DESIGNING SELF-MATCHING LINACS*

R. S. Mills, K. R. Crandall, AT-1, and J. A. Farrell, AT-2, MS-H817
Los Alamos National Laboratory, Los Alamos, New Mexico 87545 USA

Summary

The present trend in ion-linac design is to begin with a radio-frequency quadrupole (RFQ) linac followed by one or more drift-tube linac (DTL) tanks in which permanent-magnet quadrupoles are used for transverse focusing. The lack of adjustable elements (knobs) strongly suggests that one should seek linac designs with intertank matching solutions that are insensitive to beam currents and emittances, which can be accomplished if there are no sharp discontinuities in the focusing properties along the entire linac.

In this paper, we present guidelines for linac design and describe techniques for longitudinal as well as transverse matching between tanks. For a wide range of beam currents and emittances, a beam matched at the entrance to the RFQ should remain well matched throughout the entire linac.

Introduction

The objective is to find matching solutions between the RFQ and the DTL and between DTL tanks that are insensitive to beam current and emittance. We hypothesize this will be possible, provided adjacent structures and the matching region between have similar values for their average focusing strengths. If this hypothesis is true, the analysis can be done without considering the space-charge forces. Then the solutions in the longitudinal and transverse planes are essentially independent, but they must be compatible. Matching must be considered when designing the entire linac system. There should be no large discontinuities in focusing properties, which often means that the initial accelerating gradient in the DTL must be lowered to reduce the longitudinal focusing strength to be compatible with the final RFQ focusing. Also, obtaining longitudinal matching between tanks may require us to make changes in the geometry of the end cells.

In the discussion that follows, we will make use of some well-known beam-dynamics properties. The matched ellipse parameters (α, β, and γ) for a periodic structure are obtained from the transfer matrix for one period:

$$R = \begin{bmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{bmatrix} , \qquad (1)$$

where μ is the phase advance for one period, and $\beta\gamma - \alpha^2 = 1$.

The beam may be represented in a phase plane by a beam matrix

$$\sigma = \epsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix} , \qquad (2)$$

where ϵ is the beam emittance.

If the beam matrix at one location is σ_1, the beam matrix at any other location is obtained from the transfer matrix between the two locations:

$$\sigma_2 = R\,\sigma_1 R^T , \qquad (3)$$

where R^T is the transpose of R.

Equation (3) is equivalent to

$$\begin{pmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{pmatrix} = \begin{bmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -R_{11}R_{21} & R_{11}R_{22}+R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{bmatrix} \begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix} \qquad (4)$$

Matching Between RFQ and DTL

Typical matched conditions at the output of an RFQ and at the input to the DTL are shown schematically in fig. 1. Note the transverse ellipses at the exit of the RFQ, A, are similar to those at midcell, C, in the DTL. This similarity suggests that the distance between A and B should be approximately $(n + 1/2)\beta\lambda$. If no additional quadrupoles are to be used, n should be zero. A longitudinal match requires a waist-to-waist transfer between A and B or D.

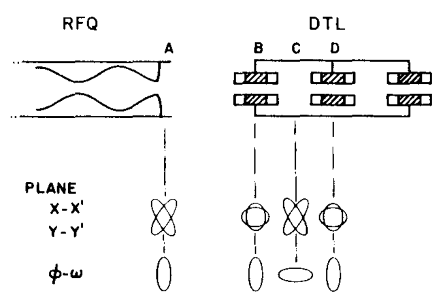

Fig. 1. Schematic representation of matched phase spaces at end of RFQ (A), and at midquad (B and D) and gap (C) in DTL.

Transverse Matching

Referring to fig. 1, the matched ellipses at A in the two transverse planes have the same β's and γ's and equal and opposite α's. At C the matched ellipses have these same characteristics, although not necessarily the same values.

Ignoring the effect of the accelerating gaps on the transverse motion, a DTL can be approximated by a sequence of drifts and thin lenses having alternating gradients. By constructing the transfer matrices for one period, one can show that the matched conditions at the point midway between lenses of focal length f satisfy the condition

$$\frac{|\alpha_x|}{\gamma_x} = \frac{|\alpha_y|}{\gamma_y} = f , \qquad (5)$$

as well as $\alpha_x = -\alpha_y$, $\beta_x = \beta_y$, and $\gamma_x = \gamma_y$, suggesting that transverse matching between the RFQ and DTL should be simplified if the ratios α/γ are the same at A and C.

*Work supported by the US Department of Defense, Defense Advanced Research Projects Agency, and Ballistic Missile Defense Advanced Technology Center.

