TRACE: AN INTERACTIVE BEAM-TRANSPORT CODE[*]

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Summary

TRACE is an interactive, first-order, envelope-tracing, beam-dynamics computer code with space charge. It includes some unique features as well as a number of elements not commonly found in other beam-transport programs such as the permanent-magnet quadrupole (PMQ), radio-frequency quadrupole (RFQ), rf gap, accelerator column, and accelerator tank. The code also has a number of fitting capabilities, allowing almost any element parameter in the beamline to be varied, including space charge. TRACE calculations provide immediate graphic display, including the beam envelope and the phase-space ellipses in the transverse dimensions. The program is easy to use and contains its own help package that lists all instructions necessary for input, calculations, and graphic output.

Introduction

The original version of TRACE[1] was written in 1973 and was developed for use on the controls computer of the Los Alamos Meson Physics Facility (LAMPF). It facilitated beam-transport calculations, starting with measured properties of the LAMPF beam, and served as an aid in tuning the beam transport. A modified version[2] was written for the controls computer for the CERN linac in 1977; in 1979, the CERN version was adapted to the PIGMI controls system, and was used in tuning the transport for the test of the RFQ linac. Because of its usefulness as a design tool as well as a tuning aid, it was expanded and adapted for use on the CDC 7600 and the VAX-11/750 computers.

General Description

TRACE follows the evolution of the beam, represented by beam matrices, through a set of transport elements. Each element is divided into an integer number of segments, where the integer is determined by a specified maximum step size. For each segment, 2-by-2 transfer matrices are calculated for the x-x' and y-y' planes. Each transfer matrix is that of a thick lens, either focusing or defocusing depending on the sign of the total force "constant" generated from the sum of the externally applied fields and the space-charge forces. As the beam is followed through the elements, the transfer matrices for the complete distance are accumulated. Useful information can be obtained from the transfer matrices, examples of which are the phase advance and the matched ellipse parameters for a periodic system.

This computational technique is valid only for linear forces. For the space-charge forces to be linear, one must postulate a beam having a uniform charge distribution. Although real life is rarely so accommodating, it has been shown that, for distributions having ellipsoidal symmetry, the evolution of the rms beam envelope depends almost exclusively on the linearized part of the self-forces.[3] Consequently, for calculational purposes, the "real beam" may be replaced by an "equivalent beam" having the same rms properties and a uniform charge distribution. The emittance of the equivalent beam should be four times the rms emittance.

Normally, at the beginning of the calculation, the initial beam ellipses are displayed on the graphics terminal. At each step, the x- and y-profiles are calculated and displayed, and at the end, the final beam ellipses are displayed.

Transport-System Elements

The transport elements are defined by a "type-code" and by five or fewer parameters (except for a bending magnet). The 16 elements recognized by TRACE are given in table I along with their type-code and associated parameters. We cannot define all of the parameters here; however, the meaning of many of them are obvious. Five of the more unusual elements are described briefly.

TABLE I

TRACE TRANSPORT-SYSTEM ELEMENTS AND PARAMETERS

Element	Type Code	Parameters
Drift	1	l
Quadrupole	2	B', l
Symmetric Doublet	3	B', l, s
Bending Magnet (con't)	4	α, ρ, β_1, β_2, g θ, n, b, k_1, k_2
Symmetric Triplet	5	B'_o, l_o, s, B'_1, l_1
Buncher/Debuncher	6	$\Delta\phi_i$, $\Delta\phi_f$, d, ΔW, V_r
Solenoid	7	B_s, l
RF Quadrupole	8	B_1, B_2, n_{RA}, n_s, ϕ
RF Gap	9	E_oT, ΔW, ϕ_s
Thin Lens	10	1/f
Accelerator Tank	11	l_t, ϕ_s, n_c, W_1, W_o
Accelerator Column	12	r_a, l, ΔW, n_s, I
Current Change	13	ΔI
R-matrix (Horiz.)	14	R_{11}, R_{12}, R_{21}, R_{22}
R-matrix (Vertical)	15	R_{33}, R_{34}, R_{43}, R_{44}
Permanent Magnet Quadrupole	16	B', l_p, r_1, r_o, s_t

Radio-Frequency Quadrupole

This element is extremely useful in determining the matched conditions at the input of an RFQ linac in which there is a time-varying quadrupole field. The x-component of force from the external field is proportional to B(z) cos [(2πz/βλ) + φ]x, where B may be a function of z. TRACE can accommodate three forms for B: a constant, a linear taper, and an S-shaped curve. This latter form has particularly good characteristics to be used in a radial matching section, whose function is to adapt the continuous beam to the time-varying characteristics of the RFQ.

