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AN EXPLORATION OF PHASE STABILITY IN THE "SURFATRON" ACCELERATOR*

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Summary

Proton and electron motion in a laser "beat-wave" accelerator with a transverse magnetic field is explored. Parameters of stable acceleration are determined analytically and by simulation. The effects of synchrotron radiation on electron acceleration are also explored.

Introduction

Katsouleas and Dawson[1,2] have developed a modification of the plasma beat-wave accelerator,[3] called a surfatron, that adds a magnetic field to the plasma beat-wave field to maintain synchronism between the particle motion and the wave. The beat-wave longitudinal electric field is

$$\vec{E} = E_0 \sin(k_p x - \omega_p t)\,\hat{x} \tag{1}$$

and develops from plasma-wave oscillations excited by laser pulses of frequencies ω and $\omega + \Delta\omega$ and wave vectors $\vec{k}_1$, $\vec{k}_2$, where $\Delta\omega = \omega_p = \sqrt{4\pi Ne^2/m_e}$ is the plasma frequency and $k_p\,\hat{x} = \vec{k}_1 - \vec{k}_2$. E_0 can be expressed as

$$E_0 = \alpha\, m_e c\, \omega_p \tag{2}$$

with α, the plasma-wave amplitude, $\lesssim 1$. In the surfatron, a constant transverse magnetic field is added

$$\vec{B} = B\,\hat{z} \quad , \tag{3}$$

where $\hat{z}$, $\hat{x}$ are unit vectors.

The equations of motion for a particle with mass M in these fields are

$$\frac{d}{dt}(\gamma\beta_x) = \frac{q E_0}{Mc}\sin(k_p x - \omega_p t) + \frac{q B}{Mc}\beta_y \quad , \text{ and}$$

$$\frac{d}{dt}(\gamma\beta_y) = -\frac{q B}{Mc}\beta_x = -\omega_c\,\beta_x \quad , \tag{4}$$

where ω_c is the cyclotron frequency and $\vec{\beta} = \vec{V}/c$.

Combining these equations, an expression for the rate of energy gain of a particle in a surfatron is found:

$$\frac{d\gamma}{dt} = \frac{qE_0}{Mc}\beta_x \sin(k_p x - \omega_p t) \quad . \tag{5}$$

Here $\gamma = (1 - \vec{\beta}^2)^{-1/2}$.

Surfatron Oscillations

In synchronous injection into a surfatron, β_x is initially set near the wave velocity β_p:

$$\beta_x \simeq \omega_p/(k_p c) = \beta_p \quad .$$

Particles "trapped" by the magnetic field oscillate in β_x about β_p while β_y, the y velocity, increases following (from eq. 4):

$$\beta_y \simeq \frac{-\omega_c\,\beta_p t}{\gamma_p(1 + \omega_c^2\beta_p^2 t^2)^{1/2}} \quad . \tag{6}$$

In the relativistic limit $\beta_y \simeq -1/\gamma_p = -(1-\beta_p^2)^{1/2}$ with $\beta_x \simeq \beta_p$, eqs. (4) and (5) can be combined to obtain

$$\frac{d^2x}{dt^2} = \frac{q E_0}{M\gamma\gamma_p^2}\sin(k_p x - \omega_p t) + \frac{\omega_c c\,\beta_y}{\gamma} \tag{7}$$

or, with $\phi = k_p x - \omega_p t$ and $\beta_y \simeq -1/\gamma_p$,

$$\frac{d^2\phi}{dt^2} \simeq \frac{k_p q E_0}{M\,\gamma\gamma_p^2}(\sin\phi - \sin\phi_0) \tag{8}$$

with

$$\sin\phi_0 = \frac{\gamma_p M\omega_c c}{q\,E_0} = \frac{\gamma_p B}{E_0} \quad .$$

The motion is stable if $|\sin\phi_0| < 1$ or $\gamma_p B < E_0$, which is precisely the condition for "magnetic trapping" in ref. (1). Equation (8) is the expression for phase oscillations about a central stable phase ϕ_0 at which particles gain energy at a rate given by

$$Mc^2\frac{d\gamma}{dt} = q\,E_0\,\beta_p\,c\,\sin\phi_0 = q\,\gamma_p\,\beta_p\,c\,B \quad , \tag{9}$$

and energy oscillations about the central energy are found from

$$\frac{d\,\Delta\gamma}{dt} = \frac{q\,E_0\,\beta_p}{Mc}(\sin\phi - \sin\phi_0) \quad , \tag{10}$$

$$\text{whence } \Delta\gamma \simeq \frac{\beta_p\,\gamma_0\gamma_p^2}{k_p c}\frac{d\phi}{dt} \quad .$$

