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NEW FORMULATION OF THE RFQ RADIAL MATCHING SECTION*

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Summary

The electric field potential function in an RFQ radial matcher has been formulated, adopting the lowest order potential function expanded in a Fourier-Bessel series. The focusing strength varies sinusoidally with the distance along the beam axis. The overlap between the time-dependent RFQ acceptance and the injected beam phase space area has been calculated. The overlap more than 90% can be attained with a several cell long radial matcher, and it is found that the length must be chosen accurately at (integer) × $\beta\lambda$.

Introduction

The radial matcher at the entrance of an RFQ linac plays an important role to attain a high capture efficiency, especially in a machine, of which the acceptance is not large enough. As the ellipse parameters of the RFQ acceptance change with the rf phase, it is necessary to match the time-independent injected beam emittance ellipse through a radial matcher. In the radial matcher the beam is focused and de-focused alternately along the beam axis, where the focusing strength increases from zero to its final value.

In spite of the significance of the radial matcher, the electric field distribution in it has never been formulated. We have derived the electric potential function imposing a ground plane and line on the cavity end wall and the beam axis, respectively, and adopting the lowest order potential function expanded in a Fourier-Bessel series. The resultant focusing strength increases sinusoidally with the distance along the beam axis, which is appreciably different from the linearly increasing one proposed by LASL.[1)]

In this paper we describe the derivation of the potential function and the performance of the new radial matcher, i.e. the capability of achieving a good overlap between the phase space areas to be matched.

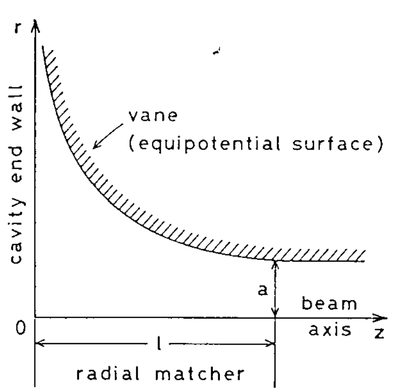


Fig. 1. Geometry of the radial matcher.

* The main part of this work was done at Lawrence Berkeley Laboratory, University of California, USA.

Formulation of the Potential

The potential of the electric field in a four conductor quadrupole cavity is generally expanded in a Fourier-Bessel series.

$$U(r,z,\psi,t) = \frac{V}{2}\left[F_o(r,\psi) + \sum_{n=1}^{\infty} F_n(r,\psi)\sin nkz \right] \times \cos(\omega t+\phi) , \quad (1)$$

where

$$F_o(r,\psi) = \sum_{m=0}^{\infty} A_{om} r^{2(2m+1)} \cos 2(2m+1)\psi , \quad (2)$$

$$F_n(r,\psi) = \sum_{m=0}^{\infty} A_{nm} I_{2m}(nkr)\cos 2m\psi , \quad (3)$$

$$k = \pi/L , \quad (4)$$

and r and z are defined as in Fig. 1. The length, L, is half of the periodic length. To determine the coefficients we impose boundary conditions: The potential vanishes on the beam axis and the cavity end wall,

$$U(r=0) = 0 , \quad (5)$$

$$U(z=0) = 0 . \quad (6)$$

From the condition of Eq. (6), the potential must be an odd function with respect to z, so $F_o(r,\psi)$ vanishes. As the potential is quadrupole symmetric, only the term of m = 1 in Eq. (3) is taken. Then the potential is

$$U(r,z,\psi) = \frac{V}{2}\left(\sum_{n=1}^{\infty} A_{n1} I_2(nkr)\sin nkz \right)\cos 2\psi . \quad (7)$$

This potential satisfies the condition of Eq. (5). Taking the lowest order potential, n = 1, we have

$$U(r,z,\psi) = \frac{V}{2}\frac{I_2(kr)}{I_2(ka)}\sin kz\cos 2\psi . \quad (8)$$

