COLLIDER CONSTRAINTS IN THE CHOICES FOR WAVELENGTH AND GRADIENT SCALING

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SUMMARY

During the last few years many suggestions have been made for novel accelerating schemes, with the aim of making centre of mass energies of 1 TeV and above available for e^+e^- collisions at a socially acceptable cost. A wavelength range from microns to centimetres, and field gradients from tens to thousands of MeV per metre have been considered. It is now evident that the need for high luminosity with acceptable power cost in the face of destructive collider constraints imposes severe restrictions on the choices available. An attempt is made to examine these in general way, but with emphasis on wavelength and gradient scaling. Non-harmonic schemes, where an equivalent wavelength must be assigned, will be included in the survey, and the use of multiple bunches considered. The need for essentially new inventive suggestions will be emphasized.

INTRODUCTION

The essential features of a large accelerator installation can be conveyed by a 'parameter list'. This is a list of numbers quantifying its properties, such as final energy, power consumption, length and cost. One can divide this list into 'key parameters', and a vast number of 'secondary parameters', which include all the information on detailed drawings. The division is not sharp, and a useful description for most purposes might contain between 10 and 100 numbers. These parameters are interdependent, and subject to overall constraints, dictated by the availability of various commodities such as money, manpower, specialized equipment, and space. A common situation is that some particular design is known to be technically feasible, but not optimized; the question then arises, suppose we alter one parameter, what does this imply for the others? To answer this requires a knowledge of how the constraints are related; easing some problems makes others worse, and the art of juggling the 'trade-offs' is quite familiar to anyone who has built a machine.

Another common situation, illustrated by a tokamak fusion reactor for example, is that the relation between the constraints is not yet understood. Indeed it is not even known that an acceptable parameter list for a power station based on a tokamak can be found.

The functional relations between these parameters may be termed 'scaling laws', and in undertaking any radically new venture an important task is to elucidate the underlying structure of these scaling laws as clearly as possible. Commonly we find fairly transparent connections between some groups of parameters, but how these groups are linked with one another cannot be ascertained without the acquisition of new knowledge.

In the field of linear colliders with energies in the TeV range some of the connections that are needed to arrive at a credible design are missing, and I should like to emphasize the importance of concentrating attention on these unclear links, rather than refining understanding of the connections within a particular group.

One approach to a high luminosity 1 TeV collider might be to extrapolate directly through SLAC and SLC, and build accelerators of essentially the same type using the same power sources. This has obvious problems, and new schemes have been proposed to circumvent them. When this is done, all the complications must be investigated, and any possible weaknesses exposed for examination, not glossed over! Only in this way can one really see where inventive suggestions are needed. One must either work within the constraints, or invent something essentially rather than trivially new that circumvents them.

LINEAR COLLIDER CONSTRAINTS

General

In this section some groups of constraints that at present seem to be inevitable in linear colliders will be discussed. Later, the way these groups interact will be explored. The central problem is that, contrary to the case with storage rings, it seems possible only to use the beams for one collision, or at least a very limited number. For acceptable beam power and high luminosity this immediately implies a very dense beam. Producing dense bunches is difficult, but that is not all. Destructive side-effects emerge that introduce features hitherto unfamiliar to accelerator designers. While details may still be obscure, the main features are by now understood, and have often been described. References 1a - 1d are reviews which contain references to earlier original work.

Here we quote some well known relations between the principal parameters. Since the object is to exhibit the structure of the constraints, some approximations and simplifications, discussed later, are made. The usual symbols are used, L denotes luminosity, and equal cylindrical bunches of r.m.s. radius σ_r and length σ_z are assumed. The number of particles per bunch and pulse frequency are N and f respectively, and $\gamma m_o c^2$ is the energy of the particles in each beam. Numerical factors and fundamental constants are grouped together in brackets. The classical electron radius is denoted by r_c.

The luminosity

$$L = \left(\frac{1}{4\pi}\right) \frac{N^2 f}{\sigma_r^2} \; H(D) \qquad (1)$$

where H(D) is the pinch enhancement factor [1a]. The total power per beam

$$P_b = \tfrac{1}{2}\eta_T P_o = N f \gamma m_o c^2 \qquad (2)$$

where P_o is the total power to the accelerator and η_T the total conversion efficiency, to be discussed later.

