

SOME PERFORMANCE OF THE INTERLACED ACCELERATOR STRUCTURE

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Summary

The performance of the interlaced standing wave accelerator structure has been reviewed in detail with a testing model and a computer program.

Introduction

Nowadays medical linear accelerators have found wide applications in radiotherapy in the world. According to the form of the trajectory of the electron beam, medical linac can be divided into two types: bent beam and straight beam machines. The straight beam machines have many advantages over the bent beam ones owing to its small size and compact structure. But the length of the accelerating tube in straight beam machines should not exceed 30 cm otherwise the isocentric height will be unacceptable. In order to obtain higher energy electron beam in such a short tube, it must have the possibility to withstand very high electric field gradient. The interlaced side coupled standing wave structure (IL) proposed by V. Vaguine of Varian Associates has obtained the highest accelerating gradient in comparison with other accelerating structures. But since the efficiency of IL structure is approximately 20 percent lower than the common side coupled structure (SC), the relative advantage of IL structure becomes apparent only in the range of higher accelerating gradient (above 40 MV/m). Meanwhile the 3 mm beam hole diameter seems too small to make a longer accelerating tube. So it would be worthwhile to study the performance of the IL structure in more detail.

Influences of the geometrical parameters of the IL cavity

The typical one quadrant cross sections of IL, IL with nose cone and SC are shown in Fig. 1. The web wall thickness t_w is determined by mechanical rigidity and thermoconductivity. In IL cavity the web wall thickness also affects the coupling between two cavity chains. A web thickness of 3 mm which was adopted by Vaguine has been kept constant in our calculations. Each time when we change one parameter the other parameters will remain unchanged.

Calculated data for each parameter are shown graphically in Fig. 2-3. The resonant frequency is kept at 2998 MHz. The relationship between resonant frequency and various geometrical parameters in shown in Fig. 4. It can be seen that the effect of the variation of geometrical parameters on the effective

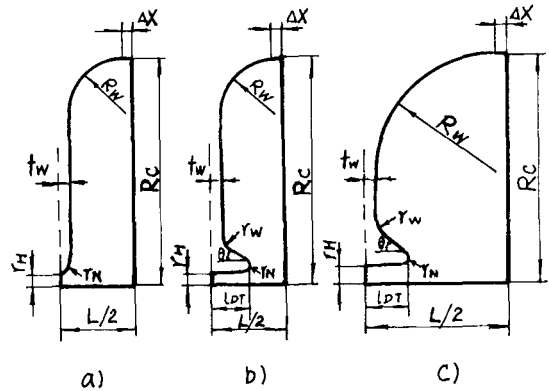


Fig. 1. One quadrant cross section of various cavities.

- a). IL
- b). IL with nose cone
- c). SC

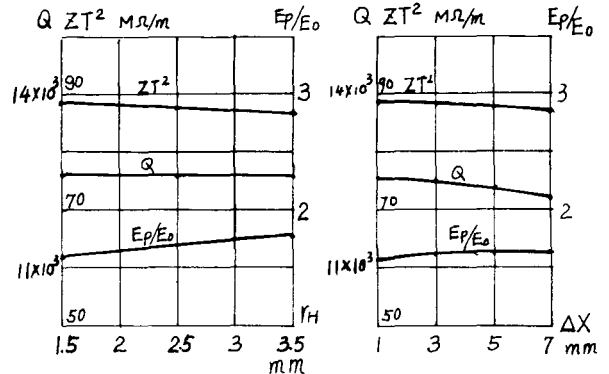


Fig. 2. $Q, ZT^2, Ep/E_0$ vs r_H and X (IL)