To obtain a smooth connection of the radial matcher and the modulated vane, the z-component of the electric field must be vanishing at the end of the radial matcher, $z = \ell$. So the periodic length L should be equal to 2ℓ, and

$$k = \pi/2\ell . \quad (9)$$

The electric field components are derived as

$$E_r = -\frac{V}{2}k\frac{I_1(kr) - (2/kr)I_2(kr)}{I_2(ka)}\sin kz\cos 2\psi , \quad (10)$$

$$E_z = -\frac{V}{2}k\frac{I_2(kr)}{I_2(ka)}\cos kz\cos 2\psi , \quad (11)$$

$$E_\psi = V\frac{1}{r}\frac{I_2(kr)}{I_2(ka)}\sin kz\sin 2\psi , \quad (12)$$

The focusing strength is obtained from the coefficient of the linear term of E_r,

$$B(z) = \frac{1}{8}\frac{k^2a^2}{I_2(ka)}B_o\sin kz , \quad (13)$$

$$\simeq B_o\sin kz , \quad (14)$$

The vane shape of the radial matcher is given by equipotential surface obtained from Eq. (8):

$$\frac{I_2(kr)}{I_2(ka)} \sin kz \cos 2\psi = 1 \ , \qquad (15)$$

$$0 \le z \le \ell. \qquad (16)$$

Figure 2 shows the vane shape and electric field lines in the x-z plane calculated for LITL (Lithium Ion Test Linac)[2]; ℓ = 5.876 cm (12 cells), a = 0.041 cm. Practically the vane is chopped at z = 0.4 cm, so that the electric field in a beam passing region (r $\lesssim$ 0.25 cm) might not change so much. The gap distance between the end wall and the vane should be determined also considering the electric surface field and the convenience of attaching end tuners.

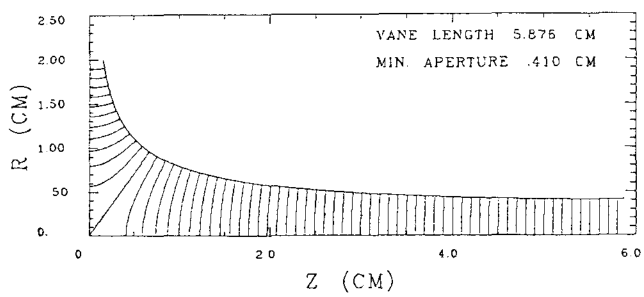


Fig. 2. Electric field lines in the radial matcher.

Beam Dynamics Study of the Radial Matcher

With the sinusoidally increasing focusing strength, the overlap between the input beam phase space area and the RFQ acceptance has been calculated. It has been found that the length of the radial matcher effects on the overlap significantly.

The ellipse parameters of the RFQ acceptance are first calculated for various phases in the modulated vane, where the focusing strength is constant: B_o = 5 according to the LITL design. Figure 3 shows such obtained ellipses, corresponding to phases 90° apart. The normalized emittance is 0.6 π mm·mrad (β = 0.00326). The parameters of the phase spaces, at the input of of the radial matcher, to be matched with the above acceptances at the output are calculated by use of a matrix. This is derived from the product of transfer matrices of segments, into which a cell is divided. Phase space ellipses with such parameters are shown in Figs. 4 (a) and (b) for two cases of 11 and 12 cell long radial matchers, respectively. As two cells comprise one focusing unit, it is expected that a good overlap among the ellipses is attained with a radial matcher of even-number cell length. This is graphically shown in the figures. For the case of 12 cells it is possible to draw an ellipse over the phase space ones to attain an overlap with tem more than 90%. The time-independent phase space of the input beam must be shaped to the ellipse. With 11 cells the overlap reduces to 50%.

Figure 5 shows the dependence of the overlap on the length of the radial matcher. Sharp peaks appear at every even-number cell length. This means that the length of the radial matcher must be accurately set at (integer) × $\beta\lambda$. In the figure are plotted transmissions through RFQ structures, of which the modulated vane part is same as that of LITL. The computer code PARMTEQ is used in the calculation. The transmissions agree well with overlaps. Generally, however, the overlap is not always equal to the transmission, as it depends on the whole RFQ structure.