Calculating a radial match with TRACE is simple. A single command entered by the user instructs TRACE to take the matched beam in the periodic portion of the RFQ, backtrace it through the radial matching section to the entrance of the RFQ (once for each phase section of the first periodic cycle), and finally average all of the output beams to obtain the best starting beam

parameters at the entrance to the RFQ. The code displays this graphically.

Permanent-Magnet Quadrupole

PMQs are typically short in length and have strong pole-tip fields, resulting in "soft edge" fringe fields that effectively extend out along the beamline a distance two to three times the inner quad radius. The fringe field of a rare-earth cobalt PMQ of standard design, depicted in fig. 1, is known analytically and has been verified experimentally.[4] The fringe fields are calculated as a function of distance F(z) where F(z) is constructed from two semi-infinite quadrupoles whose finite ends overlap one quadrupole length. TRACE searches the beamline for any PMQs to superimpose the fringe fields on any adjacent elements by adding the squares of the force constants. On the graphic output, the extent of the fringe fields are shown with dotted lines (see fig. 2).
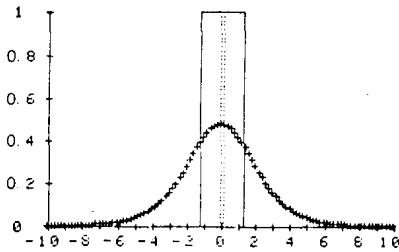
Radio-Frequency Gap (rf gap)

This element is intended to simulate the behavior of an acceleration gap in a drift-tube linac. It acts as a thin defocusing lens in the transverse planes, and its inverse focal length is

$$\frac{1}{f} = -\frac{\pi q E_0 T \sin \phi_S}{m_0 c^2 \beta^2 \gamma^3} \quad ,$$

where $E_0 T$ is the acceleration gradient and ϕ_S the synchronous phase.

Accelerator Tank

This element simulates the effects of any coupled-cavity linac tank on the transverse beam dynamics. One tank consists of a sequence of n_C identical cells having a π phase shift between adjacent accelerating cells. Examples of such structures are side-coupled linacs and disk-and-washer linacs. The parameters are the tank length ℓ_t, the synchronous phase ϕ_S, the number of cells n_C, and the input and output energies W_i and W_0. An additional term is included in the equations of motion because of the continuous energy gain:

$$x'' + \frac{(\beta\gamma)'}{\beta\gamma} x' + k^2 x = 0 \quad .$$

Assuming the coefficient of x' to be constant, one can construct transfer matrices for any arbitrary distance along the tank.

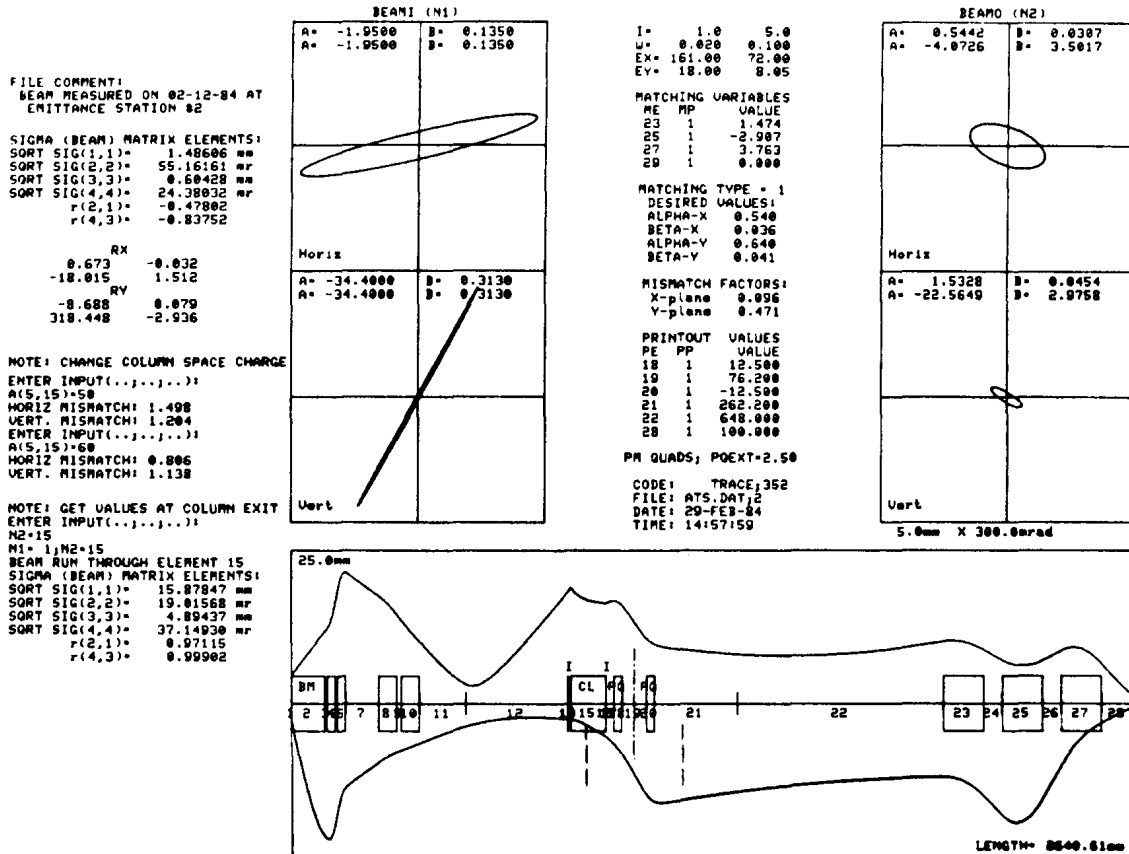Fig. 1. Permanent-magnet quadrupole. Fringe field--2-in. inner radius.