The two constraining factors are the disruption parameter

$$D \cong (r_c) N\sigma_z / \gamma\sigma_r^2 \qquad (3)$$

and the beamstrahlung parameter

$$\delta \cong \left(\frac{r_c^3}{5}\right) N^2\gamma / \sigma_r^{\,2}\sigma_z \qquad \text{(classical regime)} \quad (4a)$$

$$\delta \cong (10^{-3})\,(N^2 \sigma_z/\gamma\sigma_r^2)^{1/3} \quad \text{(quantum regime)} \qquad (4b)$$

The general effect of these phenomena is by now fairly clear, though many details remain to be sorted out.

There are recent studies by Yokuya[2] and Noble[3]. Moderate values of D improve L by a factor of several, but if D is too large, the beams blow up before the collision is completed, and L is reduced. If δ is too large an excessive amount of electron energy is converted into radiation; besides complicating the experiments this introduces an energy loss of order $\delta\gamma$ and energy spread, $\delta\gamma/\gamma$ of about $\delta/3$.

So far, nine parameters have been introduced. These may be divided into three groups

1) L and γ are specified.

2) P_b, D and δ must not be too large, and for simplicity we assume that upper limits can be assigned. The limit on P_b, through P_o is economic, D must be less than about 2 if luminosity is to be maintained, and δ, partly for experimental and partly for economic reasons can hardly exceed about 0.3.

3) N, f, σ_r, σ_z are 'technical' parameters, and their value is determined by the design of the accelerator.

Parameters in the first group are fixed, and if those in the second are set at their upper limits, then the four technical parameters are completely determined by equations (1) to (4)! At first sight this appears a highly restrictive state of affairs. Before commenting further it is necessary, first, to see how the numbers work out and second, to decide whether one is in the classical or quantum regime. This has been considered by various authors, (see references 1a - 1d) and it is generally thought that below ½ TeV (per beam) it is easiest to work in the classical regime. By 1 TeV quantum effects would be evident but not dominant, whereas at 5 TeV it would be necessary to accept the demanding constraints of the quantum regime.

In the next sections we note some typical parameters in both regimes implied by the constraints, and then to examine their implications. We also enquire whether we can invent some way round them that will increase the technical choices available.

Typical parameters a) classical beamstrahlung regime.

In this short review there is no room for detailed discussion of acceptable limits on P, D and δ. These have been discussed in the quoted references. Here we merely quote some order of magnitude figures, first for a 1 + 1 TeV machine with $L = 10^{33}$ cm^{-2} sec^{-1} working in the classical regime.[4]

Classical Regime Parameters, $L=10^{33}$, $\gamma=2\times10^6$

P_b = 5MW		
D = 2	}	Acceptable upper limits
δ = 0.3		
$\sigma_r \cong 10^{-4}$mm		
$\sigma_z \cong$ 2mm	}	Derived from constraints
$N \cong 1.5 \times 10^{10}$		
f = 2000 sec^{-1}		

The following comments can be made. First, both N and f look reasonable. Second, the small σ_r demands a high quality beam, and good alignment techniques. Third, σ_z looks very reasonable for an accelerator with $\lambda \cong$ 10cm, but there will clearly be problems with energy spread if λ is much reduced. Fourth, a power level of 5MW per beam requires at least a few per cent efficiency. From these observations it is evident that we need to be clear about constraints and connections associated with the following quantities.

Table of secondary parameters

1) Intrinsic beam quality after acceleration

   a) transverse emittance ε
   b) energy spread $\Delta E/E$

2) Efficiency of energy transfer to beams

Consideration of these again requires some knowledge of the following tertiary parameters

Table of some tertiary parameters

1) Accelerating gradient, E_z
2) Energy spread associated with longitudinal wakefields
3) Emittance growth associated with transverse wakefields
4) The form of transverse wakefields associated with transverse alignment errors
5) etc etc

Already the complexity of the relationships is becoming evident. We do not yet know enough to specify a credible design. Every effort must be made to elucidate these relationships before going into too much detail on any one aspect! Later, we discuss connections between gradient, efficiency, energy spread,and wavelength. We note the important questions already raised. 1) Within the constraints discussed, how short in wavelength can we go while preserving sufficiently small energy and emittance to enable the beam to be focused? 2) What methods may there be of evading the constraints? Some of these constraints are eased, but others made more severe when we enter the quantum beamstrahlung regime, considered next.