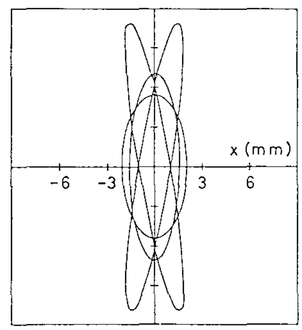
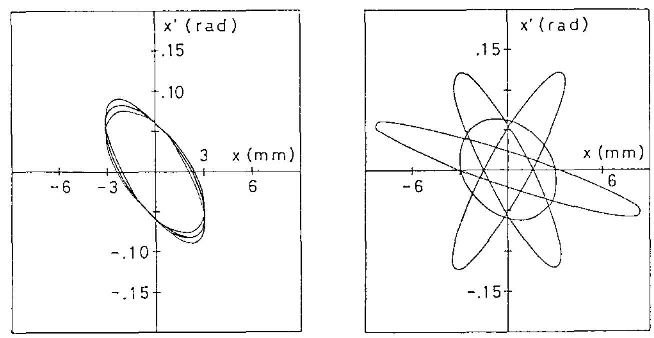


Fig. 3. Phase space ellipses of the RFQ acceptance. The rf phases are apart by 90°.



(a) (b)

Fig. 4. Phase space ellipses at the input of the matcher. They are matched with the RFQ acceptance ellipses through a radial matcher of 12 cells (a) or 11 cells (b).

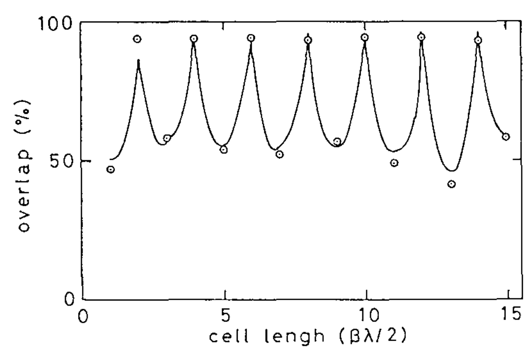


Fig. 5. Dependence of the overlap, between the input beam phase space area and the RFQ acceptance, on the length of the radial matcher (solid line). Circles denote transmissions.

Figure 6 shows the reduction of transmission due to space charge effect for two extreme cases; an 11 cell long radial matcher and a 12 cell long one. With 11 cells the plateau extends to a higher current than with 12 cells. This pattern agrees qualitatively with the experimental result of POP at LASL.[3]

From the above considerations, a radial matcher part of an RFQ linac should be designed so that its length can be regulated to correct the slight change of the length due to the effect on a hole for beam entrance and the chop of the vane.

Conclusion

The potential of the electric field in the radial matcher of an RFQ has been formulated. The derived focusing strength increases sinusoidally from zero to its final value with the distance along the beam axis. The length of the radial matcher is measured from the end wall of the rf cavity, but not from the truncation point of the vane. From the view point of beam dynamics, the length must be accurately $n \times \beta\lambda$ (n : integer). The overlap between the time-independent injected beam phase space area and the time-dependent RFQ acceptance has been calculated, and a high value more than 90% has been attained.

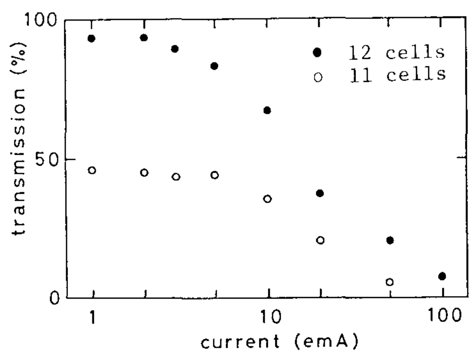


Fig. 6. Space charge effect on the transmission for two cases of radial matchers with 11 and 12 cells.

Acknowledgment

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