Let ℓ_1 denote the distance from A (where $\alpha_x = -\alpha_y = \alpha_1$, $\beta_x = \beta_y = \beta_1$, and $\gamma_x = \gamma_y = \gamma_1$) to a thin lens having focal length f. When space-charge forces are ignored, γ remains constant throughout any drift space. Let us calculate values of f and ℓ_1 that make $\gamma_x = \gamma_y = \gamma_2$ in the drift space following the lens. After transforming the ellipses through a drift followed by a thin lens,

$$\gamma_x = \beta_1/f^2 + 2(1 - \ell_1/f)\alpha_1/f + (1 - \ell_1/f)^2\gamma_1 \quad ; \tag{6}$$

$$\gamma_y = \beta_1/f^2 + 2(1 + \ell_1/f)\alpha_1/f + (1 + \ell_1/f)^2\gamma_1 \quad . \tag{7}$$

For $\gamma_x = \gamma_y$,

$$\frac{1}{f} = -\frac{\gamma_1}{\alpha_1} \quad . \tag{8}$$

The focal strength of the lens is determined by this ratio regardless of what value is wanted for γ_2. The drift length ℓ_1 determines the value of γ_2:

$$\gamma_2 = \gamma_1(1 + \gamma_1^2\ell_1^2)/\alpha_1^2 \quad . \tag{9}$$

Therefore, to obtain a specified γ_2, the drift length must be

$$\ell_1 = \frac{1}{\gamma_1}\sqrt{\frac{\gamma_2}{\gamma_1}\alpha_1^2 - 1} \quad . \tag{10}$$

From the symmetry of the situation one can show, by going backward from the point at which $\alpha_x = -\alpha_y = \alpha_2$, that the focal strength of the lens must be

$$\frac{1}{f} = \frac{\gamma_2}{\alpha_2} \quad , \tag{11}$$

and the drift length ℓ_2 from this point to the lens is

$$\ell_2 = \frac{1}{\gamma_2}\sqrt{\frac{\gamma_1}{\gamma_2}\alpha_2^2 - 1} \quad . \tag{12}$$

Comparing eq. (8) with eq. (11), we must have $-\alpha_1/\gamma_1 = \alpha_2/\gamma_2$ for transformations of this type to be possible.

Longitudinal Matching

We need a transfer matrix R that transforms an upright ellipse characterized by β_1 to another upright ellipse with β_2:

$$\begin{bmatrix} \beta_2 & 0 \\ 0 & 1/\beta_2 \end{bmatrix} = R \begin{bmatrix} \beta_1 & 0 \\ 0 & 1/\beta_1 \end{bmatrix} R^T \quad . \tag{13}$$

Three matching options are

1. to adjust the RFQ to move the waist at A downstream;
2. to adjust the first cell of the DTL to move the waist at B upstream;
3. to use a buncher cavity between A and B.

We have been unable to find a satisfactory solution for Option 1. Anything we did to move the longitudinal waist downstream made a transverse match difficult or impossible. Option 2 is successful with rigid constraints and is discussed later. Option 3, discussed below, makes it easier to find longitudinal matches compatible with transverse solutions and also allows small energy shifts.

Consider a matching section with a drift ℓ_1, followed by an rf gap or buncher, followed by a drift ℓ_2. The rf gap is represented by a thin lens having a focusing strength

$$\frac{1}{f} = -\frac{2\pi E_0T \sin\phi_s}{m_0c^2\beta^2\gamma} \quad . \tag{14}$$

The transfer matrix for this matching section is

$$R = \begin{bmatrix} 1 - \ell_2/f & \ell_1 + \ell_2 - \ell_1\ell_2/f \\ -1/f & 1 - \ell_1/f \end{bmatrix} \quad , \tag{15}$$

The waist-to-waist transformation requires

$$R_{11}R_{21}\beta_1 + R_{12}R_{22}/\beta_1 = 0 \quad , \tag{16}$$

and

$$R_{21}^2\beta_1 + R_{22}^2/\beta_1 = 1/\beta_2 \quad . \tag{17}$$

Putting the elements of the R-matrix in terms of ℓ_1, ℓ_2, and f, and solving for ℓ_1 and ℓ_2 in terms of β_1, β_2, and f:

$$\ell_1 = f(1 - qB) \quad , \tag{18}$$

$$\ell_2 = f(1 - q/B) \quad , \tag{19}$$

where $$B = \sqrt{\frac{\beta_1}{\beta_2}} \quad ,$$

and $$q^2 = 1 - \frac{\beta_1\beta_2}{f^2} \quad .$$

These equations imply certain restrictions:

1. $f^2 \geq \beta_1\beta_2$ because q^2 cannot be negative;
2. $0 \leq qB \leq 1$ because ℓ_1 cannot be negative;
3. $0 \leq q/B \leq 1$ because ℓ_2 cannot be negative.