Typical parameters b) quantum beamstrahlung regime

The form of equation (4b) indicates that in the quantum regime δ is proportional to $\sigma_z^{1/3}$ rather than $1/\sigma_z$, so that a very short bunch is indicated. It is interesting to display δ as a function of γ, L, P and the bunch dimensions

$$\delta_{CL} \sim \frac{\sigma_r^2}{\sigma_z}\left(\frac{\gamma^3 L^2}{P_b^2}\right) \qquad (5a)$$

$$\delta_{QM} \sim \left\{\sigma_r^2 \sigma_z \left(\frac{\gamma L^2}{P_b^2}\right)\right\}^{1/3} \qquad (5b)$$

For given γ and L, both require large P_b and small σ_r, but the role of σ_z changes. Typical parameters are given in the table, appropriate to a 5+5 TeV machine with $L=10^{34}$.[5]

The power used is very much less than in the first example; if this is sufficiently increased it may be possible to work in the quantum regime even at 1 TeV.

Quantum Regime Parameters, $L=10^{34}$, $\gamma=10^{7}$

$\delta = 0.3$		Acceptable upper limit
$D \cong 0.1$	}	No longer at
$P_b \cong 0.5$ MW	}	upper limits.
$\sigma_r \cong 2.5\text{-}5$A		
$\sigma_z \cong 0.5\mu$		
$N \cong 10^8$		
$f \cong 5000\ \text{sec}^{-1}$		

In this case D and P_b are not at the upper limits, so that there is more choice. Nevertheless at this energy it is not possible to avoid the extremely small bunch sizes. These ease the wavelength constraints, but make demands on beam quality and alignment that are far beyond present experience.

The construction of a machine with such parameters, if ever realized, is evidently in the distant future. In the remainder to this paper the emphasis is placed on the classical regime, though some of the discussion applies to either.

SCALING OF GRADIENT, WAVELENGTH, AND EFFICIENCY

We continue with examination of some quite general relationships between accelerating gradient E_z, wavelength, and efficiency. In order to include pulsed schemes, with non-harmonic accelerating fields, we introduce the concept of 'equivalent radius' a of the accelerating device. In a Stanford type waveguide a is about $\lambda/3$. If W is the energy stored per unit length and E_z is the accelerating gradient, assumed constant over a distance equal to the length of the bunch, then the equivalent radius a is defined by

$$W_s = \tfrac{1}{2}\varepsilon_o \pi a^2 E_z^2 \qquad (6)$$

(In terms of the shunt impedance per unit length, Z_s and the structure Q, $a^2 = 2Q/\pi\varepsilon_o \omega Z_s$.) A bunch of electrons passing through the accelerator gains energy per unit length

$$W_b = \alpha N e E_z \qquad (7)$$

where α is a coefficient less than unity. For a group of b bunches passing in a time small compared to the decay time of the field the efficiency with which energy is extracted, denoted by η_e is, therefore,

$$\eta_e = \frac{bW_b}{W_s} = \left(\frac{2e}{\pi\varepsilon_o}\right)\alpha\left(\frac{bN}{E_z a^2}\right) \qquad (8)$$

For bN=1, obviously η_e is very small and α=1. As N increases, the first electron in the bunch always acquires energy eE_z, but subsequent electrons acquire less because, by the time they arrive to where the first electron was slightly earlier, the field is depleted. The trailing electron gets least energy; for a field initially uniform with z the mean energy deficit is $(1-\alpha)eE_z$, and it is conventional to describe this as due to a longitudinal wake field. The fractional energy spread is $(1-\alpha)$, and it is not difficult to show that

$$\frac{\Delta\gamma}{\gamma} = (1-\alpha) \geq \tfrac{1}{2}\eta_e \qquad (9)$$

For an accelerating field which, in the absence of the bunch, decreases with z, (for example in the region in front of the crest of a sinusoidal field,) good compensation for this energy spread can in principle be obtained.[6] Furthermore, when b>1 compensation can in principle be obtained by sending earlier bunches before the accelerator is filled. In addition, energy recycling might greatly improve efficiency for given energy spread.[7]

