BACKGROUND ION TRAPPING IN RFQs

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Summary

The beam in an RFQ accelerator can ionize some of the residual background gas molecules producing low energy background ions. These low energy ions can be transversely stable in the RFQ beam bore-hole. When this happens the ions can only escape longitudinally from either end of the RFQ. If the rate of longitudinal diffusion is low compared to the ionization rate, a space charge build-up from these background ions could occur which might have significant effects on the RFQ current limit under long pulse and cw operation. This paper examines ion trapping and estimates its effect on RFQ beam dynamics.

Introduction

When an ion-electron pair is produced within a beam by ionization of the residual gas, the effects on the beam dynamics will depend on the production rate and the length of time the background ion or electron remains near the beam. For example, space charge neutralization of a positive ion beam occurs when the time of electron confinement by the beam's potential well significantly exceeds the escape time of background ions.

In RFQs the opposite result occurs. Electrons have very unstable trajectories within the RFQ bore and are lost to the vanes within one rf cycle, whereas the background ions are usually stable against loss in the transverse direction and are weakly stable longitudinally. If these ions were completely stable they would accumulate, increasing space charge in the bore until the beam itself became unstable. This does not occur however. The background ions eventually escape from the RFQ by diffusing longitudinally. Nevertheless, sufficient charge may accumulate to have an effect on the beam.

The following approach is used to estimate the significance of the trapped ions. First the stability of background ions in an RFQ is examined. Then the effective loss rate is estimated, based on a model of longitudinal diffusion produced by multiple-scattering of the ions by the background gas. Finally the equilibrium charge density of background ions is calculated and its effect on the RFQ beam dynamics determined.

Single Ion Stability in RFQs

When an atom or molecule is ionized by a charged particle beam, the residual ion usually receives very little kinetic energy, less than a few eV is typical. Thus, for simplicity, the ion can be considered to have been created at rest in an RFQ. Thereafter, its motion is determined by the equations of motion in the RFQ potential Φ[1] given by:

$$\Phi(x,y,z,t) = \frac{V}{2}\sin(\omega t - \phi_s)\left[C_0\left(\frac{x^2-y^2}{R_1^2}\right) + \frac{A_1}{2}\cos kz(\cosh kx + \cosh ky)\right] \quad (1)$$

where: V is the maximum inter-vane potential difference

ω is the rf angular frequency,

ϕ_s is the synchronous phase,

R_1 is the minimum bore radius,

$k = 2\pi/\beta\lambda$ is the longitudinal wave number, and

A_1, C_0 are coefficients which depend upon the geometrical dimensions of the RFQ cell.

For charged particles initially at rest, there are stable nodes along the RFQ axis located at $kz=n\pi$. The equations of motion, when they are linearized about these points, can be placed in the form of Mathieu equations giving:

$$\frac{d^2u}{d\tau^2} + (a_u - 2\,q_u\cos 2\tau)u = 0 \quad (2)$$

where $u = x,y,z$ (z now represents the longitudinal displacement from the nearest stable node) and $\tau = (\omega t - \phi_s - \pi/2)/2$ is a dimensionless variable corresponding to time. For background ions:

$$q_x = \frac{2eZV}{m\omega^2}\left[\frac{C_0}{R_1^2} \pm \frac{A_1k^2}{4}\right] \quad (3a)$$

$$q_y = -\frac{2eZV}{m\omega^2}\left[\frac{C_0}{R_1^2} \pm \frac{A_1k^2}{4}\right] \quad (3b)$$

$$q_z = \pm\frac{eZVA_1k^2}{m\omega^2} \quad (3c)$$

where the signs are +, - for n even, odd respectively and eZ/m is the charge to mass ratio of the residual or background ion. If the modulation term with A_1 is small, equation (2) places the same transverse stability requirements on these ions as the usual linearized RFQ beam dynamics equations places on the beam. Consequently, if the RFQ has been designed to accelerate beam with a specific charge to mass ratio, then all residual ions with ratios less than or equal to that of the beam will also be transversely stable. However ions with higher ratios may not be stable. A similar stability analysis shows that all electrons in the bore of an RFQ are unstable for transverse motion and are rapidly lost to the vanes.

a_u is the ion space charge term which in the linear approximation is represented by:

$$a_u = \frac{-3eZ\,Q\,F_u}{\pi\,\varepsilon_0 m\omega^2\,ABC} \quad (4)$$

and corresponds to the effect of a uniformly charged elliptical bunch with total charge Q and semi-axes A, B, C in the x,y,z directions respectively. F_u is a dimensionless form factor[1].

When q_u and a_u are less than 1 the oscillation frequencies ν_u (in units of τ) about the nodes are approximately[2]:

$$\nu_u^2 = \frac{q_u^2}{2} + a_u \quad (5)$$

This is analogous to the smooth approximation for the phase advance per focusing cell in RFQ beam dynamics[1].

Since a_u is negative the effect of space charge is to reduce the ν_u at each node. As Q is increased, loss of stability occurs when ν_u is zero in any direction. Loss of longitudinal stability occurs first since usually $\nu_z \ll \nu_x, \nu_y$. This loss of stability does not mean that the ions are accelerated out, only that the longitudinal potential modulation may be neglected at higher charge densities. The maximum amount of charge which can be trapped at a node is:

$$Q_{max} = \frac{\pi \varepsilon_o eZ\ V^2\ A_1^2\ k^4\ ABC}{6\ m\ \omega^2 F_z} \tag{6}$$

Longitudinal Diffusion of Background Ions

Assume that sufficient charge, Q_o, has accumulated in all RFQ cells so the longitudinal electric fields can be neglected. Then additional ions produced in the bore hole simply oscillate transversely and drift longitudinally. While oscillating, the ion can scatter off of the background gas. The net effect is to transfer some of the ion's transverse momentum to the longitudinal direction, increasing the ion's longitudinal momentum. Additional similar random scatterings can occur until the ion gains enough momentum to escape from the RFQ longitudinally.