For any practical buncher field that results in a focal strength $f^2 \geq \beta_1\beta_2$, solutions can be found for ℓ_1 and ℓ_2. From this range of solutions, we choose one that is compatible with a transverse solution, and has an average focusing strength that is midway between those in the RFQ and DTL.

Option 2 requires shifting the waist from position B to position A (see fig. 1) by modifying the focusing strength in the first gap. The focusing strength depends on $E_0T \sin\phi_s$ and is most easily changed by displacing the gap to change ϕ_s. In eqs. (18) and (19), ℓ_1 is now the distance between A and the displaced gap, and ℓ_2 is the distance between the gap and D. A phase shift $\Delta\phi$ defines both ℓ_2 and focusing strength 1/f. For any given $\Delta\phi$, one can calculate ℓ_2 by

$$\ell_2 = (1 - \frac{\Delta\phi}{180})\beta\lambda/2 \tag{20}$$

and f from eq. (14), then use eq. (19) to solve for β_1, which specifies the size of the waist at A. If β_1 is not the correct value, modify $\Delta\phi$ until the correct value is found, if possible. Then ℓ_1 is calculated from eq. (18). If ℓ_1 is not compatible with a transverse matching solution, with mechanical constraints, or with average focusing constraints, then the DTL must be modified to change β_2.

There are obvious limitations on how much ϕ_s can be modified. Also, for Option 2 to work ℓ_1 must be larger than ℓ_2, implying $\beta_2 > \beta_1$, which can be seen by dividing eq. (18) by eq. (19). Consequently, E_0T must be less than it would otherwise need to be so that the focusing strength is reduced. The synchronous phase cannot be used for this purpose without reducing the phase acceptance.

Matching Between DTL Tanks

We are limiting our solutions to designs that have small discontinuities in focusing strengths in all planes, and to those with intertank spacings about equal to $\beta\lambda$. Multicurrent transverse solutions can be obtained when the distance is any integer of $\beta\lambda$, but longitudinal matching without a buncher is limited to shorter distances.

Transverse Matching

If the quadrupole strengths in the two tanks are the same, and if the quadrupole periodicity has been retained, the transverse beams will be matched. The two end quads in each tank can be adjusted to find a solution for almost any differences in focusing strengths between tanks. However, unless the discontinuity is small (we found a 10% change to be acceptable), the solution will not be valid for a wide range of currents and emittances. A matching code (such as TRACE 3-D) can be used to find optimum quadrupole gradients for the four end quads for any specific design.

Longitudinal Matching

Longitudinal matching can be accomplished by changing the synchronous phase in the end cell in each tank. Referring to fig. 2, the object is to find a waist-to-waist transfer between A and E. The waists normally occurring at B and D are moved into the intertank space to coincide at C. The procedure is as described for Option 2 in the RFQ to DTL matching, except that we use both end cells and work toward the middle. The size of the waist at C and its precise location are free parameters. The constraints are that the distance between A and D is specified and that the waist produced at C by the upstream cell is the same size as that produced at C by the downstream cell.

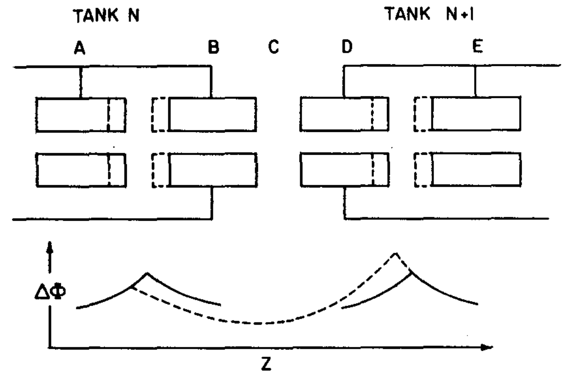

Fig. 2. Longitudinal matching between DTL tanks. Solid lines show normal drift tubes and phase profiles. Dashed lines show modified drift tubes and resulting change in phase profile.

The synchronous phase in a cell is shifted by making one drift tube longer and the other shorter, thereby displacing the accelerating gap. Because the length of the cell remains at approximately $\beta\lambda$, the quad spacing is preserved.

Results of a Test Case

We have used these techniques successfully in the paper design of a 353-MHz linac capable of accelerating 100 mA. The matching solutions were satisfactory for all currents from 0 to 100 mA. If longitudinal matching is not used, the longitudinal mismatch soon develops into a transverse mismatch for high-current beams because of the space-charge coupling.