THE WAKE FIELD ACCELERATION USING A CAVITY OF ELLIPTICAL CROSS SECTION

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Summary

Discussed are the possibility and the capacitance of the wake field acceleration by means of a cavity of elliptical cross section. A computer code WELL is developed for the calculation of wake fields in that structure. The mathematical method is basically an extention of that of BCI to the 3-dimensional computation, i.e., the electromagnetic fields are calculated by numerically solving the Maxwell equations in the time domain with initial and boundary conditions. Open boundary conditions simulating infinitely long beam pipes are used to save the memory and the CPU time. An example of computation in an elliptical structure gives a reasonable wake field pattern at the focus for an accelerated beam, and points out that there exists the rather strong transverse wake field deflecting particles' tracks. Structures to cure this disturbing field are necessary.

Introduction

The energy of interest for high energy physics, being about to reach the region of TeV, demands us, accelerator physicists, for the successive construction of huger and huger machines. Synchrotrons are all right with protons because the synchrotron radiation loss is negligible up to ∿ 100 TeV, while linear colliders are the unique choice for electrons, they need the construction cost proportional to the length, though. The problem is therefore how to get higher energy of particles in terms of shorter linac, i.e., higher gradient linac of, say, 200 MeV/m, so that the construction cost would remain in our hands. Various ideas of new acceleration methods for this high gradient field have been proposed so far, including the extention of conventional microwave linac. Of those, one of the most prominant methods is the idea using wake fields generated by a bunch of electrons passing a structure of varing cross section which has been proposed by Voss and Weiland.[1,2] MPE group at DESY is now preparing the experiment to put the principle into practice, aiming at the first experiment by the end of this year.[3] In general, possible structures for wake field acceleration must have two beam holes, one for driving beam to excite wake fields and the other for an accelerated beam; therefore one might call them wake field transformers. Here at KEK, exployed was a cavity of elliptical cross section with two beam holes at the two focal points, like fig. 2. This is based on the anticipation that if a high current bunch goes along one focus, electromagnetic fields radiated from that in all the directions will get focussed to the other at the same time. If so, since the strength of the field is proportional to the current in the driving bunch, one might obtain a high gradient field from the driving bunch of high current but low energy. BCI[4] and TBCI[5] by Weiland have shown their great power to calculate wake fields in cavities, vacuum pipes and so on, however their applications are restricted to axisymmetrical structures. The present problem is completely 3-D so that it was impossible so far to estimate how high fields are obtained how much phase lag after the driving bunch goes through, and consequently to optimize the structure. In order to overcome this obstacle and push experiments at KEK forward, the computer code WELL (wake fields in a cavity of elliptical cross section) was developed[6] and is presented in this paper. WELL is basically an extention of BCI, i.e., solves the Maxwell equations directly in the time domain with the 3-dimensional meshes and boundary conditions when a primary Gaussian beam passes an elliptical structure. By means of adoption of "quasi" open boundary conditions[7] which simulate inifinitely long beam pipes, the number of mesh points for an ordinary calculation goes within a reasonable range (say, 40000 mesh points), and so does the CPU time (∿ a few minutes). The program can be applied to cylindrically symmetrical structures as well simply by inputing the same length for the major axis and the minor one. The results were confirmed to agree with those of BCI within a few percents, which is described in Ref. 6 in detail. In a next section, the computational method is briefly described. The rest is devoted to an example of computation in an elliptical structure and some discussions on the problems of the elliptical wake field transformers.

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Computational Method

The mathematical method used here follows closely the method of BCI except for 3-dimensional meshes and assosiating items. For further details, the reader should refer to Ref. 7 and 8 by Weiland and Ref. 6 by the author.

Each mesh cell has a cubic shape, and field components lie on it as shown in fig. 1.

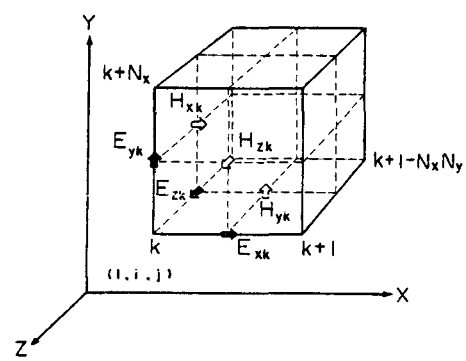

Fig. 1 Position of the field components in a unit cell.

