COLLISIONAL HEATING OF BEAMS IN STRONG FOCUSING ACCELERATORS

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1. This is a brief report of the results obtained in our papers /1,2/. The subject is the heating of the beam, i.e. increasing of its phase space volume or partial temperatures, due to intra beam scattering (IBS). Though consideration in Refs /1,2/ was done for ring accelerators, some of the effects discussed there probably can be important in linear accelerators. Especially in machines with strong focusing, which give additional modulation of fields induced by particles.

It is well known that in the simplest case IBS equalizes transverse and longitudinal temperatures, which cause a redistribution of emittances in the beam. If such redistribution is not desirable, this effect limits the phase space density of the beam.

The effect of focusing fields makes the kinetics of IBS more complex /3,4/. In this case all collisions can be divided into fast ($\tau_c \omega_\alpha \ll 1$, τ_c – typical collision time, ω_α – oscillation frequencies in the focusing fields) and adiabatical ($\tau_c \omega_\alpha > 1$) collisions. For adiabatical collisions the energy transfer to oscillatory degrees of freedom is depressed exponentially. So, the redistribution of partial phase space volumes (the thermolization of the beam) in an external field is caused by fast collisions.

The nature of forces, which are leading, dependes on beam parameters. For low energies and high beam density relaxation can be caused by Coulomb scatterings. For high energy and high beam currents collision of particles with wave fields, can be more significant.

It was shown in Ref. /4/ that in a strong focusing machine Coulomb scattering blowups the total phase space volume of the beam. This was studied more straightforward in Ref. /1/. It turms out, that this is a specific feature of collisional relaxation provided that the interaction of two particles in the beam is modulated by motion along the equilibrium trajectory.

Before discussion of results obtained in Ref. /1/ let us introduce some definitions. Evolution of the distribution function in the beam is governed by stoss integral. In our case the calculation of the stoss integral should take into account that motion of particles in an accelerator is a combination of motion along some central trajectory and oscillations around this trajectory. The interaction of particles leads to small disturbances of this unperturbed motion. We shall also assume that collision times are always much shorter than the period of phase oscillations. Then the motion around the central trajectory can be described /5/ by

$$X = X_b + \eta \frac{\Delta p}{p_s},\ \Delta p = p - p_s,\ \vec{p}_\perp = p_s \frac{d\vec{r}_\perp}{d\sigma},$$
$$(X_b, z) = \sqrt{\varepsilon\beta}\cos(\psi + \chi(\sigma))\Big|_{x,z}, \tag{1}$$
$$\sigma = \sigma_s + \sigma_o + \sigma_b,\quad \sigma_s = v_s t,$$
$$\sigma_b = \frac{dx_b}{d\sigma}\eta - \frac{d\eta}{d\sigma}x_b,\quad \dot\psi_{x,z} = \omega_{x,z},$$
$$I_{x,z} = \sqrt{\tfrac{1}{2} p_s \varepsilon_{x,z}},$$

which make canonical transformation from variables ($\vec{r}$, $\vec{p}$) to integrals of unperturbed motion ($I_\perp$, ψ°, Δp, σ_o). We use $p_s = \gamma M v_s$ – the momentum of synchronous particle, $\varepsilon_{x,z}$ betatron emittances, the betatron functions $\beta_{x,z}(\sigma)$ and $\chi_{x,z}(\sigma)$ have the period of the lattice

$$\beta(\sigma + L_o) = \beta(\sigma),\quad \chi(\sigma + L_o) = \chi(\sigma).$$

Instead of σ it is more convenient to introduce the angular variable $\psi_{\parallel} = 2\pi(\sigma - \sigma_b)/L_o$. Then $\dot\psi_\parallel = \omega_o(p)$, $\omega_o = 2\pi V/L_o$, $\omega_s = \omega_o(p_s)$, $R_o = 2\pi/2\pi$, $I_\parallel = R_o \Delta p$, $\omega_{x,z} = \omega_o(p)\nu_{x,z}$ and the betatron phase advance on the lattice period is $2\pi\nu$. In variables (I, ψ) the hamiltonian of unperturbed motion is especially simple

$$\mathcal{H}_o - E_s = \omega_s I_\parallel + \mathcal{H}_T = \omega_s I_\parallel + \frac{I_\parallel^2}{2\mu_\parallel} + \vec{\omega}_\perp \vec{I}_\perp, \tag{2}$$

where E_s is the equilibrium energy and $\mu_\parallel = (R_o\, d\omega_o/dp)^{-1}$ is the effective mass of longitudinal motion. In linear accelerators there is no contribution from dispersion items ($\sim \eta$ and η'). Due to nonlinearity of focusing, the tunes $\nu_{x,z}$ depend on $I_\parallel$ and $I_\perp$.