The phase oscillation eq. (8) is quite similar to the equations for synchrotron oscillations familiar to accelerator scientists, and the properties of surfatron oscillations may be developed by analogy.[4] The surfatron oscillation frequency is

$$\Omega_s = \sqrt{\frac{q\,E_0\,k_p\,|\cos\phi_0|}{M\,\gamma_0\,\gamma_p^2}} \quad . \tag{11}$$

Trapped particles are found within boundaries of a stable accelerating bucket with boundaries determined by

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$$\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + \frac{q\, E_o k_p}{M\, \gamma_p^2 \gamma}\left[\cos\phi - \cos\phi_1 + \phi\sin\phi_o - \phi_1\sin\phi_o\right] , \tag{12}$$

with ϕ_1, the extreme stable phase, $= \pi - \phi_o$.

The bucket area (in ϕ - $\Delta\gamma$ space) increases as $\sqrt{\gamma}$ in acceleration, so trapped particles remain trapped. In adiabatic acceleration, the area of a trapped bunch remains constant. The phase width $\Delta\phi$ decreases as $\gamma^{-1/4}$ while the energy width $\Delta\gamma$ increases as $\gamma^{1/4}$. The major difference between surfatron and synchrotron oscillations is that surfatron acceleration is fast and not entirely adiabatic; the approximation condition

$$\dot{\gamma} << \Omega_s\, \gamma \tag{13}$$

is not always valid.

The oscillations are substantially different from linac phase oscillations. In an ultrarelativistic linac, the oscillation frequency ($\Omega \sim \gamma^{-3/2}$) becomes quite slow and particles approach an asymptotic phase as $t \to \infty$. In the surfatron, ($\Omega_s \sim \gamma^{-1/2}$) oscillations continue, although slowed, and no asymptotic phase is reached;

$$\theta = \int_0^\tau \Omega_s\, dt \quad \text{increases indefinitely.}$$

The features of surfatron oscillations described above have been confirmed in simulations integrating the exact equations of motion.[4]

Synchrotron Radiation in a Surfatron

Radiation in a transverse magnetic field limits electron acceleration in a synchrotron to <100-GeV final energy. It is important to determine whether a similar limit exists in a surfatron.

The power radiated by a particle is given by[5]

$$P = \frac{2}{3}\frac{e^2}{c}\, \gamma^6 \left[(\dot{\vec{\beta}})^2 - (\vec{\beta} \times \dot{\vec{\beta}})^2\right] , \tag{14}$$

which can be combined with the equations of motion to obtain

$$P = \frac{2}{3}\frac{e^2}{c}\, \frac{\omega_c^2\, \beta_x^2}{1 - \beta_x^2} + \frac{2}{3}\frac{e^2\gamma^2}{c}\left[\sqrt{1 - \beta_x^2}\, \frac{eE_o}{Mc}\, (\sin\phi - \sin\phi_r)\right]^2 , \tag{15}$$

where

$$\sin\phi_r \equiv \frac{-\beta_y\, B}{(1 - \beta_x^2)E_o} \cong \sin\phi_o$$

at high energies.

An exactly synchronous particle has a constant phase at $\phi = \phi_o$ with only the first term of eq. (15) nonzero. This first term can be expressed using the rate of energy change $\dot{\gamma}$ as

$$P = \frac{2}{3}\, \frac{e^2}{c}\, \dot{\gamma}^2 . \tag{16}$$

This same result is obtained in the corresponding linac, and in both cases is quite small and does not change as particle energies increase.

The second term of eq. (15), which corresponds to radiation from the transverse B-field modified by the E-field, is nonvanishing with finite phase errors and increases as γ^2, becoming important in the ultrahigh energy acceleration of electrons. In the high-energy limit, this term becomes

$$P \approx \frac{2}{3}\frac{e^2}{c}\, \omega_c^2\, \gamma^2 \left(\frac{\sin\phi - \sin\phi_o}{\sin\phi_o}\right)^2 , \tag{17}$$

and acceleration requires that this term be less than the synchronous rate of energy gain. This implies (for electrons)

$$\frac{E^2\ (\mathrm{GeV})\ B(\mathrm{T})}{\gamma_p}\left(\frac{\sin\phi - \sin\phi_o}{\sin\phi_o}\right)^2 < 2.3 \times 10^5 , \tag{18}$$

where E is the electron energy.

