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A DESIGN CONCEPT FOR THE SNQ ALVAREZ LINEAR ACCELERATOR FOR LOW **E**MITTANCE GROWTH

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# Summary

As a part of a spallation neutron source SNQ a 200 MHz Alvarez linear accelerator has been designed. Because of the high peak current of 200 mA space charge is important; because of the high average current of 5 mA particle losses must be kept low. Considerations and requirements concerning the Alvarez linac parameters for minimum emittance growth are presented. As a result from multiparticle calculations our design shows no transverse emittance growth. The longitudinal emittance growth is caused by a too small longitudinal acceptance. For the longitudinal case a detailed analysis is presented.

# Introduction

In this paper we present the design concept for the SNQ Alvarez linear accelerator. The main parameters are listed in table I.

#### Table I

PARAMETERS OF THE SNQ ALVAREZ LINAC

n	roton
2	MeV
102	MeV
200	mΑ
5	mA
250	μs
100	Ηz
201.2	5 MHz
	p 2 102 200 5 250 100 201.2

The choice of the listed parameters shows the similarity with existing Alvarez linear accelerators, e.g. at BNL, FNAL, CERN, LANL. The essential difference is the large average current of 5 mA of the SNO linac. Because of this high average beam current the activation by beam losses is a severe problem. Therefore the main task of the beam dynamics is the determination of the machine and beam parameters for minimum particle loss. Up to now no theoretical method exists which can predict particle losses. Our philosophy is to approach this problem by designing the linac for minimum emittance growth. This is the main content of the second part of this paper.

#### Properties of the Structure

The design of the structure of the SNQ Alvarez linac was based mainly on the design procedure of the CERN new 50 MeV linac<sup>1</sup>. We used the computer programs CLAS and GENLIN. The linac consists of 9 tanks each consuming 3 MW peak power. This leads to a cavity length of roughly 10 m. The synchronous phase  $\phi_{\rm e}$  has been chosen -32° along the whole linac.

The average accelerating field  ${\rm E}_{_{\rm D}}$  has a tilt in the first tank. It starts with 1.30 MV/m at the beginning and increases up to 1.84 MV/m at the end. For all other tanks  $E_{o}$  is kept constant. Its value is around 1.7 MV/m. The discontinuity of  ${\rm E}_{\rm o}$  is needed to keep E.T continuous. T is the transit time factor. Because of the high duty cycle E, has been chosen rather low to garanty a save operation of the machine. For reasons of flexibility the first tank is designed to consume less power than 3 MW. It allows to vary the synchronous phase  $\phi$  down to -50°. Keeping the particles' energy gain per gap constant the resulting  $E_o$  is 1.7 MV/m at input. With this option an increase of about a factor 4 of the longitudinal acceptance without current is possible. Concerning the problem of keeping the particles losses low the variability can be very helpful.

The geometrical dimensions of the cavities and the drift tubes are very similar to the CERN new linac ones.

## Beam Dynamics

#### General Considerations

Our goal for the beam dynamics is to design the SNQ Alvarez linac with minimum emittance growth. The emittance growth we are considering here results from resonances and instabilities due to high space charge. We do not discuss the influence on the emittance caused by gas scattering, intrabeam scattering and imperfections of the machine. For completeness we shortly list the effects we are concerning.

- (i) mismatch
- (ii) longitudinal-transverse resonances caused by rf
- (iii) envelope resonances caused by space charge forces
- (iv) instabilities caused by collective space charge effects
- (v) too small longitudinal acceptance

For keeping emittance growth as low as possible one tries to fix the beam parameters in such a way that all of the listed effects can be avoided. It turns out that at the beginning of the SNQ Alvarez linac the conditions (i) to (v) can not be avoided simultaneously. Due to the high space charge the longitudinal acceptance is smaller than the input emittance. This results in emittance growth. In the following we study this effect in some more detail.