If one introduces the one particle distribution function as a statistical average of the microscopic phase space density of the beam

$$f(\Gamma, t) = \langle F(\Gamma, t)\rangle \equiv \Big\langle \sum_{a=1}^{N} \delta(\Gamma - \Gamma_a(t))\Big\rangle \tag{3}$$

$\Gamma \equiv \{I, \psi\}$, N – number of particles in the beam, the stoss-integral is determined by simultaneous correlator of fluctuations F and induced by the beam fields /6/. Provided by necessary hierarchy of times /6/, the dynamic of fluctuations of F is described by Vlasov's equations. If also conditions of coherent stability are valid, one can believe that for long scale times f is independent of phase variables. With such assumptions the kinetic of the beam is described by the system of equations:

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \vec{I}}\Big\langle \frac{\partial \delta\mathcal{L}}{\partial \vec\psi}\,\delta F\Big\rangle,$$
$$\Big(\frac{\partial}{\partial t} + \vec\omega\frac{\partial}{\partial\vec\psi}\Big)\delta F = -\frac{\partial\delta\mathcal{L}}{\partial\vec\psi}\cdot\frac{\partial f}{\partial \vec I}. \tag{4}$$

Here $\delta\mathcal{L} = \mathcal{L} - \langle\mathcal{L}\rangle$ is the fluctuation of the interaction lagrangian, $\delta F = F - f$,

$\vec{a}\vec{b} = a_x b_x + a_z b_z + a_\parallel b_\parallel$. If the interaction energy of a couple is $V(1,2)$, $\delta\mathcal{L}$ is

$$\delta\mathcal{L}(\Gamma_1, t) = -\int d(2)\, V(1,2)\, \delta F(2,t) \tag{5}$$

$(1,2) \equiv (\Gamma_1, \Gamma_2)$. The functions contributing to eqs (3÷5) are periodical functions of phases $\{\varphi\}$. The periodical dependence on longitudinal coordinates is caused by periodicity of the lattice or by periodical placement of elements providing the interaction of particles. Thus there are expansions

$$X(I,\varphi,t) = \sum_{\vec{m}} X_{\vec{m}}(I,t)\, e^{i\vec{m}\vec{\varphi}},$$

where $\vec{m} = \{m_x, m_z, n\}$ integer numbers, X is any function from F, $\mathcal{L}$, etc.

The calculations with eqs (4) in the lowest order of the perturbation theory on the interaction yield equation, which is analogous to that obtained by S.T.Belaev /3/:

$$\frac{\partial f}{\partial t} = -\frac{\partial \vec{j}(I,t)}{\partial \vec{I}},$$

$$\vec{j}(I,t) = -\pi \sum_{\vec{m}_1, \vec{m}_2} \vec{m}_1 \int d(2)\, \delta(\omega(1) - \omega(2)) \times \tag{6}$$
$$\times |V(1,2)|^2 \left\{ \left(\vec{m}_1 \frac{\partial f(1)}{\partial \vec{I}_1}\right) f(2) - \left(\vec{m}_2 \frac{\partial f(2)}{\partial \vec{I}_2}\right) f(1) \right\},$$

with $\omega(1) = \vec{m}_1 \vec{\omega}_1$ and

$$V(1,2) = \int_0^{2\pi} \left(\frac{d\varphi_1}{2\pi}\right)^3 \left(\frac{d\varphi_2}{2\pi}\right)^3 V(1,2)\, e^{-i\vec{m}_1\vec{\varphi}_1 + i\vec{m}_2\vec{\varphi}_2}$$

When obtaining (6), one should assume that coherent tune shifts due to interaction are smaller that the frequency spread in the beam ($|\vec{m}\Delta\vec{\omega}|$).

Eq. (6) yields two important results - - the conservation of average hamiltonian

$$\frac{d\mathcal{H}_0}{dt} \equiv \frac{d}{dt}\left(\frac{1}{N}\int d\Gamma\, \mathcal{H}_0\, f(\Gamma,t)\right) = 0 \tag{7}$$

and the H - theorem of Boltzmann

$$\frac{dS}{dt} \equiv \frac{d}{dt}\int d\Gamma \ln f(\Gamma,t)\, f(\Gamma,t) \gtrless 0, \tag{8}$$

where S - is the beam entropy. By definition /7/, eq (8) determines the rise time of the total phase space volume of the beam $\Delta\Gamma$:

$$\frac{1}{\tau_t} = \frac{d \ln \Delta\Gamma}{dt} = \frac{dS}{dt}. \tag{9}$$

So, relaxing of the beam to equilibrium due to IBS any way blowups $\Delta\Gamma$, i.e. heats the beam.

