LONGITUDINAL BUNCHING OF ELECTRONS IN THE ADVANCED TEST ACCELERATOR*

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Introduction

The Advanced Test Accelerator (ATA) is a linear induction accelerator for electrons with design goals of 50 MeV and 10 kA. The accelerator consists of a 2.5 - 3 MeV injector and 170 accelerating units, each unit capable of an applied voltage up to 350 kV. At a number of positions in the accelerator, the return current in the beam pipe is measured as a function of time as the beam passes that position. The measuring device, a four quadrant addition, is somewhat sensitive to the transverse position of the beam centroid as well as transverse distribution of current in the beam. Therefore a time-dependent signal may indicate time-dependent transverse motion or beam structure as well as an axial modulation of the beam current. These signals sometimes exhibit spikes and notches on the leading and trailing edges of the beam pulse. The presence of these rapidly varying signals during the rise and fall of the current stimulated this work.

We investigate a possible mechanism for axial modulation, or "bunching," of the electron beam. This mechanism is the path length difference between particles executing transverse motion and particles that remain on axis in the solenoidal transport system. The particles remaining on axis and leaving the injector at time t_0 will arrive at a position z downstream from the injector at a time $t = z/v + t_0$. All particles have the same speed v, but those particles leaving the injector at time t_0 and executing transverse motion have a longer path length and arrive at position z at a later time. Thus all particles leaving the injector at time t_0 arrive over an interval of time at a downstream position. The spread in arrival time is a function of particle energy (i.e., voltage on the accelerating units), and is not monotonic. If the voltage changes linearly with time, the spread in arrival time varies in an oscillatory fashion and causes a modulation of the beam current at the downstream position.

The mechanism is similar to that which gives rise to current modulation in a Klystron amplifier, in which the variation of arrival time is achieved by a modulation of axial speed, not variation of path length.

Computation

Two computer codes were used to study the spread in path length vs voltage on the accelerating units in ATA. Our theory is based primarily on results of the code TRANSPORT.[1] This is a matrix code and in the form used in these calculations, coherent self-forces are not included. A few calculations were also performed with the particle simulation code WIRE,[2] which includes coherent radial self-forces as well as potential suppression of particle's kinetic energy. In addition, the WIRE code treats the solenoidal magnetic guide field in greater detail than does the TRANSPORT code, and requires much more computer time. Results of the WIRE code predict a greater spread in path length than results of the TRANSPORT code, mainly because of the finer detail of the guide field. We describe the TRANSPORT calculations in detail, and point out significant additional results from the WIRE calculations.

The TRANSPORT code treats a volume in six-dimensional phase space. Four dimensions are x, x', y, and y' in which prime denotes d/dz. The other two dimensions are spread, S, in longitudinal distance and percent momentum deviation $\delta p/p$. In our calculations all particles have the same total momentum so we consider only five dimensions. At the end of the injector the calculation begins with a four-dimensional volume that has no extent in the longitudinal direction. There is no assumption regarding how particles are distributed in this volume. The code then calculates the length S of this volume as a function of distance down the accelerator with a given voltage V on each accelerating unit. The size of the initial 4-D volume is set by the beam emittance, and for our calculations a normalized emittance of 0.5 rad-cm was used in both transverse planes. This value agrees with measured values at the end of the injector. However, the measured values probably correspond to a rms emittance, and larger values should be used in TRANSPORT. The length S increases as the initial emittance is increased, and generally the dependence is linear.

The calculation is repeated with different values of V on the accelerating gaps, and the curves in Fig. 1 are obtained. These curves show the axial extent S as a function of V at various positions in the accelerator. Since S is a function of position z as well as accelerating voltage V, there is a spread in arrival time $\Delta t \equiv \tau$ with τ given by