We return later to the question of energy spread, and consider the scaling implied by equation (8). We make a guess of 10% for η_e; at present it is not known what compensation can be achieved in practice, nor what energy spread is tolerable if final focusing is not to be upset by chromatic aberrations, this is thought to be of order 1%.[4]

Equation (8) then becomes

$$bN \cong 10^7 E_z a^2, \ \text{(volts, metres)} \qquad (10)$$

In the classical regime $N\cong 1.5\times10^{10}$; a wavelength of λ=0.1m implies $a^2 \cong 10^{-3}\text{m}^2$ and for E_z=20 MeV/m we find b≅12. For a pulsed linear accelerator this implies 12 bunches per r.f. pulse. If the gradient were increased to 100 MeV/m at the same wavelength this would require 300 bunches per pulse, which is much more difficult technically. Of course for a superconducting accelerator continuous operation becomes possible, and the bunches can be spread out uniformly rather than concentrated into a series of short bursts. At shorter wavelengths b is decreased, but cannot go below unity. Values of parameters satisfying equation 10 are tabulated.

Table showing rough values of b, E_z and a that satisfy equation 8 with η_e=0.1 and $N=1.5\times10^{10}$

No of bunches b	Gradient E_z,GeV/m	Equiv. radius a metres	Wavelength λ, mm
12	0.02	0.03	100
300	0.1	0.03	100
5	0.08	0.01	33
1	0.02	0.01	33
1	0.02	3×10^{-3}	10
1	1	1.3×10^{-3}	4
1	20	3×10^{-4}	1
1	2000	3×10^{-5}	0.1

We might conclude from this table that an 'optimum' wavelength, giving a gradient of about 1 GeV/metre, might be about 4mm. The figures in the table are for a $\cong \lambda/3$, but for schemes making use of external guiding structures[8] a might be much larger, and the field, therefore, smaller. Similarly, for plasma accelerators, $a \cong c/\omega_p$.

Within the context of the constraints so far assumed for the classical regime, it is not possible to go as low as a = 0.0013m (λ=4mm) because a bunch with σ_z=2mm would occupy a phase range of more than $\pm\pi$! The question of dividing the bunch into bunchlets is considered later. If we allow (optimistically) 5% energy spread at $\pm 1\tfrac{1}{2}\sigma_z$ from the peak of the wave, hoping to find some way of compensating for this, then the minimum wavelength is

$$\lambda_{min} = 1.5\pi\sigma_z/\cos^{-1}(1-0.05) \qquad (11)$$

Optimistically, squeezing σ_z down to 1 mm, this is 1.5 cm. This is already less than we are entitled to assume from the assumptions made earlier. It is extremely important, therefore, to see whether it is possible to break out of the suffocating constraints assumed so far. Before attempting to find an escape route we look at a further constraint, not yet considered.

In the quantum beamstrahlung regime N can be reduced by a factor of about 100. This has the effect of reducing by 100 the fields E_z in the table. This implies wavelengths below 1 mm if 1GeV/metre can be achieved. The problem of energy spread due to finite σ_z disappears because of the extreme shortness of the bunch, $\sigma_z \cong 0.5\mu << 1$ mm, and furthermore, the reduced beam power requirement means that the efficiency need not be so high. This allows the use of higher fields.

CONSTRAINTS IMPOSED BY THE REQUIREMENTS OF FINAL FOCUSING

The constraints on beam quality imposed by the requirements of final focusing have been examined by several authors; see especially ref 1b. Very roughly, a waist of radial extent σ_r over a length σ_z requires a beam with emittance