The distribution functions corresponding to equilibrium states of beams should have a form $f_{st} = f_{st}(\mathcal{H}_T)$ which is possible when $\mathcal{H}_T$ is an invariant of collisions. One can easily obtain from eqs (6,7) that for such equilibrium distributions

$$f \sim \exp(-\mathcal{H}_T/T)$$

$$\frac{d\mathcal{H}_T}{dt} = \frac{\pi\omega_s^2}{NT}\sum_{m_1,m_2}\int d(1)d(2)f(1)f(2)\delta(\omega(1)-\omega(2)) \times$$
$$\times |V(1,2)|^2 (n_1 - n_2)^2. \tag{10}$$

The r.h.s. of eq. (10) is not zero unless $V_{n,n'}$ is diagonal ($V_{nn'} \neq \delta_{nn'}$), i.e. when interaction of two particles is modulated by average longitudinal motion. In such conditions eq (8) also yields

$$\frac{dS}{dt} > 0 \tag{11}$$

the result of Ref. /4/. The heating effect is caused by energy transfer from average longitudinal motion to thermal degrees of freedom due to sum resonances ($\vec{m}_1\vec{\nu}_1 + \vec{m}_2\vec{\nu}_2 = n$, $\vec{m}_1$, $\vec{m}_2$, n - are integers) taking place for colliding particles. As we said for Coulomb scatterings the necessary modulation is a result of the modulation of betatron functions ($\beta(\sigma)$ and $\chi(\sigma)$) specific for strong focusing machines. For interaction via surroundings this modulation can be caused by finite size of the interaction region.

2. One more important feature of the kinetic due to IBS is connected with the long range interaction of colliding particles. Such interaction yields collective phenomena of a beam, i.e. possibility for perturbation to propagate like waves in the phase space of the beam. If the problem of coherent stability is solved (which certainly is the problem itself) the life times of these waves τ_K are determined by frequency spread (and, probably, by interaction with dissipative surroundings). If τ_K is comparable with relaxation time τ_t these coherent oscillations can exchange energy with particles. The energy transfer is a result of successive emission and absorption of waves by particles. Such reemission equalizes the temperatures of coherent oscillations and beam particles, i.e. particles fall into equilibrium with the coherent noise of the beam. The mean number of oscillations in the equilibrium N_{ex} dependes on the beam temperature T ($N_{ex} = N_{ex}(T)$) . For nondissipative surrounding, the total energy of thermal motion

$$\frac{NT}{2} + \text{energy of fluctuating fields}$$

is conserved when relaxing to equilibrium. So if the initial level of the beam coherent noise was above the thermal level, the thermalization of particles and coherent oscillations will heat the beam /2/. The relaxation rate for such process $1/\tau_t$ is equal to the sum of the decrements of coherent modes taking part in the thermalization (divided by N) and can be close to $1/\tau_K$, if the frequency spread in the beam is small enough.

Described mechanism is the leading when the main part of collisions turns to be adiabatical ($\tau_c\omega_\alpha > 1$). In the strong focusing machines this condition is valid as better as higher betatron tunes ($\nu_{x,z} \gg 1$).

The heating of beam by nonequilibrium coherent noise can enhance the heating rate of the beam from coherent kicks. The last was discussed, for example, in Ref./8/ as one of the heating mechanisms for the beam in VLEPP. After thermalization in the beam of the initial coherent noise, such coherent kicks produce new portions of nonequilibrium noise with the energy of the order of that of induced coherent oscillations. If the period Δt_o of such kicks exceeds the relaxation time $\Delta t_o > \tau_2$, absorption of this additional energy in the beam will enhance the diffusion due to kicks. One should expect that additional diffusion due to interaction of particles will be proportional to coherent tune shifts and so, will increase as the beam current increases.

Let us point out one more phenomenon, which is interesting from a general physics point of view. The presence in the equilibrium of coherent oscillations yields the ordering of the particle motion in the beam, which change the properties of equilibrium itself. The level of this ordering increases with increase of N_{ex} and for small enough frequency spread in the beam this can reach the crystal ordering /2/.

The symmetry of such crystal beam dependes on the focusing properties of the lattice and on the beam parameters. In fact in Ref. /2/ was shown, that in nonrelativistic coasting proton beam, moving in a synchrotron with the particle energy below transition the collective interaction of particles yields the exact longitudinal ordering of the betatron oscillation planes if $N_{ex} = N$. The average distance between neighbour planes in this state tends to Π/N (Π - orbit perimeter). In conditions considered in Ref. /2/ the occurrence of the crystal structure in the beam was connected with cooling to very small longitudinal temperatures. More general is a requirement of occurrence in the beam of a very small frequency spread. The last besides temperature is determined by nonlinearity of the focusing and space charge fields. For relativistic beam the contribution to tunes from space charge is depressed. Thus in the machines with small nonlinearities provided by wide enough spectrum of coherent oscillations the beam can get more complex (compearing with described) crystal structures.

The depression of the relative mobility of particles due to action of collective fields can depress the thermalization in the beam due to IBS. Such an effect was observed experimentally on Schottki noise measurements at NAP-M /9/ and on measurement of the longitudinal temperature in the magnetized electron beam /10/.

If the interaction of particles yields coherent instabilities but these are damped by Landau damping, the process of noise thermalization should be more complicated. In Ref. /9/ (and actually in /2/) it was shown that this time the equilibrium noise level (if it exists) tends to blow-up while reaching threshold ($T \to T_{th}(N)$). This means, that the level of the coherent noise in such a beam can exceed the thermal noise, and so can yield additional heating of the beam.

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