$$\tau(z, t_0) = S[z, V(t_0)]/v. \qquad (1)$$

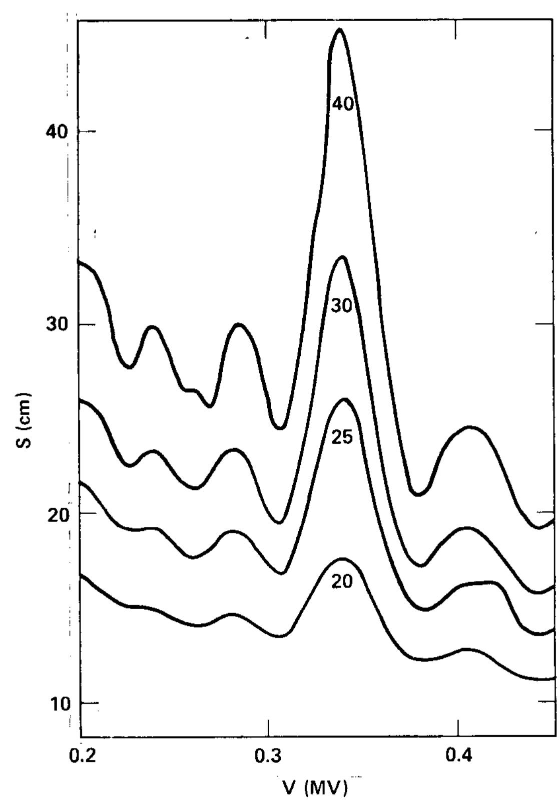

Fig. 1 Curves of maximum additional path length S vs accelerating voltage V. The numbers on the curves indicate axial position (e.g., after 20 accelerating units).

The spread in arrival time depends on the particle energy because the solenoidal transport system generally is tuned for best transport of the peak beam current. (The accelerating voltage depends on the beam current as discussed below.) If the tune (current in the individual solenoids) is appropriate for a given energy, the beam envelope varies smoothly down the machine. Such a beam is said to be well matched to the tune. For the same tune but with a different particle energy, the beam envelope will oscillate, particles undergo larger transverse oscillations, and the spread in arrival time will be greater than that for the matched beam. The relation between the voltage V, applied voltage V_a, and beam current I_b is approximately

$$V = V_a - ZI_b, \qquad (2)$$

in which Z is the loading impedance with a value of about 8Ω. Even if the applied voltage V_a is constant in time, the particles' energy varies with time during the rise and fall of the current pulse. As the beam current rises from zero to 10 kA, the voltage on the accelerating units varies from 330 kV to about 250 kV. In this energy interval there are well matched and badly matched conditions.

In addition to the beam loading, a time variation of V can arise from a time varying applied voltage V_a. The applied voltage is not constant during the beam pulse, especially if the beam arrives at an accelerating unit before V_a has reached the nominally constant value, or if the tail of the beam is still traversing the unit when V_a drops from the nominally constant value. Therefore, timing is very important in maintaining a constant V_a during the beam pulse. In the following sections both beam loading and a time variation of V_a will be considered.

The tune used to calculate the curves in Fig. 1 approximates the settings commonly used in ATA. The axial magnetic field B_z is a few hundred Gauss at the injector and rises in a stairstep fashion to 3 kG at the 20th accelerating unit. This value is then maintained through the rest of the accelerator. In the accelerator, the individual solenoids give rise to a "ripple" in B_z of about 20 percent to 30 percent with a period of 28 cm. This ripple was not included in the TRANSPORT calculations, but was included in particle simulation with the WIRE code.

During the course of this work, some errors were found in the TRANSPORT code,[1] which had never been used to calculate path length difference in a solenoidal transport system.

A particle leaving the injector at a time t_0 will arrive at position z at a time t given by

$$t - (z/v) = t_0 + s(z, t_0)/v \, . \qquad (3)$$

The additional path length will be such that $0 < s < S$, and the exact value is determined by the particle's initial position in x – x' and y – y' phase space. If we suppose (as is not true in our calculation) that all the particles leaving at t_0 have the same path length difference s we can calculate the current I(t) at position z from the relation

$$I(t)dt = I(t_0)dt_0$$

$$I(t) = I(t_0)\left[1 + \frac{1}{v}\frac{\partial s}{\partial t_0}\right]^{-1} \, . \qquad (4)$$

This relation shows that the current I(t) depends on the time derivative of $s(z,t_0)$.

