UNIFIED FORMULATION FOR LINEAR ACCELERATOR DESIGN*

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ABSTRACT

Expressions for peak and average powers required to produce a given average gradient in an accelerator section are given. They are valid for both lossy and lossless (superconducting) sections, for both traveling wave and standing wave sections, and for pulsed or continuous wave rf input. The expressions are given in terms of structure parameters that are equally applicable to traveling wave or standing wave. These parameters delineate the effect of wall losses and energy required to build up the field. For both traveling wave and standing wave sections it is possible to make the rf pulse length short enough to make the wall losses negligible at the expense of increased peak power requirement. Therefore the expressions will include the effects of pulse compression.

SECTION PARAMETERS

The necessary and sufficient parameters that can characterize traveling wave (TW) or standing wave (SW) accelerator sections are

$$s = \frac{E^2}{w}; \qquad T_o = \frac{2w}{p_d} = \frac{2W}{P_d}; \qquad v_g = \frac{P}{w} \; . \tag{1}$$

For SW section only: $s = \dfrac{V^2}{WL}; \qquad T_e = \dfrac{2W}{P_e}$.

The names of the symbols in the above definitions with a consistent set of units are:

s	elastance per unit length,	$M\Omega \; m^{-1}\mu s^{-1}$
T_o	unloaded (internal) time constant,	μs
v_g	group velocity,	$m \; \mu s^{-1}$
E	local accelerating gradient,	$MV \; m^{-1}$
w	energy stored per unit length,	$joule \; m^{-1}$
V	particle voltage,	MV
W	energy stored in the section,	$joule$
L	section length,	m
p_d	power dissipated per unit length,	$MW \; m^{-1}$
P_d	power dissipated in the section,	MW
P	power transmitted,	MW
T_e	SW section external time constant,	μs
P_e	power emitted by the SW section,	MW

The elastance is the reciprocal of capacitance. It depends on such factors as the crossectional area of the section, the concentration of the electric field at the axis, and transit time. T_o depends on the ratio of cross section area to surface area. As both stored energy and power dissipated vary linearly with section length, T_o is the same for both SW and TW, if we neglect the power dissipated in the SW section end walls. The group velocity, elastance, internal time constant can be functions of distance along the section.[1] But in this note only uniform group velocity sections are considered.

Both TW and SW sections can be characterized by the same parameters s, T_o, and fill time T_f. For both structures the fill time can be defined as the pulse duration for which the ratio of section energy to pulse energy is maximum. For a TW structure the fill time is the section length divided by the group velocity. For a SW structure, it is a function of T_e and T_o and is independent of v_g. v_g depends on coupling between cells and T_e depends on coupling between the section and the generator. For both TW and SW sections, a single bunch beam is injected at one fill time after the beginning of the input RF pulse. For a SW section, however, a long beam pulse is injected when the rate of increase in field due to the RF is balanced by the rate of increase in the field due to the beam so that the field remains constant.[2] For a SW section the time the beam pulse is injected is an alternative definition of fill time.

PEAK AND AVERAGE POWER

The peak power P_p and average power P_a required to produce a section average gradient E_a are:

$$P_p = \frac{E_a^2 L}{\eta_s s T_f M} \quad , \quad P_a = f_r P_p T_f = f_r \frac{E_a^2 L}{\eta_s s \eta_{pc}} \quad . \tag{2}$$

Here f_r is the pulse repetition frequency and must be less than $1/T_f$. If $f_r = 1/T_f$, we have the continuous wave (CW) case. The peak and average powers are then identical and their expressions coalesce into a single expression.

With no pulse compression the peak and average powers required to produce a given gradient depends only on s, T_o and T_f. The peak power decreases as the internal time constant increases. Therefore both the elastance and the internal time constant should be as large as practicable. The fill time is determined by the desired trade-off between the peak and average powers.

