A PROPER CANONICAL ALGEBRAIC MAPPING TRANSFORMATION

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Summary

Integrating the equations of motion for particles moving from sector to sector in an accelerator can require enormous computer time if a) the overall length is extremely large as in an FFAG-synchrotron or b) the space charge effects between the particles are important (as in the SNQ high current linac with peak current 200 mA) and one is interested in particle losses. Therefore, we have tried for both cases to fit the motion of particles by canonical algebraic transformations based on an expansion of the generating function up to 4^{th} order.

At present the accuracy of the fit and the accuracyof the computer programs are of the same order of magnitude.

The Method

In principle, algebraic transformation always exist describing the final values (P,Q) of a particle (e.g. after passing a sector of an accelerator) as functions of the initial values (p,q). Using an algebraic transformation is in general not easier than solving the equations of motions by numerical integration. In circular and linear accelerators, however, the algebraic transformations are more or less linear. One can therefore hope that an expansion exists, taking into account linear terms, quadratic terms etc. as functions of appropriately chosen initial coordinates. On the basis of this assumption several codes like "TRANSPORT"[1] and "MARYLIE"[2] have been developed.

Instead of expanding the algebraic transformation we are using here an expansion of the generating function S in terms of initial and final "space" coordinates Q,q:

$$S = S(Q,q), \quad \frac{\partial S}{\partial q_i} = p_i, \quad \frac{\partial S}{\partial Q_i} = -P_i \tag{1}$$

$$S = \sum_{i_1 \le i_2 \le i_3 \cdots} S_{i_1 i_2} X_{i_1} X_{i_2} + S_{i_1 i_2 i_3} X_{i_1} X_{i_2} X_{i_3} + \cdots$$
$$\underset{\sim}{X} = (Q_1 \cdots Q_{N_d}, q_1 \cdots q_{N_d}) \tag{2}$$

The number of independent coefficients N_c in n^{th} order can be seen from (2)

$$N_c = \binom{2N_d - 1 + n}{n} \tag{3}$$

with N_d being the number of degrees of freedom. The order of the corresponding implicit algebraic transformation is reduced by one, cf Eq (1); the number of independent coefficients for an implicit algebraic transformation are given in table I.

A well known disadvantage of the generating function formalism is due to the fact, that not for all canonical coordinates an S with the properties of Eq(1) exists. But as long as the algebraic transformation is essentially linear (and only then do we expect a rapidly converging series (2)) is it easy to make a linear canonical transformation that leads to an S fulfilling Eq (1).

The big advantage of the generating function formalism is that approximating the infinite series (2) by a finite, arbitrarily truncated one leads to an exactly canonical algebraic transformation.

It should be noted that in our ansatz any particle is treated as being independent of others but moving in a space charge field. If this approximation is no longer justified, i.e. if the particle density is so high that collision terms become important, then our method as well as similar procedures fail. For the parameters in which we are interested, this does not occur.

The use of a higher order transformation that is exactly canonical is of great interest in the study of the essential nonlinear properties of a machine. Truncating the generating function (at about 4^{th} order) to exclude higher order terms has the advantage that the detailed complexities, induced by such terms, that have little bearing on the machine design are avoided (assuming the absence of a direct higher-order resonance). We would then have a tool similar to the matrix methods used in the design of more linear machines that could be fast and should also provide more insight into the effects of various nonlinear terms.

Results

On the basis of expanding the generating function up to 4^{th} order we fitted the motion of particles by optimizing the coefficients in Eq (2).

Results for the high energy part of the designed SNQ linac: The code "MOTION"[3] was used to calculate the motion of 250 quasiparticles over one period at 100 MeV. For simplicity, acceleration was not taken into account. To test the accuracy of the several programs a bunched beam with current of 1 mA was fitted first. In that case the rms error was identical to that of a linear fit: 0.5 %. Next a current of 200 mA was fitted. Now the rms error was 1 % compared to an accuracy of 3 % for the linear fit. From this we conclude that at present the fit and accuracy of the computer programs are of the same order. Further calculations with higher precision are needed to determine whether or not an expansion of the generating function up to 4^{th} order is sufficient. An expansion up to 5^{th} order can be used without running into numerical problems.

Results for the FFAG-Synchrotron: We have integrated sets of orbits covering the FFAG acceptance through one sector. The integration was done with an 8^{th} order algorithm using scaling fields where the azimuthal field dependence is given through Fourier harmonics. Given about 400 such orbits in the median plane only, a fit through terms of 4^{th} order in the generating function reproduced the orbit with a rms error of 0.2 %. We anticipate a substantial reduction in these errors in future work by means of a more realistic choice of the orbits fit, more attention to the details of the numerical methods, and through the use of a more realistic and smoother azimuthal field profile in the integration program. On the other hand an expansion of the generating function up to 8^{th} order can be fitted since only two degrees of freedom are coupled.

TABLE I

NUMBER OF n^{th}-ORDER COEFFICIENTS DEFINING THE ALGEBRAIC TRANSFORMATION

		1(=linear)	2(=quadratic)	3(=cubic)
degrees of				
freedom	1	3	4	5
(dimension of motion	2	10	20	35
in space)	3	21	56	126

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