This way of defining the mesh functions together with proper boundary and initial conditions assures that only Faraday's law (1) and Ampere's low (2) are necessary and the rests in the Maxwell's equations are fulfilled automatically because the continuity equation is satisfied:

$$\mathrm{rot}\ \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \ , \tag{1}$$

$$\mathrm{rot}\ \vec{H} = \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \ , \tag{2}$$

where $\vec{E}$, $\vec{H}$ and $\vec{j}$ are the vectors of the electric, and magnetic fields and the current density of the primary beam, respectively, and ε_0 and μ_0 are permitivity and permeability of the vacuum, respectively. For the time derivatives, we use finite-difference approximation and "alternating explicit time scheme",[8] i.e., we evaluate $\vec{E}$ and $\vec{H}$ at alternate half-time steps:

$$\vec{H}^{n+1} = \vec{H}^{n} - \Delta t c Y_0 \ \mathrm{rot}\ \vec{E}^{n+1/2} \ , \tag{3}$$

$$\vec{E}^{n+3/2} = \vec{E}^{n+1/2} + \Delta t c Z_0 (\mathrm{rot}\ \vec{H}^{n+1} - \vec{j}^{n+1}) \ , \tag{4}$$

where, following Yee's notation, we denote the time by index n

$$\vec{H}^{n} = \vec{H}(t = n\Delta t) \ . \tag{5}$$

All the other notations are: Δt = unit of time step, c = velocity of light, Z_0, Y_0 = characteristic impedance, admittance of the vacuum ($Z_0 = \sqrt{\mu_0/\varepsilon_0}$, $Y_0 Z_0 = 1$).

Also for the space derivatives, we use finite difference expressions around each position of field component. The following relation is then necessary between the time step Δt and the space step $\Delta s (s = x, y, z)$ to assure the charge conservation:

$$M \beta_s c \Delta t = \Delta s \quad , \tag{6}$$

where M must be an integer, and β_s is velocity of particles in the s direction.

Finally we write down the full set of equations for computation:

$$H^{n+1}_{xk} = H^{n}_{xk} - \frac{Y_0}{M\beta_z} (E^{n+1/2}_{yk-N_x N_y} - E^{n+1/2}_{yk}) - \frac{Y_0}{M\beta_y} (E^{n+1/2}_{zk+N_x} - E^{n+1/2}_{zk}) \quad , \tag{7}$$

$$H^{n+1}_{yk} = H^{n}_{yk} - \frac{Y_0}{M\beta_x} (E^{n+1/2}_{zk} - E^{n+1/2}_{zk+1}) - \frac{Y_0}{M\beta_z} (E^{n+1/2}_{xk} - E^{n+1/2}_{xk-N_x N_y}) \quad , \tag{8}$$

$$H^{n+1}_{zk} = H^{n}_{zk} - \frac{Y_0}{M\beta_x} (E^{n+1/2}_{yk+1} - E^{n+1/2}_{yk}) - \frac{Y_0}{M\beta_y} (E^{n+1/2}_{xk} - E^{n+1/2}_{xk+N_x}) \quad , \tag{9}$$

$$E^{n+3/2}_{xk} = E^{n+1/2}_{xk} + \frac{Z_0}{M\beta_y} (H^{n+1}_{zk} - H^{n+1}_{zk-N_x}) + \frac{Z_0}{M\beta_z} (H^{n+1}_{yk} - H^{n+1}_{yk+N_x N_y}) \quad , \tag{10}$$

$$E^{n+2/3}_{yk} = E^{n+1/2}_{yk} + \frac{Z_0}{M\beta_x} (H^{n+1}_{zk-1} - H^{n+1}_{zk}) + \frac{Z_0}{M\beta_z} (H^{n+1}_{xk+N_x N_y} - H^{n+1}_{xk}) \quad , \tag{11}$$

$$E^{n+3/2}_{zk} = E^{n+1/2}_{zk} + \frac{Z_0}{M\beta_x} (H^{n+1}_{yk} - H^{n+1}_{yk-1}) + \frac{Z_0}{M\beta_y} (H^{n+1}_{xk-N_x} - H^{n+1}_{xk}) \quad , \tag{12}$$

where k is the serial number representing the position of a cell $(z, y, z) = (\ell\Delta x, i\Delta y, j\Delta z)$, $1 \le \ell \le N_x$, $1 \le i \le N_y$, $1 \le j \le N_z$ and numbered as $k = \ell + (i-1)N_x + (j-1)N_x N_y$. The symbol $\beta_x = \beta_z \frac{\Delta x}{\Delta z}$ and $\beta_y = \beta_z \frac{\Delta y}{\Delta z}$. If $E^{n+3/2}_{zk}$ is located on the primary beam axis, the source term is added:

$$E^{n+3/2}_{zk} = E^{n+3/2}_{zk} - \frac{\lambda^{n+1}_k c\Delta t}{\varepsilon_0 \Delta x \Delta y} \quad , \tag{13}$$

where λ^{n+1}_k is the line charge density (coulomb/meter) at the location where the component $E^{n+3/2}_{zk}$ is.

