
0.50000	0.82537	0.82495	0.00038
0.50761	0.79596	0.79586	0.00010
0.51282	0.77642	0.77643	-0.00001
0.52083	0.74720	0.74737	-0.00017
0.52632	0.72779	0.72799	-0.00020
0.54054	0.67953	0.67980	-0.00027
0.55555	0.63166	0.63189	-0.00023
0.57143	0.58422	0.58432	-0.00010
0.58824	0.53726	0.53721	0.00005
0.60606	0.49082	0.49087	0.00015
0.62500	0.44497	0.44474	0.00023
0.64516	0.39977	0.39953	0.00024
0.66667	0.35530	0.35511	0.00019
0.68966	0.31166	0.31158	0.00008
0.71429	0.26897	0.26904	-0.00007
0.74074	0.22737	0.22758	-0.00021
0.76923	0.18706	0.18733	-0.00027
0.80000	0.14828	0.14849	-0.00021
0.83333	0.11135	0.11138	-0.00003
0.86956	0.7676	0.07652	0.00024
0.90910	0.4524	0.04488	0.00036
0.95238	0.01819	0.01821	-0.00003
0.98039	0.00540	0.00578	-0.00038
0.99009	0.00215	0.00244	-0.00029
1.00000	0.00000	-0.00043	0.00043

The coefficients are:

6.55530 -23.52025 34.91341 -25.22779 7.27889

The coefficients a_n of $V = \sum_{n=0}^{n=4} a_n r^n$ are

a_0	6.5530
a_1	-23.52025
a_2	34.91341
a_3	-25.22779
a_4	7.27889

Differences between the two values for V as a function of r are given also. Most of these differences are a few parts in ten thousand, so the fit is adequate for our purpose.

The idealized electrode shapes are those of the corresponding equipotential surfaces.

The example given below is that used in the first design for the PIGMI injector.[2] This is for a 50-mA uniform proton beam from an idealized spherical plasma surface. The beam is accelerated to 250 000 eV and converges spherically from an 8- to 4-mm diam. The point of convergence would be 9.354 cm from the plasma surface. The 4-mm-diam exit aperture at -250 kV is 4.677 cm from the above mentioned center of convergence. The extraction electrode, at -50 000 V with respect to the plasma surface, is at 7.38 cm from the same center. The half-angle of the beam, from the axis to the edge is 2.3985°. The above numbers are also shown in Table II. A zero emittance beam is considered for the purpose of the calculation.

The shapes for the equipotential lines, those on which the shape of the column electrodes are based, are shown in Fig. 1. Table III gives numerical values for the shape of the electrode meeting the plasma surface from which the beam is

Table II

VOLTAGES AND DIMENSIONS

Electrodes	v	$V = \frac{v}{v_{max}} V_{max}$	R_{cm} beam	r	d
Accelerating column anode	0	0.00000	9.354 cm	1.0	8.00 mm
Extractor	- 50 kV	0.16500	7.380 cm		6.31 mm
Exit aperture	- 250 kV	0.82537	4.677 cm	0.5	4.00 mm

The angle between the axis of the beam and the beam edge is 2.398°. The proton beam current is 50 mA; R_0 = 9.354 cm. v_{max} = 250 kV.

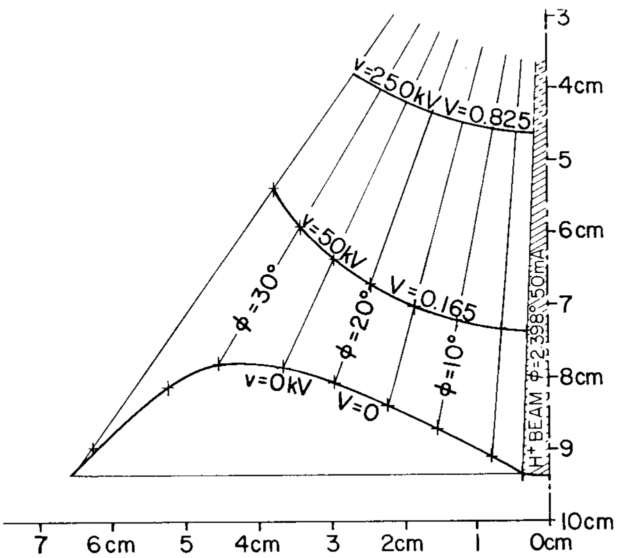

Fig. 1. Equipotentials used for electrode surfaces.

Table III

SHAPE FOR THE COLUMN ANODE

	Ycm	Xcm
0°	0.00000	9.35400
Beam edge at 2.398°	0.39147	9.34500
5°	0.79800	9.12123
10°	1.53951	8.73101
15°	2.24870	8.39226
20°	2.94945	8.10356
25°	3.67532	7.88175
30°	4.54203	7.86703
35°	6.27522	8.96194

extracted. This electrode is the anode of the accelerating column.

Table III gives the shape calculated for the column anode, that electrode from which the beam is extracted. Usually we have called this the Pierce Anode.

For the extraction electrode (at -50 kV) and the exit aperture (at -250 kV), the surfaces were close enough to spherical, near the edge of the beam, so that these surfaces were made as concentric spheres with the center at the point where the beam would converge, if the protons were to continue along their radial lines after emerging from the exit aperture.

Application of the simple-lens formula at the exit aperture, where the beam enters a field free space, indicates that the beam would still have adequate convergence after taking the exit-aperture lens effect into consideration. Further calculations by Ralph Stevens, Jr., indicated that a waist would occur at a favorable position for a buncher that was to be placed close to the exit aperture.

The criteria for the converging PIGMI beam were set up by D. A. Swenson. E. P. Chamberlin called attention to the existence of a suitable program for the least-squares fit and the least-squares-fit calculation was carried out by R. W. Hamm. Ralph Stevens, Jr. called attention to the Langmuir-Blodgett work and did the calculations regarding the formation of the waist following the exit of the beam from the -250-kV, 4-mm-diam exit aperture.

References

1. Focusing of Charged Particles, Albert Septier, Ed. (Academic Press, New York, New York, 1967), V. II, p. 32.

2. R. W. Hamm, R. R. Stevens, D. W. Mueller, J. N. Leavitt, H. M. Lederer, "A Compact 250-kV Injector System for PIGMI," IEEE Trans. Nucl. Sci. 26, p. 1493 (1979).