From a statistical analysis[3] of multiple scattering, it can be shown that the probability density distribution, $P(z,t)$, of the longitudinal ion position at time t for an ion formed at z=0, t=0, is given approximately by the Gaussian distribution:

$$P(z,t) = (\pi\ s(t))^{-1/2} \exp\ (-z^2/s(t)) \tag{7}$$

where s(t) is given by:

$$s(t) = \frac{2\pi N}{3mp_T} \left(\frac{m+M}{m}\right)^2 \left[\frac{e^2\ Z_1\ Z_2}{2\pi\ \varepsilon_o}\right]^2 \ln(\theta_{max}/\theta_{min}) t^3 \tag{8}$$

and where: p_T is the transverse momentum
eZ_1, eZ_2 are respectively the ion and background gas atom's nuclear charges
m,M are respectively the ion and background gas atom's nuclear masses
N is the particle density of the background gas
θ_{min}, θ_{max} are the minimum and maximum scattering angles for which the Coulomb scattering approximation implicit in (8) can be used.

Since p_T for each ion varies in time, and the maximum value of p_T varies with the ion's radial position at creation, the average value of p_T is used in (8) as an approximation. Assuming the transverse motion is sinusoidal, and the ions are produced with uniform density up to radius R, then the average of p_T over time and the transverse plane is:

$$\bar{p}_T = 2\ v_T\ m\ \omega\ R/(3\pi) \tag{9}$$

The characteristic time, t_c, for an ion to escape longitudinally is when the probability of the ion being found anywhere within an RFQ of length L has decreased to 1/e. For an ion created at z_o this requires that:

$$\int_{-L/2}^{L/2} P(z - z_o,\ t_c)dz = 1/e \tag{10}$$

For z_o not near the RFQ ends, equation (10) leads to the approximation:

$$s(t_c) \approx L^2/2.6 \tag{11}$$

that can be solved for t_c.

Charge Accumulation

The evolution of the total background ion charge, Q_T, can be written as:

$$\frac{dQ_T}{dt} = I\sigma NL - (Q_T - Q_o)/t_c \tag{12}$$

where the first term on the right represents the rate of ion production by a beam current I on a gas of density N with an average ionization cross section σ. The second term gives the approximate longitudinal loss rate of ions in excess of Q_o. The total charge reaches an equilibrium value of

$$Q_T = I\sigma NLt_c + Q_o \tag{13}$$

Effect on Beam Dynamics

The main effect caused by this accumulation of charge is to decrease the RFQ transverse phase advance, σ_T. This is given by a modification of the usual expression[1] for σ_T when space charge is present:

$$\frac{\sigma_T^2}{\pi^2} = \frac{\sigma_{oT}^2}{\pi^2} + a_B + a_c \tag{14}$$

where σ_{oT} is the transverse phase advance in the absence of space charge and a_B, a_c represent respectively the effect of beam and background ion space charge. At a current I, a_B is given by:

$$a_B = \frac{-e\ Z_B\ I(3b-r)}{\varepsilon_o m_B \omega^3\ b^2 r^2} \tag{15}$$

for a beam bunch of radius r, half length b and charge to mass ratio eZ_B/m_B. The contribution to (14) from background ions is:

$$a_c = \frac{-eZ(3b'-r')(2N\ \sigma\ I\ t_c\ b' + Q')}{2\pi\ \varepsilon_o m\ \omega^2\ r'^2\ b'^2} \tag{16}$$

where r', b' are the radius and half length of the background ion charge distribution at the node and Q' is the amount of background charge necessary to remove longitudinal ion stability. The beam's average space charge at the node also reduces this stability. Thus at high currents, beam space charge alone is often sufficient to cause longitudinal ion instability and then $Q' \approx 0$.

The current in (15) which reduces σ_T to zero is called the transverse current limit. Since a_B and a_c usually have the same sign, the effect of non-zero background charge is to reduce the RFQ's transverse current limit.

As an example of the effect of trapped ions, the decrease in current limit from background protons formed inside RFQ1[4] has been evaluated. Assuming a background gas pressure of 0.13 mPa (10^{-6} Torr) and an average slow proton production cross section[5] of 3×10^{-21} m^2, the characteristic escape time, t_c is 75 µs. This results in a reduction of the current limit of only ≈ 3%. However, the limit is reduced by 13% if the pressure increases by a factor of 10. This reduction is probably not significant under normal operating conditions. In some other designs of long pulse and cw RFQs these effects could be much more significant if the gas pressure or ionization cross sections are higher.

Comments

The effect on RFQ beam dynamics of background ions being trapped in the RFQ bore hole has been examined. Approximate analytic expressions for the size of this effect have been derived. For the case of RFQ1 this could result in a reduction of the current limit by several percent.

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References

1. P. Junior, "Space-Charge Limits in Heavy-Ion RFQ Linacs", Particle Accelerators, 13 (1983) 231.
2. M. Abramowitz and J.A. Stegun, "Handbook of Mathematical Functions", National Bureau of Standards, Applied Mathematical Series AMS 55, 1964.
3. B. Rossi and K. Greisen, "Cosmic-Ray Theory", Reviews of Modern Physics, 13 (1941) 240.
4. B.G. Chidley and G.E. McMichael, "RFQ1 Design Parameters", proceedings of this conference.
5. C.F. Barnett et al., "Atomic Data for Controlled Fusion Research", Oak Ridge National Laboratory report ORNL-5206.