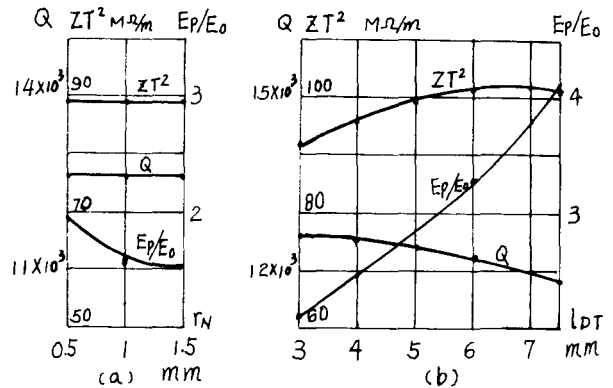


Fig. 3. $Q, ZT^2, Ep/E_0$ vs r_N, l_{DT}
 a). IL
 b). IL with nose cone

Table 1. Comparison of IL cavity with and without nose cone with SC cavity.

	L mm	tw mm	θ	r_H mm	r_N mm	r_W mm	l_{DT} mm	ΔX mm	Z M Ω /m	T	ZT ² M Ω /m	ZT ² /Q	Ep/Eo
SC	50	1.5	30°	3	2	1	10	3.5	162.6	0.822	109.7	6213.9	3.64
IL with nose cone	25	1.5	30°	3	2	1	6	1	110.4	0.958	101.4	8115.8	3.14
IL	25	1.5	-	1.5	1.5	-	0	0.5	104.8	0.921	88.9	7050.9	1.35

shunt impedance is very slight while the effect on the parameter Ep/Eo is quite apparent. The effects of beam hole and rounded corner radius on resonant frequency are also very slight. The only sensible parameter on resonant frequency in IL cavity without nose cone is the cavity radius Rc.

The IL cavity with nose cone has been calculated in order to be compared with the SC cavity. The results indicate that there is also an unsharp optimum length of nose cone. (see Fig. 3b) As in the case of IL cavity, the IL cavity with nose cone can not attain the same efficiency level of SC cavity either. Meanwhile in this case the parameter Ep/Eo increases rapidly. The comparison of physical parameters is listed in Table 1.

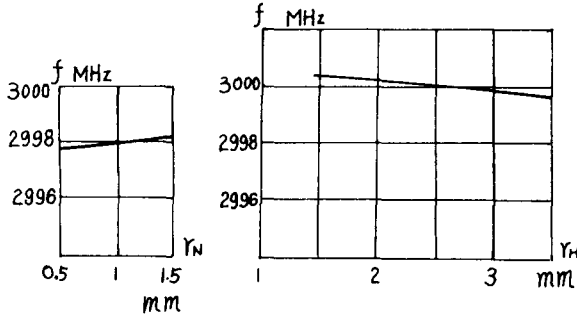


Fig.4. resonant frequency vs r_N , r_H (IL)

Since there are not sensible dimensions in IL on resonant frequency except Rc, we have tested mechanical compressive method. It is observed that a total frequency shift of only 0.4 MHz can be obtained under appropriate pressure.

Influence of the direct coupling between two cavity chains

The IL structure is composed of two separately independent cavity chains. The beam hole at the web wall center is used only for transmission of accelerated electron beam. When the beam hole radius increases, the direct coupling between two cavity chains also increases. The relationship between the direct coupling coefficient K' and the beam hole radius has been measured as shown in Fig. 5.

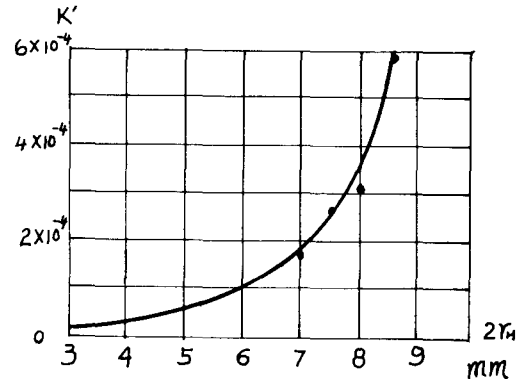


Fig.5. K' vs $2r_H$. (IL)

The equivalent circuit of IL structure with direct coupling between two cavity chains is shown in Fig. 6. The diagrammatic sketch with unequal number of cavities in two chains is shown in Fig. 7.