$$\varepsilon < C\sigma_r^2/\sigma_z \qquad (12)$$

where C ($=\sigma_z/\beta^*$ where β^* is the 'beta function') is a constant of order unity. The requirements on energy spread are not known at this time, but a value ±½% is often quoted [4]. For a 1 TeV machine the parameters quoted earlier require a value less than 10^{-11} π metre radians, and a value $2\pi \times 10^{-12}$ was quoted. [4] This is difficult to achieve in the first place, and to maintain during acceleration. The problem of obtaining low emittance by the use of storage rings has been studied, and the limitations are reasonably well understood. [1b] Mechanisms for emittance growth in the accelerator are also understood, and scaling laws for wavelength, gradient and charge taking into account known wakefield scaling can be quite simply found from dimensional arguments. There are many factors to be taken into consideration, however, if bunch size and technically achievable tolerances are to be included. One of these is the focusing wavelength, which is dependent on the configuration of the accelerating system. It is too early to obtain a clear picture of the interrelation of all these factors, and how they influence the choices of those already discussed. Nevertheless, some further systematic exploration in this direction would be profitable.

HOW CAN WE ESCAPE FROM THE CONSTRAINTS?

To operate within the constraints assumed for the classical beamstrahlung regime seems to require wavelengths greater than 1 cm, though in the quantum regime it may be possible to go much lower. If high accelerating fields are to be used, then short wave-lengths are very desirable to avoid too many bunches per r.f. pulse. We ask first how it may be possible to avoid the constraints discussed earlier. First, how well established are the formulae for disruption and beamstrahlung? There is room for further calculation which may modify present understanding, though this is unlikely to transform the apparent prospects. Perhaps the most radical suggestion, already eight years old, [9] is to use two pairs of beams, each pair consisting of an e^+ and e^- at a very small angle. The four beams would interact in the collision region, greatly reducing the effects of disruption and beamstrahlung. This would still require the production of extremely high quality beams, but would allow a much greater freedom in the choice of bunch length, and hence of r.f. frequency, N, and repetition rate. This scheme would seem to need four rather than two accelerators, though outline suggestions of how one might merge e^+ and e^- beams accelerated in the same machine have been made by Schnell [10].

The beamstrahlung constraint is also eased if flat rather than round beams are used. A further variable that could have been introduced earlier is R, the aspect ratio of the beams. Producing a flat beam of the same area as a circular one requires reduced emittance in one plane, however, and the implications of this must be faced. Merely increasing one dimension by a factor R does not help, since this increases the beam area and reduces L. Detailed argument shows that the gain from a lower δ is lost.

A further suggestion that at first sight looks helpful is to split up the bunches into a succession of bunchlets. In this way it might be possible to have these separated by one r.f. wavelength, and therefore to use a shorter accelerating wavelength. As an extreme example one could keep the overall length of the bunch train the same, (σ_z), but enhance the density of each bunchlet by S, the space-to-mark ratio. Assuming that the energy spread is kept to ±1%, this implies a bunch length of $\pm\cos^{-1} 0.99 = \pm 8°$, so that $S = 180°/8° = 22.5$. Unfortunately, since the beamstrahlung radiation is proportional to the square of the self-field of the bunch, this increases δ by S, which is unacceptable.

An alternative is to lengthen the train of bunches by a factor S, so that the longitudinal density remains the same. This keeps δ constant, but now D, if defined in the conventional way, increases by a factor S. Furthermore, a bunch train of length $S\sigma_z$ does not satisfy the inequality (12) required for the final focus. These arguments are discouraging but perhaps oversimplified. Further examination of this problem, in conjunction with studies of the experimental utilization of the machine are clearly needed[11]

CONCLUDING REMARKS

The apparent constraints on electron collider operations are severe. To acquire the knowledge necessary to produce a design within them clearly is a major task. The complications are such that systematic thought and action is needed. At the same time one must vigorously challenge the very simplified arguments set down in this paper. Are the constraints properly understood? What is the range of validity of the formulae? What can we gain from pinch effect enhancement, neglected in the analysis? Can we really do better by splitting the bunch into bunchlets? What are the possibilities for inventive suggestions such as 'Super Disruption' and 'Grand Disruption?'[10] The only firm conclusion at this time is that much needs to be done before the feasibility of a collider in the energy range 1 TeV or above can be established. We need not only inventions, but also a clearer perception of the underlying framework of constraints, extending to areas not discussed here, such as the specifics of linac design, factors affecting beam quality and especially the strategy for utilizing the beams for experiments. We must not neglect the grind in favour of the glamour of 'new ideas' that do not address the central problems.

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