In our calculation, all particles do not have the same value of s. Particles leaving the injector at time t_0 arrive in a time interval between $t_0 + (z/v)$ and $t_0 + (z/v) + \tau(z,t_0)$. The situation is shown in Fig. 2. Just how particles are distributed between these two arrival times depends on the distribution of particles in transverse phase space at the end of the injector. This distribution is not known, however, the WIRE code results indicate that for reasonable distributions of particles in the initial 4-D phase space, at a position z the distribution in s is heavily weighted toward S . This suggests a model in which all particles in the beam are distributed uniformly in a band of width Δt = T between the two curves $t-(z/v) = \tau$ and $t-(z/v) = \tau - T$. Under certain conditions, which are well satisfied in our calculation, the results are independent of T, and we obtain

$$I(z,t) = I(t_0)[1 + (\partial\tau/\partial t_0)]^{-1} \qquad (5)$$

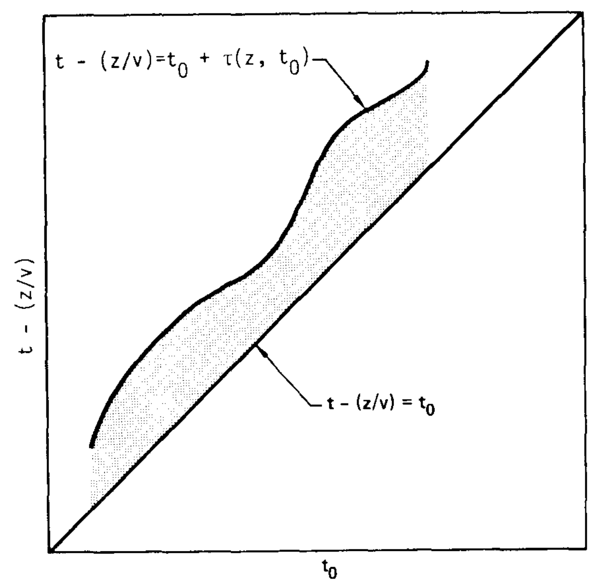

Fig. 2 Plot of arrival time vs departure time. Particles occupy the region between the two curves.

Current Modulation in ATA

We first consider a time variation of V resulting from a time variation of the beam current I_b through the relation, Eq. (1), with V_a constant in time at a value of 330 kV. The current I_b rises from 0 to 10 kA in 10 ns. We obtain τ' from the relation

$$\tau' = \frac{1}{v}\frac{\partial S}{\partial t_0} = \frac{1}{v}\frac{\partial V}{\partial t_0}\frac{\partial S}{\partial V} \, . \qquad (6)$$

The value of ∂S/∂V is found by differentiating polynomials fit to the curves. Care must be taken in plotting I(t) given by Eqs. (5) and (6). First we choose a value of t_0 and obtain a value for I. This value is assigned an arrival time $t-(z/v) = t_0 + \tau$. Results of this procedure are shown in Fig. 3 for a z position just beyond the 40th accelerating unit. There is some current modulation, but not enough for qualitative agreement with experimental data.

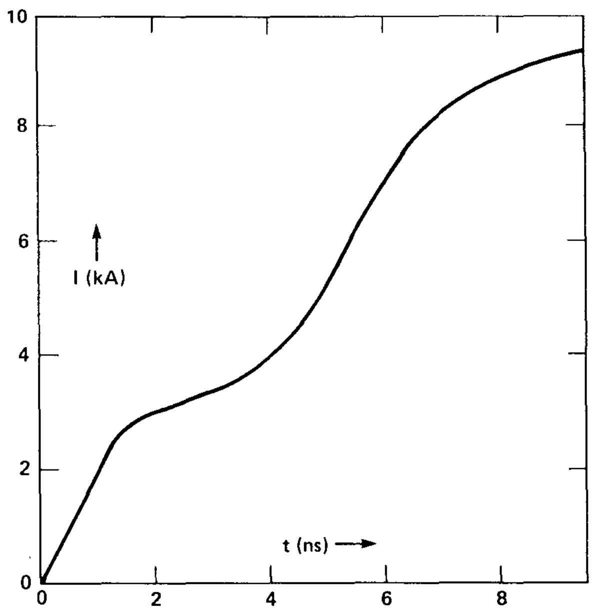

Fig. 3. Current vs time after 40 accelerating units with time variation of V arising from beam loading.