Fig. 2. TRACE output graphics.

Accelerator Column

This accelerator column models the column electrodes as two thin lenses separated by incremental energy-change gaps. The same cubic function used in the code SPEAM[5] is used to smooth the potential function ϕ in calculating the fringe field:

$$\phi = V_0 + \left(\frac{z}{3r_0}\right)^3 \frac{r_0}{L} \Delta W \quad .$$

This function is integrated a distance twice the aperture radius before the column, and one aperture radius into the column. The reverse is true as you exit the column. The function is converted to a focal length for a thin-lens approximation. TRACE can also handle the multielectrode column by making two or more columns contiguous.

Types of Matching

Eight types of matching criteria are allowed by TRACE and are listed in table II. Six of these types are concerned with finding proper values for a specified number of transport-system parameters (such as quad strengths) that cause the beam to have the desired characteristics at a given location. The other two types are concerned with finding the proper input beam ellipse parameters at a certain location. All of these situations involve finding the solution to a set of either two or four nonlinear equations.

TABLE II

TYPES OF MATCHING

Match Type	Desired Values to Match	Comments
1	α_x β_x α_y β_y	Fits desired values at any point in beamline
2	X' at X max Y' at Y max	
3	X max Y max	
4	X max X' max Y max Y' max	
5	α_x β_x (n1=n2)	Limiting section: beam clipping (X and X')
6	Input and output beam parameters	For periodic system (sets output equal to input)
7	α_x β_x	For solenoids
8	α_y β_y	

The method used is that of regula falsi, an iterative procedure that, starting with an initial guess for the solution, hopefully converges to a solution in a reasonable number of trials. A solution is assumed to have been found if a convergence criterion has been met. If a solution has not been found after a given number of iterations, the procedure is terminated with the best solution that has been found. The procedure is easily restarted, taking the best solution as the initial guess.

Using the Code

TRACE was designed to be used on small computers and was intended to address small or limited beam-dynamics problems that require no more than simple first-order approximations. Therefore, running TRACE is different from most other beam-transport codes in that all work is done interactively. From a user's viewpoint, this is often highly desirable. Typing a single keyboard character can effect calculations, print-outs, and/or graphic display. The versatility of the command procedure allows one to easily exchange data, run the beam from any one point to another, run forward or backward, do fitting calculations at any point, etc. The output may prompt the user to alter the beamline or to modify a parameter value and recalculate the output beam. There is no need to open and/or alter a data file; all data manipulation and calculations are performed on-line in an interactive mode, giving immediate results. Data files can be read, altered, or rewritten as desired from the interactive mode.

A list of commands recognized by TRACE can be seen in fig. 3 below, which is itself part of the on-line help package accessible from the interactive mode by typing "H." Output beam parameters can be requested at any point along the transport system by typing "O"; in similar fashion, the user can obtain the beam matrix, the transformation matrix, the length of the beamline, the beam profiles or phase ellipses, and the mismatch factor. All information is printed beside the graphic display, leaving a complete record of all transactions, modifications, and resulting calculations down the left-hand side of the page (see fig. 2). For example, while doing fitting calculations, the decreasing mismatch factor and corresponding parameter variations can be displayed after each group of iterations, recording the changes taking place in the transport system. The time, date, file name, comments stored in the data file, or comments entered interactively can all be displayed as part of the record alongside the graphical output.

Fig. 3. Command description. Portion of TRACE help package.

References

1. K. R. Crandall, "TRACE: An Interactive Beam Transport Program," Los Alamos Scientific Laboratory report LA-5332 (October 1973).
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5. W. Joho, "SPEAM, A Computer Program for Space Charge Beam Envelopes," TRIUMF, Design Note TRI-DN-73-11, March 1973.