The phase error term ($\sin\phi - \sin\phi_o$) will be excited by the beam phase spread as well as fluctuations in the plasma-wave phase and amplitude. Such fluctuations are expected at about the 10% level in a reasonably uniform plasma excitation. With $B = 10$ T and $\gamma_p = 10$ (typical values), $E < 5$ TeV is implied.

Surfatron oscillations are not directly affected by the radiation; however the plasma could be disturbed by intense particle radiation. Heavier particles (μ, p, etc.) can obtain much larger energies before encountering radiation difficulties.

Numerical Examples of Surfatrons

Previously, simulation results demonstrating particle trapping and surfatron oscillations have been presented, and parameters for high-energy and demonstration accelerators have been suggested.[4] Because a functional surfatron combines high-intensity synchronized laser pulses, a uniform medium-density plasma, an injected beam, and a uniform high magnetic field, a precise formulation of optimum parameters is not yet possible. In this section we outline two possible surfatrons to demonstrate the technique of parameter generation.

The most difficult requirements are the laser characteristics. High-power lasers with very short pulses are required. The time scale of the pulse is set by the fact that the plasma wave remains coherent only for a time less than the ion oscillation time[5] τ_I where

$$\tau_I = \sqrt{\frac{m_p}{m_e}}\, \frac{2\pi}{\omega_p} .$$

High intensity on this time scale is somewhat beyond current capabilities;[5] lasers with wavelengths from $\lambda = 10\ \mu$ (CO_2) to $0.3\ \mu$ (frequency-tripled Nd-glass) may obtain the necessary intensity. For these numerical examples $\lambda = 1\ \mu$ is chosen.

Uniform excitation of a plasma probably requires that the parameter α be much less than 1; α = 0.1 to 0.25 is chosen here. The other parameters are set by considering the equation $\alpha\, m_e\, c\, \omega_p \sin \phi_o = e\, \gamma_p\, B$, noting that $\gamma_p = \omega/\omega_p$ and $\omega_p = \Delta\omega$. Sin $\phi_o < 1$ is required for stability, and $\gamma_p \gg 1$ is also required. B is chosen in a reasonable field range (1 to 10 T) and the remaining parameters are reshuffled such that the acceleration rate is at a comfortably high level, many gigavolts/meter. The synchronous acceleration rate can be rewritten as

$$\frac{dE}{dx} = \gamma_p\ 0.30\ B(T)\ \frac{\text{GeV}}{\text{m}} \quad .$$

With $B \geq 1$ T and $\gamma_p > 10$, substantial acceleration is obtained.

Parameters for an electron and a proton surfatron are displayed in table 1. Simulations of particle trapping and acceleration have been performed for these cases. In these simulations, a particle distribution (with energy γ in a distribution about γ_p and random phase ϕ and zero V_y) is injected and the equations of motion integrated until much higher energies are reached by trapped particles. Surfatron oscillations are observed, and phase-energy distributions are measured. Some results are displayed in table I. The idealized constant amplitude sine wave of eq. (1) is used to represent the plasma-wave electric field, implying that the beam phase-space area is conserved in acceleration. Fluctuations are expected to produce substantial phase space increase but, because the stable buckets are quite large, acceleration should remain stable at high energies.

TABLE I

Parameter	Electron-Accelerator Example	Proton-Accelerator Example
B-field	10 T	1.25 T
Laser wavelength	1.0 μ	1.0 μ
Plasma-wave amplitude α	0.25	0.1
Plasma gamma $\gamma_p = \omega/\omega_p$	13.77	25
Plasma density N	5.9×10^{18} cm^{-3}	1.8×10^{18} cm^{-3}
Synchronous ($e\,Bc\gamma_p$) acceleration rate	41.3 GeV/m	9.38 GeV/m
Length for 1-TeV accelerator	24.2 m	107 m
Surfatron oscillation parameters:		
sin (ϕ_o) =	0.704	0.73
Ω_s/c ($\gamma = \gamma_p$)	3.76×10^3 m^{-1}	12.7 m^{-1}
Ω_s/c (1 TeV)	9.97 m^{-1}	1.95 m^{-1}
Simulation results:		
Capture efficiency	66%	41%
ΔE total at 100 GeV of captured bunch	0.36 GeV	2.2 GeV
$\Delta\phi$ total at 100 GeV	0.22	1.4

Conclusions

This paper has discussed some features of the surfatron, which provides the possibility of stable high-energy acceleration of particles. The nature of surfatron oscillations has been discussed and limitations imposed by synchrotron radiation have been described.

References

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