THE LONGITUDINAL SPACE CHARGE PROBLEM IN THE HIGH CURRENT LINEAR PROTON ACCELERATORS: A NEW CONCEPT

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Summary

In a linear proton accelerator peak currents of 200 mA lead to high space charge densities and the resultant space charge forces reduce the effective focussing considerably. In particular the longitudinal focussing is affected. A new concept based on linear theory is proposed that restricts the influence of the space charge forces on the longitudinal focussing by increasing a, the mean transverse bunch radius, as $a \propto (\beta\gamma)^{3/8}$. This concept is compared with other concepts for the Alvarez (1 MeV - 100 MeV) and for the high energy part (100 MeV - 1100 MeV) of the SNQ linear accelerator.

Scaling Equations

Beam dynamics is simple and beam losses do not occur as long as nonlinear forces can be neglected. Space charges reduce the linear forces by a factor of $1-\mu_l$ (in longitudinal direction) and $1-\mu_t$ (in transverse direction). Thus μ_l or μ_t growing beyond 0.8 means a strong reduction of the linear forces and the nonlinear forces have to be taken into account.

To obtain an efficient acceleration, E_oT and the phase φ_s are kept constant. Because of this in particular μ_l tends to increase during acceleration:

$$\mu_l = 1 - \sigma_l^2 / \sigma_{lo}^2$$

$\sigma_{lo}^2 = \text{const} \cdot L_l^2 \cdot (\beta\gamma)^{-3}$ for <u>constant</u> φ_s and E_oT

The longitudinal bunch radius b is connected with the longitudinal tune σ_l, the normalized emittance E_{nl} and the period length L_l via

$b^2 = \text{const} \cdot E_{nl} \cdot L_l/(\beta\gamma^3 \cdot \sigma_l)$

Therefore

$$\mu_l = 1 - \text{const} \cdot E_{nl}^2 \cdot \beta/(\gamma^3 b^4) \qquad (1)$$

In linear approximation E_{nl} is constant and thus μ_l is approaching 1 unless b is getting <u>smaller</u>.

This result is in contrast to the idea that the charge density and thus μ_l should decrease by making b larger. It can be understood as follows: Under the constraint of a constant E_{nl} an increase of b can be achieved only by increasing the betatron function i.e. by reducing the effective focussing. Since E_oT and φ_s are fixed the effective focussing can be reduced only by increasing the longitudinal Coulomb forces which means by making μ_l larger.

To prevent μ_l from growing b must vary as

$$b = \text{const} \cdot (E_{nl}^2 \cdot \beta \cdot \gamma^{-3})^{1/4} \qquad (2)$$

In the simplest approximation (bunch is an ellipsoid with constant charge density) a second equation is obtained for μ_l[1]

$$\mu_l = \text{const} \cdot M_z \cdot \beta/(a^2 b) \qquad (3)$$

where M_z is the ellipsoid form factor and a the mean transverse bunchradius. In the present case M_z can well be approximated by a constant. Then the condition for keeping μ_l constant reads

$$a \propto (\beta^3 \gamma^3 / E_{nl}^2)^{1/8} \qquad (4)$$

As long as Eq. (4) holds true the longitudinal linear forces are not further reduced.

Discussion

Comparison for High Energy part of the SNQ linac (100 MeV - 1100 MeV, peak current 200 mA, frequency 200 MHz): The high energy part of the planned SNQ linac consists of 160 periods (period length 3 m) each containing 4 single cells placed in equal distances and a FODO quadrupole structure.

Two designs are compared (cf table I): One with constant magnetic field gradient[2] (design 1) the other fulfilling eq (4) (design 2). Common parameters: $E_{nl} = 2\,\pi°$ MeV, $E_{nt} = 9\,\pi$ mm mrad

$$\varphi_s = -24° \; , \; E_o T = 3.7 \text{ MeV/m} \qquad (7)$$

Comparison for Alvarez (1 MeV - 100 MeV, peak current 200 mA, frequency 200 MHz). The Alvarez has been described elsewhere[3]. Two designs are compared (cf table II): One with constant transverse tune σ_t (design 3) the other fulfilling eq (4) for energies higher than 20 MeV (desgin 4).
Common parameters: $E_{nl} = 1\pi°$MeV, $E_{nt} = 9\,\pi$ mm mrad $\varphi_s = -35°$, $E_o T = 1{,}4$ MeV/m

The advantages of design 1 and 3 are a smaller bunch radius and a higher transverse tune. The longitudinal acceptance is decreasing because it is $\propto (1-\mu_l)^{5/2}$. The advantages of design 2 and 4 are a twice as high $1-\mu_l$ factor, a strongly increasing longitudinal acceptance and a less extreme ratio transverse energy / longitudinal energy (TE/LE) of particles in the bunch. To compare advantages and disadvantages of the designs quantitatively multiparticle calculations will be done.

Table I
Designs for SNQ High Energy Linac

	Design 1		Design 2	
energy (MeV)	100	1100	100	1100
long. accept. ($\pi°$ MeV)	9	5.3	5.2	28.0
max. bunch radius (cm)	1.5	1.1	1.2	1.8
bunch length (cm)	1.1	1.0	1.2	0.9
σ_t (°)	20.2	7.1	31.3	2.7
$1-\mu_t$	0.39	0.57	0.49	0.34
σ_l (°)	11.0	0.8	9.8	1.1
$1-\mu_l$	0.42	0.14	0.33	0.28
magn. field grad. (T/m)	3.6	3.6	4.6	2.0
TE/LE	1.6	23.8	2.6	6.0

Table II
Designs for SNQ-Alvarez

	Design 3		Design 4	
energy (MeV)	1	100	1	100
long. accept. ($\pi°$ MeV)	1.8	1.0	1.1	3.0
max. bunch radius (cm)	1.2	1.1	1.1	1.4
bunch length (cm)	0.5	1.1	0.5	1.0
σ_t (°)	13.6	13.6	18.3	8.0
$1-\mu_t$	0.36	0.48	0.41	0.36
σ_l (°)	14.6	2.4	13.3	3.0
$1-\mu_l$	0.52	0.12	0.44	0.2
magn. field grad. (T/m)	33.5	3.4	37.0	2.5
TE/LE	1.2	8.0	2.3	3.5

References

1) K. Mittag, Kernforschungszentrum Karlsruhe, KfK-Report 2555 (1978)
2) S. A. Martin et. al, presented on this conference
3) The Alvarez is described in the paper of M. Pabst, presented on this conference