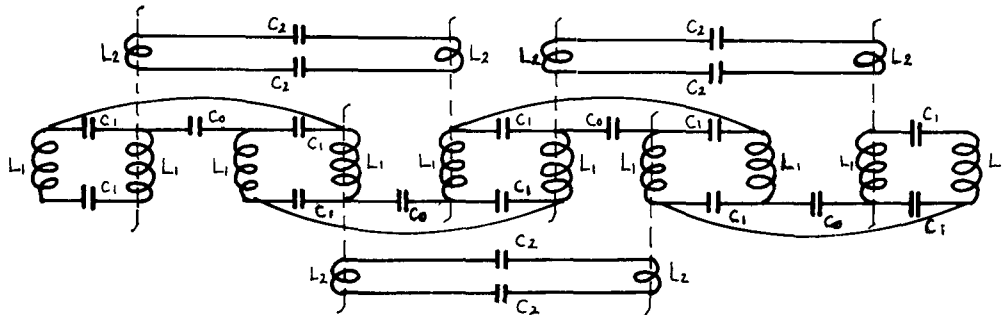


Fig. 6. Equivalent circuit of IL structure with direct electrical coupling.

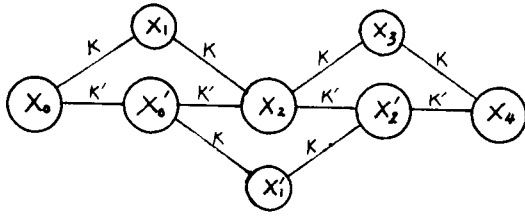


Fig. 7. Diagrammatic sketch of LL structure with direct electrical coupling. (unequal number of cavities in two chains)

For homogeneous and lossless case, the coupled resonator equations of Fig. 7 can be written as:

$$\begin{aligned}
 X_0(1 - \frac{\omega_a^2}{\omega^2}) + \frac{K}{2}X_1 + \frac{K'}{2}\frac{\omega_a^2}{\omega^2}X_0' &= 0 \\
 X_1(1 - \frac{\omega_c^2}{\omega^2}) + \frac{K}{2}(X_0 + X_2) &= 0 \\
 X_2(1 - \frac{\omega_a^2}{\omega^2}) + \frac{K}{2}(X_1 + X_3) + \frac{K'}{2}\frac{\omega_a^2}{\omega^2}(X_0' + X_2') &= 0 \\
 X_3(1 - \frac{\omega_c^2}{\omega^2}) + \frac{K}{2}(X_2 + X_4) &= 0 \\
 X_4(1 - \frac{\omega_a^2}{\omega^2}) + \frac{K}{2}X_3 + \frac{K'}{2}\frac{\omega_a^2}{\omega^2}X_2' &= 0 \\
 X_0'(1 - \frac{\omega_a^2}{\omega^2}) + \frac{K}{2}X_1' + \frac{K'}{2}\frac{\omega_a^2}{\omega^2}(X_0 + X_2) &= 0 \\
 X_1'(1 - \frac{\omega_c^2}{\omega^2}) + \frac{K}{2}(X_0' + X_2') &= 0 \\
 X_2'(1 - \frac{\omega_a^2}{\omega^2}) + \frac{K}{2}X_1' + \frac{K'}{2}\frac{\omega_a^2}{\omega^2}(X_2 + X_4) &= 0
 \end{aligned}$$

Where $\omega_a^2 = \frac{1}{L_1 C_1}$, $\omega_c^2 = \frac{1}{L_2 C_2}$

$K = \frac{M}{\sqrt{L_1 L_2}}$, $K' = \frac{C_0}{C_1 + C_0}$

$\sqrt{2L_1} in = \lambda n$, $\sqrt{2L_1} in' = \lambda n'$, $n = 0, 2, 4 \dots$