The effect of pulse compression is included in Eq. (2). Its constants are: the compression factor C_f (the klystron pulse length divided by the compressed pulse length), the power multiplication factor M (the square of the section voltage with compression divided by the section voltage with no pulse compression), and the compression efficiency η_{pc} given by $\eta_{pc} = M/C_f$. For a TW section the attenuation in nepers τ, the section efficiency η_s and the section length L are

$$\tau = \frac{T_f}{T_o}, \quad \eta_s = \frac{(1-e^{-\tau})^2}{\tau^2}, \quad L = v_g \tau T_o \; . \tag{3}$$

For a SW section, the efficiency is

$$\eta_s = \frac{2\gamma(1-e^{-\tau_e/\gamma})^2}{\tau_e/\gamma} \tag{4}$$

$$\gamma = \frac{1}{1+T_e/T_o} \quad , \quad \tau_e = \frac{T_p}{T_e} \tag{5}$$

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* Work supported by the Department of Energy, contract DE-AC03-76SF00515

The pulse length T_p when it maximizes η_s is the fill time T_f and is given by

$$T_f = 1.257T_f, \quad T_l = \gamma T_e \quad . \tag{6}$$

T_l is the loaded time constant.

Substituting for η_s and T_f from (3) into (2) we obtain for a constant impedance TW section an identical expression for the CW power and pulsed peak power

$$P = \frac{E_a^2 L/sT_o}{(1-e^{-\tau})^2/\tau} \tag{7}$$

For a fixed s and T_o, P is minimum when $\tau = 1.257$ that is $T_f = 1.257T_o$. This is similar to the condition that minimizes the average pulsed power for a SW section, except that T_l is replaced by T_o. The condition for minimum CW power for a SW section is $T_e = T_o = 2T_l$.

Plots of peak and average powers required to produce 21 MV/m gradient in the SLAC disk-loaded structure operating at 2856 MHz and having a internal time constant of 1.44μs with no pulse compression, $M = \eta_{pc} = 1$, are shown in Fig. 1. The solid lines are for constant group velocity and hence constant elastance, which is assumed to be 76.4 $M\Omega/m-\mu$s. The dotted lines are for a constant length and variable group velocity and hence elastance. The dashed lines are for a SW section having the same elastance and internal time constant as the constant group velocity TW section. At each fill time the section efficiency was maximized with respect to T_e. The value of T_e is given in Ref. 1. In this case, with E_a, s, L, held constant the average power is a constant divided by the section efficiency.

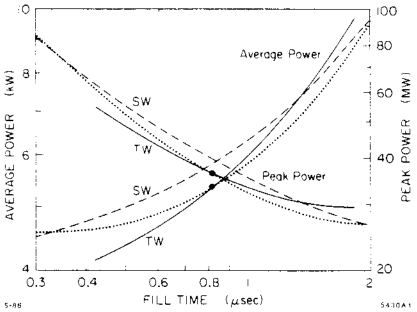

Fig. 1. TW and SW average and peak powers/section.

Define the improvement factor I_f as the section material resistivity divided by the resistivity of copper. For a superconducting section,^{3} $\tau << 1$ and the refrigerator ac power in KW is

$$P_r = \frac{2\tau_c R_f}{I_f} P_a \quad . \tag{8}$$

Here τ_c is the copper section attenuation at room temperature, R_f is the ratio refrigerator power to heat load power. If the section is cooled τ is much lower than its average power- peak power trade-off value and is determined by refrigerator AC power requirements and by the average power rating of the cooled section.

Peak and average powers vs fill time for a copper section at room temperature and at liquid nitrogen temperature 77°K, $I_f = 3$, $R_f = 10$, are shown in Fig. 2. It is apparent from the Fig. 2 that improving the internal time constant can reduce the peak or average power requirements or both. The undesirable effect of cooling the section is the required refrigerator power which is the same order of magnitude as the average power. But for niobium or lead sections at liquid helium temperature 4.2°K, $I_f = 4000$, $R_f = 400$, at a few microsecond fill times, the refrigerator power is reduced to a fraction of the average power as shown in Fig. 3.

