RFQ RADIAL MATCHING SECTIONS AND FRINGE FIELDS*

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Summary

The entrance radial matching section is an extremely important part of any radio-frequency quadrupole (RFQ) linac. It allows a beam having time-independent characteristics to become adapted to the time-dependent focusing in the RFQ. The matching sections proposed in this paper are defined by a four-term potential function and are very effective over lengths of 3 $\beta\lambda$ or longer.

The fringe field at the RFQ exit is mainly of interest because of the time-varying on-axis potential. The beam can either lose or gain energy, depending on the shape of the fringe field. The same four-term potential function can be used for shaping the vanes at the exit so that the fringe fields can be controlled. This formulation also applies to exit radial matching sections, which would be useful if the beam is to enter another RFQ operated at a higher frequency.

The Potential Function

Several different forms[1-3] have been used for the potential function in the radial matching section. I am proposing the function

$$U(r,\theta,z) = \frac{V}{2} \sum_{n=0}^{3} A_n T_n(r,z) \cos 2n\theta \quad , \tag{1}$$

where

$$T_n(r,z) = I_{2n}(kr) \cos kz + 3^{-(2n+1)} I_{2n}(3kr) \cos 3kz \quad . \tag{2}$$

In this formulation, z = 0 at the interface between the radial matching section and the rest of the RFQ, and k = $\pi/2L$, where L is the length of the matching section and V is the intervane voltage. The four coefficients A_0 through A_3 are determined from geometrical properties at the interface, namely, the displacements x_p and y_p of the horizontal and vertical vanes and their respective transverse radii of curvature, $1/x_p''$ and $1/y_p''$. This potential function has the following properties:

1. Each term separately satisfies Laplace's equation.

2. Each term is zero at the end wall (at z = -L).

3. $\partial U/\partial z$ = 0 at z = 0 for all r.

4. Because the first term in the expansion of the modified Bessel function

$I_m(kr)$ is $\frac{(kr)^m}{2^m m!}$, $\frac{\partial U}{\partial z} \approx 0$ at z = -L when r is small.

These properties provide for a smooth transition between the field-free region outside the cavity and the time-varying field within the RFQ. Inclusion of the n = 0 term allows the vanes to start with a modulation, and also allows this same potential function to be used for shaping the fringe fields at the exit of an RFQ.

Two of the boundary conditions are

$$U(x_p,0,0) = V/2,$$

and

$$U(y_p,\pi/2,0) = -V/2,$$

which lead immediately to two relationships between the four coefficients:

$$\sum_{n=0}^{3} A_n T_n(x_p,0) = 1 \quad , \tag{3}$$

$$\sum_{n=0}^{3} A_n T_n(y_p,0) \cos n\pi = -1 \quad . \tag{4}$$

The other two boundary conditions are found by differentiating the potential function twice with respect to y and evaluating the result at $(x_p,0,0)$, and by differentiating twice with respect to x and evaluating the result at $(y_p,\pi/2,0)$. The following expressions are required:

$$\frac{\partial T_n}{\partial r} = k[I_{2n}'(kr) \cos kz + 3^{-2n} I_{2n}'(3kr) \cos 3kz] \quad ,$$

where $I_{2n}'(\xi)$ denotes $\frac{dI_{2n}(\xi)}{d\xi}$;

$$\frac{\partial r}{\partial x} = \frac{y\,y' + x}{r} \quad ;$$

$$\frac{\partial r}{\partial y} = \frac{x\,x' + y}{r} \quad ;$$

$$\frac{\partial \theta}{\partial x} = \frac{x\,y' - y}{r^2} \quad ;$$

and

$$\frac{\partial \theta}{\partial y} = \frac{x - y\,x'}{r^2} \quad .$$

In the above expressions, x' denotes dx/dy and y' denotes dy/dx.

The transverse curvatures, x_p'' and y_p'' are then computed from

$$\sum_{n=0}^{3} A_n[\frac{\partial T_n}{\partial r} (x_p'' + \frac{1}{x_p}) - 4n^2 T_n (\frac{1}{x_p^2})] = 0 \quad , \tag{5}$$

and

$$\sum_{n=0}^{3} A_n[\frac{\partial T_n}{\partial r} (y_p'' + \frac{1}{y_p}) - 4n^2 T_n (\frac{1}{y_p^2})] \cos n\pi = 0 \quad . \tag{6}$$

Because x_p, y_p, x_p'', and y_p'' are specified at z = 0, these last two equations give us two more relationships between the A_n, allowing them to be completely determined. Knowing the values for the A_n's, we can use the same four equations to calculate x_p, y_p, x_p'', and y_p'' at any z (for which they exist).