The initial conditions are

$$E^{1/2}_{x,y,zk} = H^{0}_{x,y,zk} = \lambda^0_k = 0 \tag{14}$$

for all k's.

We skip the description of how to treat boudary conditions in this paper; it is only to patch fields taking into consideration all the cases, rather complicated, though.

The longitudinal field felt by a test charge following at distance s behind a bunch head is called the longitudinal wake field:

$$w_L(s) = \int_0^\infty E_z(x,y,z,t = \frac{s+z}{c}) dz \quad . \tag{15}$$

The transverse wake fields are defined as well by

$$w_H(s) = \int_0^\infty E_x(x,y,z,t = \frac{s+z}{c}) - Z_0 H_y(x,y,z,t = \frac{s+z}{c})\, dz \quad , \tag{16}$$

$$w_V(s) = \int_0^\infty E_y(x,y,z,t = \frac{s+z}{c}) + Z_0 H_x(x,y,z,t = \frac{s+z}{c})\, dz \quad , \tag{17}$$

where $w_H(s)$ and $w_V(s)$ are the horizontal and vertical component of the wake field, respectively.

It has been found that BCI gives nonzero wake fields even for a smooth beam pipe alone which should be zero. This is due to the finite size of meshes and imcompleteness of open boundary conditions. WELL has the same problem. In order to remove this erroneous contribution coming from beam pipe, WELL runs twice automatically, firstly for the total structure, and secondly for the beam pipe alone, and then subtracts the contribution from the beam pipe. This procedure is done only for a beam pipe through which a driving beam goes.

Numerical examples and discussions

Consider a cavity of elliptical cross section with the cylindrical beam pipes on the front and rear of the both foci, which is illustrated in fig. 2.

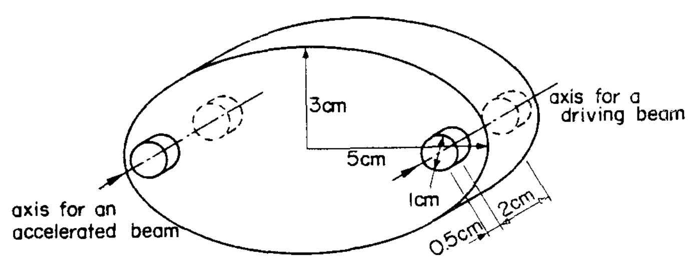

Fig. 2 A cavity of elliptical cross section.

The path length between the two foci is 10 cm if the field generated on one focus axis reaches the other after one reflection at the elliptical wall. The solid line in Fig. 3 shows the longitudinal wake field on the focus axis through which a Gaussian bunch with σ_z = 5mm goes. Note the transverse wake field arizing from the right-left asymmetry of the structure at beam axis, which is however only the horizontal wake field $w_H(s)$ owing to the up-down symmetry as far as a beam runs on the medium (y = 0) plane. The function $w_H(s)$ is shown in fig. 3 by the broken line. One can see that the wake fields become small around the middle of the figure, which reflects the fact that the radiated power outgoes towards the other focus. On the other hand, fig. 4 gives a graph of the wake fields at the focus for an accelerated beam. The large narrow beak A having a full length of about 2 cm corresponds to the very fields focussed after one bound at the elliptical wall. It is proved by the observation that this peak is at about 10 cm behind the peak D in fig. 3. The fields scattering from this focus in turn go back to the first and then come back again. One should read that the similar patterns of the wake fields appear again about 20 cm later in fig. 3. The field gradient achieved in this example is listed in table I.

TABLE I

FIELD GRADIENT GIVEN BY THE ELLIPTICAL STRUCTURE

Number of particles in the driving bunch	10^{13}
Peak current	∿ 50 kA
One standard deViation of the bunch length	5 mm
Field gradient	170 MV/m

What must be pointed out here is that the transverse wake field is not zero at the accelerating wake peak. In the present example its strength is about one-third as large as the longitudinal one. This means that the track of particles will be badly deflected (to the right hand side in this case), and as a result, continuous acceleration will fail. In order to overcome this problem, the following geometries could be thought as possible cures:

i) The right-left symmetrical structure about the common focus for an accelerated beam consisting in combined two elliptical pill boxes. See fig. 5(a). The transverse dipole wake fields are cancelled at the focus.

ii) The alternating aligned structure. See fig. 5(b). Even if particles are kicked to right at the first set of cells, they will be turned back to the center through the second set.

The first geometry, as the number of joined elliptical pill boxes increases, eventually approaches the hollow ring type structure proposed by Voss and Weiland.[1,2] If the right and left balance of the timing of charge of two bunches is out, the dipole field would be excited again. Furthermore, since the contribution from the fields reflected at the arc cut down disappears, the accelerating wake field is not doubled simply. The second geometry has the advantage that using the alternating empty space, the driving beam can be reaccelerated by ordinary RF cavities to replenish the lost energy or can be extracted from the beam line by dipole magnets.

