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COMPUTER OPTIMIZATION OF A LINAC INJECTOR TRAJECTORY*

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Summary

One can determine a computer prediction of the beam radius as a function of axial distance for a linac beam by providing a set of inputs to the computer code, ZFIELD.[1] The trajectory may be improved by varying the magnet current values in the code, but repeated trials may still not attain the best trajectory. Starting with a set of points containing the desired trajectory, one may work the problem backwards and obtain the necessary magnet currents required by the trajectory. In the examples given, a portion of the trajectory is chosen to be parabolic. The trajectory information is used with a differential equation involving beam radius and its derivatives to yield the magnetic field as a function of axial position. Matrix methods are used to obtain the magnet currents from the magnetic field.

Trajectory Simulation

ZFIELD is a computer code which propagates a beam of electrons through a linac and calculates the magnetic field produced by solenoids, iron lenses, and other elements. The beam may be started with zero or finite emittance. Optimization is performed only on the trajectory of the envelope of a zero emittance beam. Bunching is approximated by allowing the beam current to be a function of distance. Acceleration of the beam may be included by assuming an accelerating voltage as a function of axial position. The code takes into account space charge, self-fields, beam loading, cavity attenuation, and relativity. Integration of the trajectory is by means of a third order Runge-Kutta method.

The radial electron trajectory is derived in Ref. 2 for a beam of electrons moving along the z-axis in an axially symmetric system. The equation for the electron radius at a particular axial value is:

$$r'' = \frac{r'\eta \frac{\partial V}{\partial z}}{v_z^2} - \frac{r}{2}\frac{\eta}{v_z^2}\frac{\partial^2 V}{\partial z^2} - \frac{r}{4}\left(\frac{\eta B}{v_z}\right)^2 + \frac{\eta I\left(1-v_z^2/c^2\right)}{2\pi\varepsilon_o v_z^3}\cdot\frac{1}{r} + \frac{C}{r^3\beta^2\gamma^2} \tag{1}$$

where primes denote differentiation with respect to z and

$\eta = e/m_o\gamma$

$\gamma = 1/\left(1-v_z^2/c^2\right)$

$\beta = v_z/c$

r = radius of a ray at location z

B = z component of externally applied axial magnetic field

c = velocity of light

V = electric potential

ε_o = permittivity of free space

C = constant

I = beam current

The magnetic field produced by one solenoid or magnetic lens, individually, increases linearly with the current in the element. The total magnetic field produced by all elements at a particular point on the beam axis is a linear combination of a field produced when each element has a current of 1 A. This relationship may be expressed as a matrix product. The magnetic field vector, B, representing the field at ℓ fixed points, z_1, z_2, ... z_ℓ along the axis of the beam, may be calculated by multiplying an $\ell \times m$ matrix, S, by a vector of m magnet currents, C. The S matrix represents magnetic elements in fixed positions. The column of S representing the contribution of the kth magnet may be calculated by setting the current in this magnet to 1 A and to zero in all others, and determining the magnetic field produced at z_1. This would yield S_{1k}. The field at z_2 ... z_ℓ would yield S_{2k} ... $S_{\ell k}$. B_i, the ith component of the field for arbitrary current settings, may be represented as:

$$B_i = \sum_{j=1}^{m} S_{ij}\, C_j \tag{2}$$

Optimized Calculations

Once matrix S has been calculated, the desired trajectory radius may be calculated for each axial step, and the derivatives, r' and r" may be calculated for axial values. Equation (1) may be solved for the optimal B so that we have B as a function of r, r', and r" and other variables. The desired axial field vector, B, may be calculated, and eq. (2) solved for the new optimized current, C. This is done by finding an approximate (least squares) solution to the vector, C. To solve the general linear system

$$AX = Y \tag{3}$$

where A is a matrix and X and Y are vectors, the vector X_o is a least squares solution of eq. (3) if for all vectors X

$$||\ AX - Y\ || > ||\ AX_o - Y\ || \tag{4}$$

where

$$||\ X\ || = (X,\ X)^{\frac{1}{2}} \tag{5}$$

A method of solution is employed using modified Gram-Schmidt orthogonalization as defined in Reference 3. This method was implemented in a computer program developed by Buzbee and Frank,[4] contained in the Los Alamos National Laboratory computer library.