Effectiveness of Input Matching Section

The S-shaped quadrupole field produced by this radial matching section does a very good job of adapting a constant phase-space ellipse to the time-dependent ellipses required by the RFQ. At any given time, one can calculate how the input ellipse is transformed through the matching section. The resultant ellipse can be compared with the ellipse that is matched to the RFQ focusing system at that particular time. The difference between these two ellipses can be quantified by a mismatch factor. To calculate the mismatch factor between two ellipses having equal areas, transform both ellipses to a coordinate system in which the first ellipse is a circle of radius R_C. Unless the two ellipses are identical, the second ellipse will still be elliptical in the new coordinate system, and will have a semimajor axis R_e larger than R_C. The mismatch factor used in this paper is defined as $(R_e - R_C)/R_C$. It is a measure of how much the ellipse protrudes beyond the circle.

The effectiveness of a radial matching section can be measured by the maximum value of the mismatch factors calculated at various times during a complete rf cycle. Values of a few per cent indicate a very effective matching section, and any value less than 0.1 is probably acceptable. Mismatch factors produced by radial matching sections between 2 βλ and 12 βλ long for four beam currents are shown in fig. 1. The zero-current phase advance for this RFQ was 40°. The other three currents were chosen to reduce the phase advance to 32, 24, and 16°. Although some structure can be seen in these curves, there would appear to be no reason for making the radial matching section longer than 3 βλ. Short matching sections are advantageous, especially for high beam currents, because a less convergent beam is required than for longer matching sections.
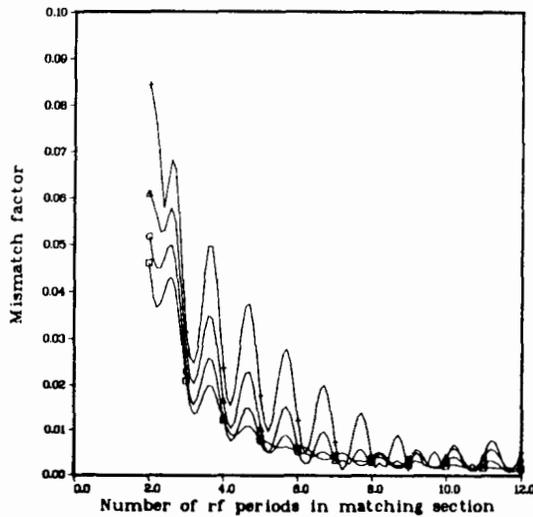
Fig. 1. Mismatch factors indicating effectiveness of radial matching sections from 2βλ to 12 βλ long for four beam currents. The phase advance for zero current (□) was 40°. The phase advances for the other three currents were 32 (0), 24 (Δ), and 16° (+).

An example of the vane profile for a 3 βλ radial matching section for the PIGMI RFQ[4] is shown in fig. 2.
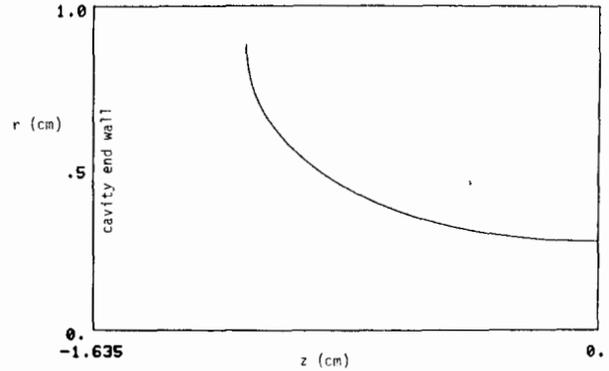
Fig. 2. Vane profile of 3-βλ-long radial matching section for a 440-MHz RFQ with 30-keV injection energy. Specified conditions at z = 0 are $x_p = y_p = 0.274$ cm, $1/x_p'' = 1/y_p'' = 0.206$ cm.

Exit Fringe Fields

At the exit of an RFQ, the vane modulation produces an oscillating axial potential. Depending on how this axial potential goes to zero, particles exiting from the RFQ can either gain or lose energy. By properly shaping the vane tips after the last accelerating cell, one can control the shape of the fringe field and therefore control the energy change.