We now consider the effect of a time varying V_a. Near the end of the accelerating voltage pulse, V_a drops from its peak value to zero in about 10 ns. If the peak value is 250 kV, then $\partial V_a/\partial t = -25$ kV/ns. Although we have not obtained curves of S(V) for V less than 220 with the code TRANSPORT, the code WIRE predicts the same oscillatory behavior for lower voltage. We assume an S of the form

$$S = S_1 + L \sin[(V - 230)\pi/20)], \qquad (7)$$

with V in kV, to be valid over the range of values 125 kV < V < 250 kV. The form of Eq. (7) is chosen to approximate the curves in Fig. 1. We choose L = 3 cm, which is appropriate for S after 40 accelerating units. We insert this value as well as $V = 250 - 25t_0$ in Eq. (7). The value of S_1 is unimportant. With the above numbers we have $\tau = -(0.4/\omega) \sin \omega t_0$ and Eq. (5) yields

$$I(t) = I(t_0)(1 + 0.4 \cos \omega t)^{-1} \qquad (8)$$

in which $\omega = (\pi/20)dV/dt = 5\pi/4 \times 10^9$, or a frequency of 625 MHz. Equation (8) is the basic relation in klystron theory, with the so-called "bunching parameter" equal to 0.4. If $I(t_0) = I_0$ is constant, I(t) over one cycle is plotted in Fig. 4. The peak current is 1.67 I_0 and the minimum current is .71 I_0.

In ATA the phenomenon is observed over a frequency range of about 300 MHz to 600 MHz. Generally, the peak to valley ratio increases with increasing frequency, first as predicted by the above model. If, for instance, the beam current remains constant for 5 ns after the voltage starts to fall, about 3 such modulations would occur.

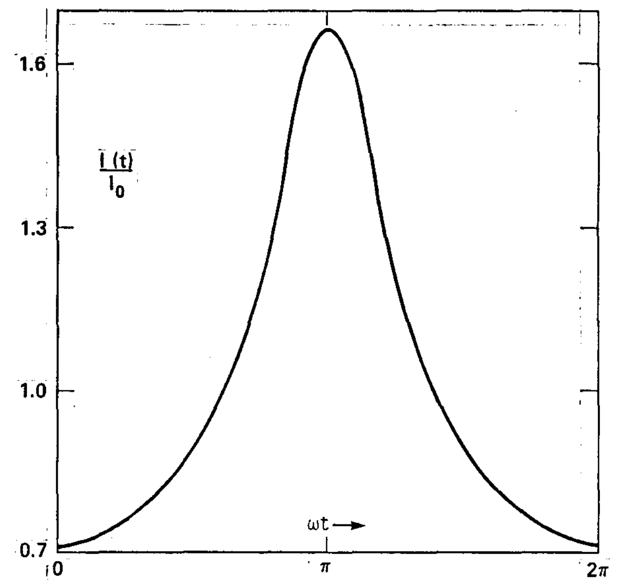

Fig. 4. Current vs time over one cycle as given by Eq. (8).

Although the above example treats the tail of the current pulse, bunching on the head of the current pulse can be explained by a rising value of the accelerating voltage. The rate of rise is, however, less than the rate of fall. We also conclude that (to the extent that Eq. (1) is a valid representation of beam loading) that beam loading is not an important effect for a beam current rise of 1 kA/ns.

References

1. A. C. Paul, "Transport, an Ion Optic Program, LBL Version." Lawrence Berkeley Laboratory Report LBL-2697, (1975). The second order path length matrix elements, Eqs. (3.11) to (3.14) were corrected for this work.

2. G. J. Caporaso and A. G. Cole, IEEE Trans. Nucl. Sci. 30 2618 (1983).

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