## Analysis of the Longitudinal Acceptance

For the analysis some formulae are needed. The relationship between the particle tunes  $\sigma$  with full current to the tunes without

2)

current is given by 1

$$\sigma_{t} = \sigma_{to} (1 - \mu_{t})^{\overline{2}}$$
 (1)

anđ

$$\sigma_{1} = \sigma_{10} (1 - \mu_{1})^{\frac{1}{2}}, \qquad ($$

Here the index t refers to the transverse and the index 1 to the longitudinal motion. The zero current tunes are denoted by  $\sigma_{to}$  and  $\sigma_{1} \cdot \mu_t$  and  $\mu_1$  are the space charge parameters. For the space charge we assumed the bunch to be represented by an uniformly filled ellipsoid with the transverse diameter 2a and the bunch length 2b. The bunch dimensions are calculated from the envelope equations in smooth approximation.

$$a = \left(\frac{E_t \cdot L_t}{\sigma_t}\right)^{\frac{1}{2}}$$
(3)

$$b = \left(\frac{E_1 \cdot L_1}{\sigma_1}\right)^{\frac{1}{2}}$$
(4)

Here E and L denote the total emittances and the period length. The representation of  $\sigma_{to}$ ,  $\sigma_{1o}$ ,  $\mu_{t}$  and  $\mu_{1}$  in form of machine and beam parameters can be found elsewhere<sup>2</sup>.

The longitudinal acceptance we express by

$$A_{1} = A_{10} (1-\mu_{1})^{\frac{5}{2}}$$
 (5)

with

$$A_{1o} = \left(-\frac{2\lambda}{3\pi} q mc^2 E_o T \beta_s^3 \gamma_s^3 \varphi_s^5\right)^{\frac{1}{2}}.(6)$$

Here q is the charge and mc<sup>2</sup> the rest mass of the proton.  $\beta$  and  $\gamma$  are the relativistic factors of the synchronous particle and  $\lambda$  is the wavelength of the rf field.

Expression (6) includes nonlinear terms of the accelerating electric field up to third order<sup>2</sup>. The zero current acceptance  $A_{10}$  describes roughly the area inside the longitudinal separatrix which is fixed by machine parameters only. Including a quadratic space charge potential which is centered at  $\varphi$  Eq. (5) results. We see that the zero current acceptance is reduced by the factor  $(1-\mu_1)^{5/2}$ which involves the beam current.

For the design of the Alvarez linac  $E_{o}$ , T,  $\varphi_{s}$ ,  $\beta_{s}$ ,  $\gamma_{s}$ ,  $\lambda$ , q, mc are input parameters and are fixed by the machine. If the emittances are known only one parameter is free in Eqs. (1) to (4). It is common then to fix  $\sigma_{t}$  to have an average beam radius which fits properly into the aperture. Then all other parameters are fixed, especially  $\sigma_{1}$ ,  $\sigma_{to}$ ,  $\mu_{1}$ ,  $\mu_{r}$ ,  $A_{1o}$  and  $A_{1}$ . Therefore  $A_{1}$  is a function of the quantities  $E_{1}$ ,  $E_{t}$  and  $\sigma_{t}$ . For a better understanding we show that with certain approximations  $\mu_{1}$  and therefore also  $A_{1}$  are functions of the function of the parameters are fixed.

$$\mathbf{x} = \frac{\mathbf{E}_{1n}^{2} \cdot \mathbf{E}_{tn}}{\sigma_{t}}$$
(7)

with the total normalized emittances E<sub>1</sub> and E<sub>tn</sub>. E<sub>1</sub> is given here in units of ° eV. We start with Eq. (2) expressing  $\sigma_1$  and  $\mu_1$  by machine and beam parameters and insert Eq. (4) into the left hand side of Eq. (2). Next we approximate the formfactor in the formula of  $\mu_1$  by a/(3 $\gamma$  b) which is a useful approximation for 0.8  $\leq$  b/a  $\leq$  3. Taking the square on both sides of Eq. (2) the result is a quadratic equation in b°. Solving this equation for b° and inserting the result into the approximation of  $\mu_1$  we get

$$\mu_{1} = \frac{2}{1 + \sqrt{1+\theta}}$$
(8)

with  

$$\theta = - \frac{L_t E_o T \sin \varphi E_{\ln}^2 E_{tn}}{450 \pi qmc^2 I^2 \lambda \beta_s^2 \gamma_s^2 \sigma_t (Ohm)^2}$$
(9)

Here I is the peak current.

In this representation all parameters are independent among one another. Eqs. (8) and (9) show that  $\mu_1$  and A can be understood as functions of the parameter x. The choice of x like in Eq. (7) is rather arbitrary. We have chosen this expression because we have varied E n, E n and o in the multiparticle calculations discussed below.