We know through these considerations that it is difficult to have the wake fields under our control like the ordinary microwave, since it is impossible to excite only a mode easy to handle like in a cavity. We then conclude that a rather strong focussing system is necessary for continual stable acceleration, probably even in almost ideal circumstances.

References

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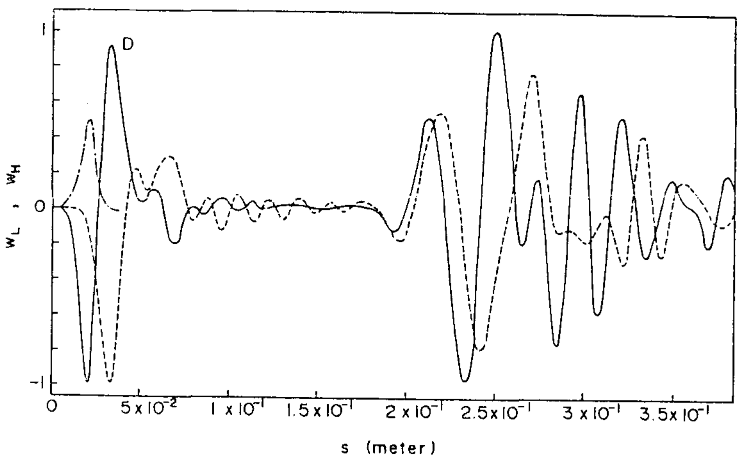

Fig. 3 The longitudinal and transverse wake fields for a driving beam in the cavity of elliptical cross section. Bunch length σ_z = 5 mm. Total Charge Q = 1 coulomn. The scale = 1.92 × 10^{12} V for $w_L(s)$, 1.48 × 10^{12} V for $w_H(s)$.

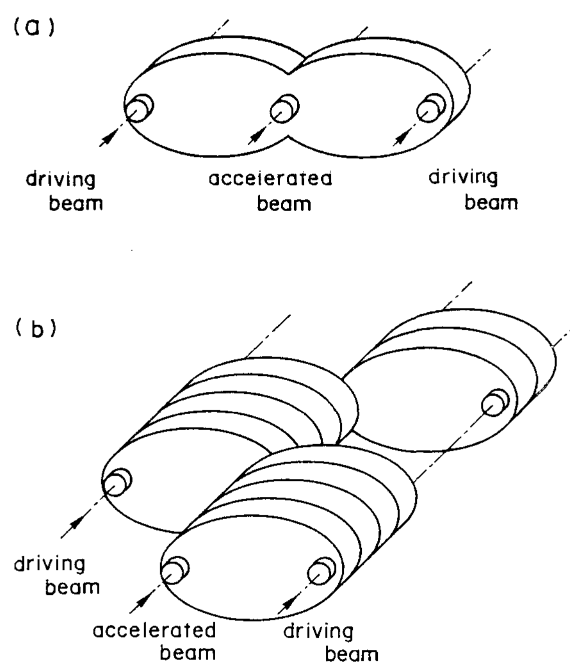

Fig. 5

(a) The right-left symmetrical structure about the common focus for an accelerated beam.
(b) Layout of an alternating aligned structure.

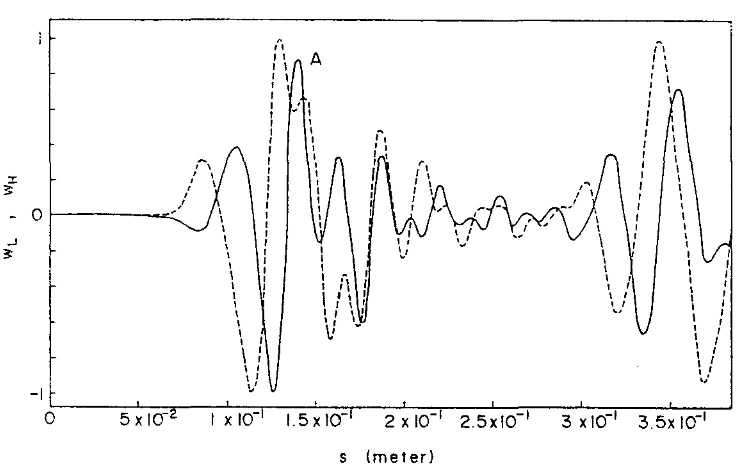

Fig. 4

The longitudinal and transverse wake fields for an accelerated beam. The scale = 2.54 × 10^{12} V for $w_L(s)$, 1.11 × 10^{12} V for $w_H(s)$.