This optimization method will yield a set of magnet currents that may be used in ZFIELD to plot the optimized trajectory. Repeated cycles of optimization may be performed, reading in data from the previous cycle and optimizing the resulting trajectory, until only small changes result between succeeding cycles.

In the linac at EG&G/EM Santa Barbara Operations (SBO), all magnets do not have independent power supplies, with two or three magnets sharing a power supply in certain cases. If C is a vector of magnet currents of

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length m with components C_j, and I is a shorter vector of independent magnet currents of length n with components I_k, they may be related by a m by n connection matrix of power supplies, P:

$$C_j = \sum_{k=1}^{n} P_{jk} I_k \tag{6}$$

For example, if magnets 1 and 2 were on the same power supply, and 3 was not, P would be expressed as:

$$P = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{7}$$

and

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \tag{8}$$

Results

The optimization technique described above was implemented for an injector test system, with a zero emittance beam, assuming 15 independent magnetic lenses and one subharmonic buncher, with maximum current occurring at the end of the system at 248 cm. Five cycles of optimization were run, using the optimized data from output from one cycle as input to the next cycle. A parabolic trajectory was selected, beginning at an axial position of 78 cm at the radius produced by the unoptimized trajectory, with radius of 1 cm at an axial position of 248 cm, the position of the faraday cup. Figure 1 shows the magnetic field produced by the optimized currents. The simulated trajectory resulting from these currents is shown in fig. 2.

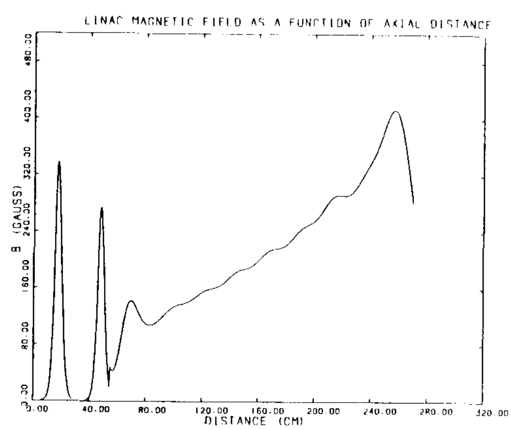

Fig. 1. Injector magnetic field produced by optimized magnet currents.

The SBO linac is now being redesigned and the optimization program was used to determine the initial settings of the solenoid coils and iron lenses, with no bunching or acceleration, and assuming a zero emittance beam. These predicted settings produced a beam that successfully traversed the length of the linac when it was turned on initially. The trajectory was required to be a parabola beginning at the axial position of the first maximum of the radius with 1-cm radius at 900 cm. Figures 3 and 4 show the resulting magnetic field and trajectory for 38 independent magnets. The ripples in the field result from the limitation on the number of magnets used to create the ideal field. Since there are only 33 independent power supplies, the optimized data for independent magnets was used as input to the

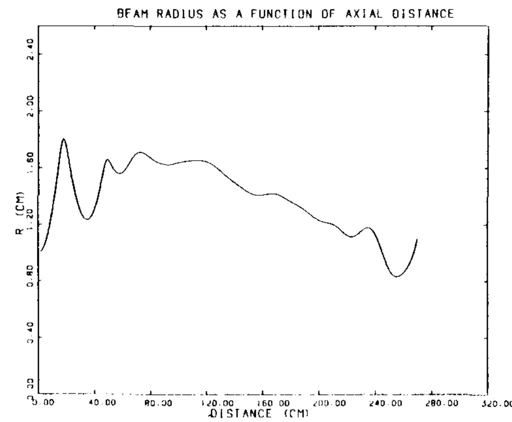

Fig. 2. Injector trajectory produced by optimized magnet currents.