$\sqrt{2L_2} in = \lambda n$, $\sqrt{2L_2} in' = \lambda n'$, $n = 1, 3 \dots$

The solutions of $\frac{\pi}{2}$ mode case will be the superposition of following two groups of solutions.

$$\begin{cases}
 X_0 = -X_2 = X_4, & X_1 = X_3 = 0 \\
 X_0' = X_2' = 0, & X_1' = 0 \quad \omega_{\frac{\pi}{2}} = \omega_a = \omega_c
 \end{cases}$$

$$\begin{cases}
 X_0 = X_2 = X_4 = 0, & X_0' = -X_2' \\
 X_1 = -X_3 = -\frac{K'}{K}X_0', & \\
 X_1' = 0, & \omega_{\frac{\pi}{2}} = \omega_a = \omega_c
 \end{cases}$$

It can be seen that a small amplitude of fields will appear in the coupled cavities which will lower the efficiency of the accelerator.

The amplitude of X_0 and X_0' will be determined by the hybrid coupler which may be designed to obtain equal amplitude of X_0 and X_0' .

Influences of the detuning effects of two cavity chains

The energy gain V can be represented by the following formula:

$V = UT$

Where U - Equivalent voltage at cavity.
 T - Transit time factor.

When the detuning of two cavity chains takes place, it leads to the decrease of field amplitude and the transit time factor which will in turn decrease the beam energy gain.

$$\frac{\Delta V}{V} = \frac{\Delta U}{U} + \frac{\Delta T}{T}$$

Let f_1, f_2 represent the $\frac{\pi}{2}$ mode resonant frequency of two cavity chains respectively, f_0 is the frequency of RF source. We can define:

$$f_0 = \frac{1}{2}(f_1 + f_2) \quad \Delta f = \frac{1}{2}(f_1 - f_2)$$

$2\Delta f$ is the frequency difference between two cavity chains.

Equivalent voltage at a high Q resonant system can be written as

$$U = \frac{U_0}{1 + jQ_L \frac{2\Delta f}{f_0}}$$

$$\frac{\Delta U}{U} = -\frac{1}{2}Q_L^2 \left(\frac{2\Delta f}{f_0}\right)^2$$

$$\cos \Delta \phi = 1 / \sqrt{1 + (Q_L \frac{2\Delta f}{f_0})^2}$$

Where U_0 is the equivalent voltage at resonance.

$\Delta \phi$ is the phase shift caused by the detuning.

When the number of accelerating cavity N is small, the electrons are not so tightly bunched that the influence of the cavity phase shift is not apparent. When the electron beams are being bunched aggressively the influences of the phase shift also increases. When the electron beams are well bunched, further increase of N would not apparently change the effect of detuning phase shift. Similar to the case in the travelling wave tube, we may suppose that the effect of detuning phase shift changes exponentially with N . When $N=1$, the LL structure has been turned into a single cavity, the effects would disappear.

So that $\Delta T/T$ can be presented in the following equation.

$$\Delta T = \frac{\left(\int_{-\lambda/8}^{\lambda/8} E(z) \cos\left(\frac{2\pi z}{\lambda} + \Delta \phi\right) dz - \int_{-\lambda/8}^{\lambda/8} E(z) \cos\left(\frac{2\pi z}{\lambda}\right) dz \right)}{\int_{-\lambda/8}^{\lambda/8} E(z) dz} e^{-\frac{1}{N}}$$

$$\text{or } \frac{\Delta T}{T} = -\frac{1}{2} e^{-\frac{1}{N-1}} Q_L^2 \left(\frac{2\Delta f}{f_0}\right)^2$$

The relative energy decrease can be written as:

$$\frac{\Delta V}{V} = -\frac{1}{2} (1 + e^{-\frac{1}{N-1}}) Q_L^2 \left(\frac{f_1 - f_2}{f_0}\right)^2$$

The above formula is very close to the dynamic calculation results obtained by computer program as shown in Fig. 8.