For the SNQ Alvarez linac we varied the normalized input emittances between the following boundaries:

0.8 
$$\pi$$
 ° MeV  $\leq$  E<sub>1n</sub>  $\leq$  3  $\pi$  ° MeV

4.5  $\pi$  mm mrad  $\leq$  E<sub>tn</sub>  $\leq$  14  $\pi$  mm mrad

It turns out that at 2 MeV the longitudinal acceptance is always smaller than the emittance. This is a quite known effect. In Fig.1 the acceptance is shown as a function of the parameter x at input and at output of the Alvarez linac. Two important facts can be observed. On one hand we have a strong dependence of A<sub>1</sub> and  $\mu_1$  on the longitudinal emittance. If e.g. the longitudinal emittance is doubled then the acceptance can increase more than one order of magnitude. On the other hand A<sub>1</sub> can decrease or grow with increasing energy. This is due to the fact that A<sub>10</sub> increases and 1- $\mu_1$  decreases for higher energies. In Fig. 1 both situations are present. For small x A<sub>1</sub> decreases and for large x A<sub>1</sub> increases. This leads to the following conclusion.

If the longitudinal acceptance is smaller than the emittance then the unstable situation will cause longitudinal emittance growth. This reduces the defocusing space charge forces longitudinally. Therefore the acceptance can grow faster than the emittance and lead to a stable situation if the x is not too small.





## Multiparticle calculations

We expect no transverse emittance growth in the multiparticle calculations if the effects (i) to (iv) have been avoided. In case of a too small longitudinal acceptance there should be an increase of the longitudinal emittance. For the SNQ Alvarez linac we studied the emittance growth with the multiparticle program MAPRO. We varied all the parameters  $E_{1n}$ ,  $E_{1n}$  and  $\sigma_{1}$  independently. The calculations were done on a CRAY computer at the KFA. All runs were done with 2000 particles. The initial filling was a constant density in the transverse and longitudinal phase space independently.

As a result of many multiparticle calculations it turned out:

- no transverse emittance growth if the effects (i) to (iv) have been avoided
- the longitudinal emittance growth is caused by a too small longitudinal acceptance
- the longitudinal emittance growth is a function of the parameter x
- no particles losses in case of a too small longitudinal acceptance

In Fig. 2 the longitudinal rms emittance growth is shown as a function of x for various parameter combinations of  $E_{1}$ ,  $E_{t}$  an  $\sigma_{1}$ . Each point corresponds to one multiparand ticle calculation along the whole Alvarez linac. Circles correspond to calculations without resonances and instabilities. Black points are calculations were some resonances are present but they cause no remarkable emittance growth. The triangles are multiparticles calculations where we kept transverse tune (25°) and longitudinal emittance (1  $\pi$  °MeV) constant and varied the transverse emittance. The strong emittance growth probably is caused by collective instabilities.



Fig. 2 Longitudinal rms emittance growth as a function of the parameter x. Each point corresponds to a multiparticle calculation. In all cases at input the longitudinal emittance is larger than the acceptance.

In general we see a strong correlation between the longitudinal emittance and the parameter x. In all calculations we had a too small longitudinal acceptance which causes emittance growth. If we remember that the space charge parameter  $\mu_1$  is also a function

of x, we can correlate the emittance growth directly to  $\mu_1$ . This means that two multiparticle runs having different longitudinal input emittances can show similar emittance growth if the  $\mu_1$ 's at input are equal. The longitudinal emittance growth does not simply depend upon the ratio  $A_1/E_1$  it also depends on the transverse dimension of the beam. We expect this functional dependence between emittance growth and  $\mu_1$  only in the case of a too small longitudinal acceptance.

We can use Fig. 2 for practical purposes. If the input emittances are known one can reduce the emittance growth by chosing a smaller transverse tune. The restrictions are only that one has to take care of the conditions (i) to (iv) and of a not too large beam radius.

#### Conclusions

We have designed the SNQ Alvarez linear accelerator with almost no transverse emittance growth. At input the longitudinal emittance is larger than the acceptance. This leads to an increase of the longitudinal emittance. Because there are no particle losses we feel not too much disturbed by the longitudinal emittance growth.

For practical purposes we could present the emittance growth as a function of a parameter combination of the longitudinal emittance  $E_{n}$  and the transverse quantities  $E_{tn}$  and  $\sigma_t$ .

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## References

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