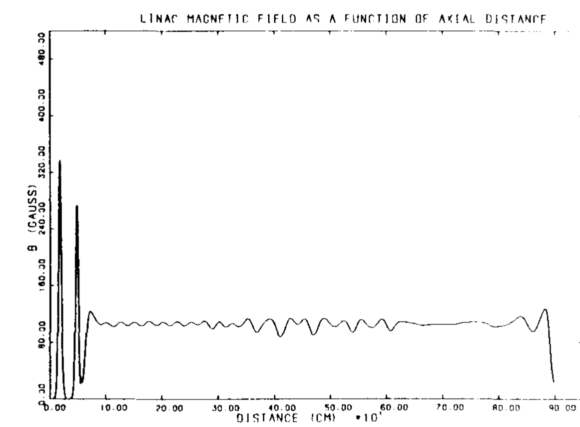

Fig. 3. Linac magnetic field produced by optimization of 38 magnet currents.

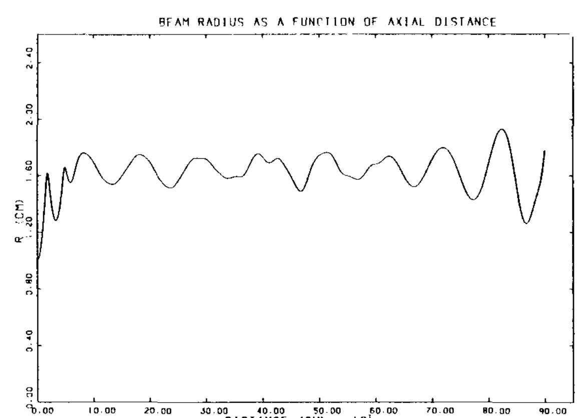

Fig. 4. Linac trajectory produced by optimization of 38 magnet currents.

optimization with given magnet connections. The resulting field and trajectory are shown in figs 5 and 6. This trajectory departs further from an ideal parabola and would be improved by manipulating the power supply connection matrix.

Conclusion

This method of trajectory optimization has proved useful in the case where space charge effects are of

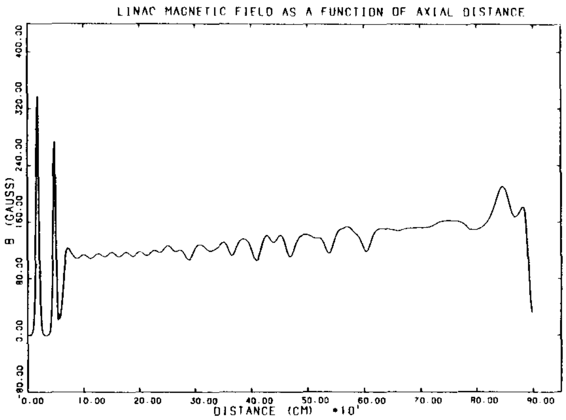

Fig. 5. Linac magnetic field produced by optimization of currents in 33 power supplies.

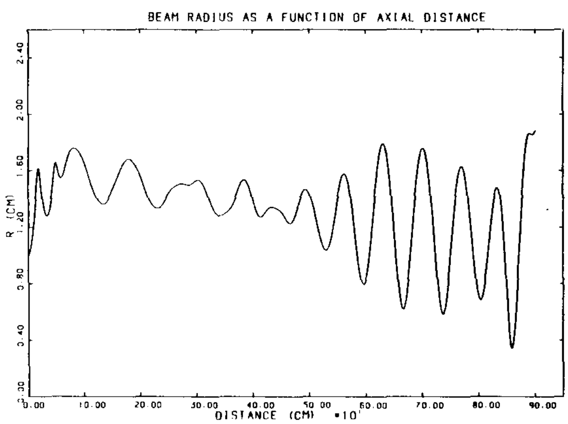

Fig. 6. Linac trajectory produced by optimization of currents in 33 power supplies

consequence. Finding initial settings for 38 magnets tied to 33 power supplies that drifted the beam to the end of the linac was successfully accomplished. Further optimization studies will be made with accelerated beams.

References

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