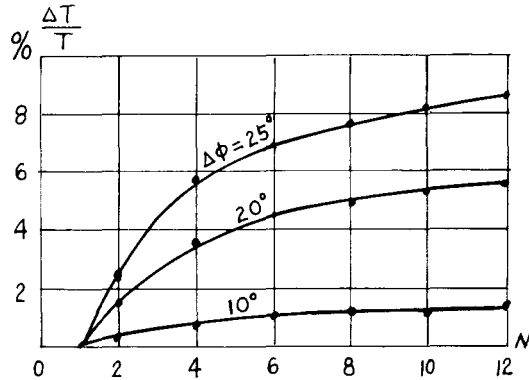


Fig. 8. Comparison of the formula with the results obtained by computer program

Power feeding into IL cavity chains with unequal number of cavities

When the two cavity chains have unequal numbers of accelerating cavities, the RF power in two chains should be in proportion to the numbers of the accelerating cavities of the two chains.

The hybrid coupler of the TEM/ H_{01} quadrature type is shown in Fig. 9.

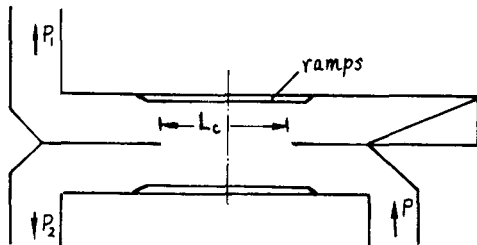


Fig. 9. The cross section of hybrid coupler

The outputs from two arms of hybrid coupler are always in quadrature but their power ratio depends on the length of coupler slot.

The relationship between the power ratio and the slot length can be presented as:

$$L_c = \frac{1}{\pi} \frac{\lambda_g \lambda_0}{\lambda_g - \lambda_0} \tan^{-1} \sqrt{\frac{P_1}{P_2}} = \frac{1}{\pi} \frac{\lambda_g \lambda_0}{\lambda_g - \lambda_0} \tan^{-1} \sqrt{\frac{N_1}{N_2}}$$

Where λ_g - H_{01} mode guide wavelength
 λ_0 -free space wavelength.

N_1, N_2, P_1, P_2 -cavity number and power needed in two cavity chains.

When $N_1=N_2, L_c = \frac{1}{4} \frac{\lambda_g \lambda_0}{\lambda_g - \lambda_0}$, it is the slot

length for 3db hybrid coupler.

The measurement results are highly consistent with the above equation. (Fig. 10)

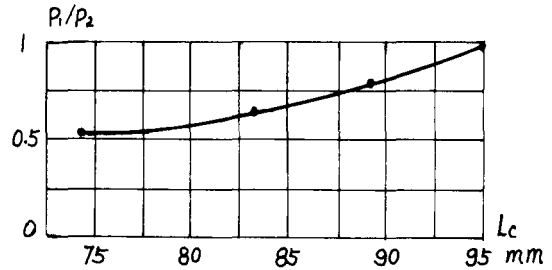


Fig. 10. Comparison of the experiment results with the formula.

Conclusion

The IL standing wave structure is a hopeful structure in medical linac with straight beam trajectory. But there are still some problems to be cleared. The following points have been analysed in this paper:

1. Except the cavity wall radius there are no other geometrical parameters sensible to resonant frequency. The mechanical compressive method can give limited frequency shift. The IL structure and IL with nose cone can not attain the same efficiency level of SC structure under lower accelerating gradient.
2. The IL structure needs two separate cavity chains. The increase of direct coupling between two cavity chains owing to the larger beam hole radius will lead to the decrease of efficiency.
3. The detuning of two cavity chains will lead to the decreasing of equivalent voltage and the transit time factor which will in turn decrease the beam energy gain.
4. A IL structure with unequal number cavities of two chains could be fed by the hybrid coupler with appropriate slot length.

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