If the vane tips are shaped to generate the potential function given by eq. (1), the on-axis potential will be

$$U(z;t) = \frac{AV}{2} \cdot \frac{3}{4} \left(\cos kz + \frac{1}{3}\cos 3kz\right) \sin \omega t \ , \qquad (7)$$

where z = 0 at the beginning of the fringe region, z = L at the cavity end wall, k = π/2L, ω is the angular frequency of the rf, and AV/2 is the amplitude of the axial potential. The longitudinal electric field is

$$E_z = \frac{AV}{2} \cdot \frac{3}{4} k (\sin kz + \sin 3kz) \sin \omega t \ . \qquad (8)$$

Let φ be the phase of the rf when a particle is at z = 0. Then ωt can be replaced by φ + k'z, where k' = 2π/βλ. Ignoring the effect of a particle's change in velocity as it crosses the fringe field, the energy change is given by

$$\Delta W(\phi, L) = \int_0^L E_z dz = \frac{AV}{2} [B(L) \sin \phi + C(L) \cos \phi] \ , \quad (9)$$

where

$$B(L) = \frac{3}{4} \int_0^{\pi/2} (\sin kz + \sin 3kz) \cos k'z \, d(kz)$$

and

$$C(L) = \frac{3}{4} \int_0^{\pi/2} (\sin kz + \sin 3kz) \sin k'z \, d(kz) \ .$$

Figure 3 shows the energy change, in units of $\Delta V/2$, plotted versus $L/\beta\lambda$ for several values of the initial phase. Notice that for $L \leq \beta\lambda/2$, the fringe field gives a slight bunching effect because particles arriving later gain more energy, or lose less energy, than do particles arriving earlier. Fringe fields longer than 1.5 $\beta\lambda$ give very little energy change.
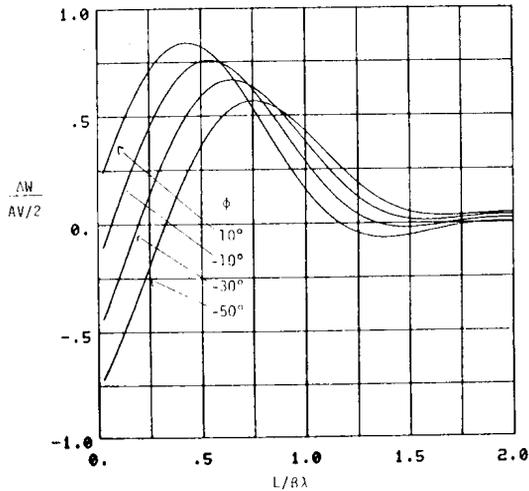
Fig. 3. Change in energy of a particle in the fringe region as a function of fringe length L plotted for several initial phases ϕ, assuming potential function from eq. (7).

If the fringe region is made longer than 2 $\beta\lambda$, this region would also be an exit radial matching section. The x-x' and y-y' ellipses would be almost identical. This feature would be desirable if the RFQ were to be followed by a solenoidal focusing system, but would not be desirable if the beam were to be injected into a quadrupole system, such as a drift-tube linac.

Figure 4 shows how the vanes should be shaped at the exit of the PIGMI RFQ to prevent an energy change. The synchronous phase is -25°, so the length of the fringe field should be approximately 0.15 $\beta\lambda$, or 0.8 cm in this case. Because this distance is so short, the transverse properties of the beam would be essentially unaffected.
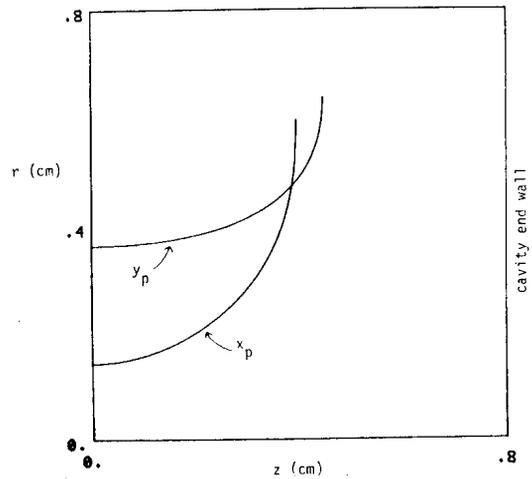
Fig. 4. Horizontal (x_p) and vertical (y_p) vane profiles for tailoring fringe field region at exit of 440-MHz RFQ at 2.5 MeV. Specified conditions at $z = 0$ are $x_p = 0.145$, $y_p = 0.363$, $1/x_p'' = 1/y_p'' = 0.206$.

References

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