If $T_f = 1/f_r$ then the rf is CW. At 4.2° operating at fill times approaching CW results in prohibitive refrigerator power and low gradient break down. But one can operate at short fill time. How short depends on the experimentally determined pulse energy and average power rating of the superconducting section. Making a section superconducting reduces the peak and average power by the reciprocal of η_s. The peak power per unit length can be further reduced by increasing the section length.

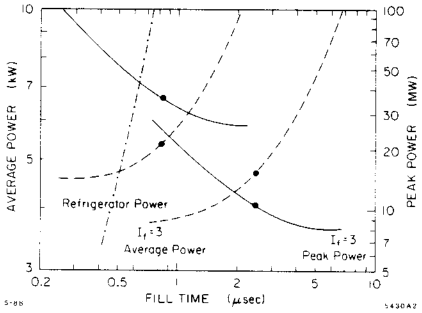

Fig. 2. Peak and average powers for a room temperature and a liquid nitrogen temperature copper TW section.

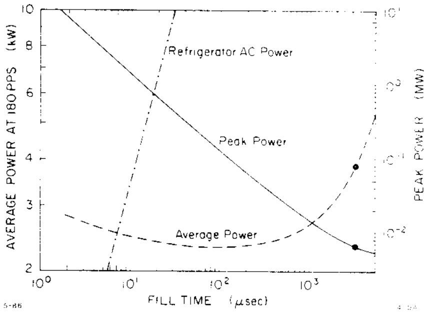

Fig. 3. Peak and average powers for a liquid helium temperature niobium or lead TW section.

High effective gradients can be obtained with reduced peak and average power into the section by using the energy that has already accelerated the beam in one accelerator section to drive a second accelerator section to accelerate the recirculated beam as illustrated in Fig. 4. Or two beams can be accelerated. This can be done because the energy loss in a SC section is essentially zero.

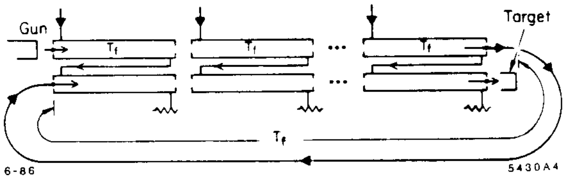

Fig. 4. Double pass recirculation scheme.

FREQUENCY SCALING

The effect of frequency when keeping either the group velocity, or the section length, or the aperture size constant is now considered. As the linear dimensions vary as the frequency f, we infer from their definitions that for the same group velocity, geometry, and mode

$$s \propto f^2 \qquad \text{and} \qquad T_o \propto f^{-3/2} \quad . \tag{9}$$

Dividing s by f^2 yields the structure parameter s_g which is invariant with frequency. It depends only on structure geometry and mode. For the SLAC disk-loaded $2\pi/3$ structure s_g and the aperture size a as a function of group velocity are[4]

$$s_g = \frac{13.24}{1 + 0.216\sqrt{v_g}}, \qquad \frac{a}{\lambda} = 0.08 v_g^{1/4} \quad . \tag{10}$$

Here s_g is in $\frac{M\Omega}{m} - ps$ v_g is in $m/\mu s$.

The SLAC section $s_g = 9.37\ \frac{M\Omega}{m} - ps$. The SLAC section fill time and length as a function of frequency and improvement factor are

$$T_f = \tau T_o = \frac{6.95 I_f \tau}{f^{3/2}}, \qquad L = v_g T_f = \frac{6.95 I_f v_g \tau}{f^{3/2}} \tag{11}$$

Here f is in GHz, T_f in μs and L in meter. The improvement factor can be due to cooling the section or any other cause. Substituting the SLAC section $\tau = 0.57$, and $I_f = 1$ we obtain

$$T_f = \frac{3.96}{f^{3/2}}, \qquad L = \frac{3.96 v_g}{f^{3/2}} \quad . \tag{12}$$

Substituting the SLC gradient and repetition rate, $E_a = 21 MV/m$, $f_r = 180pps$, and $\eta_s = 0.581$ into eq. (2) we obtain the peak power per section and the average power per meter

$$P_p\ (MW) \ = \frac{759.1 v_g}{s_g f^2}, \qquad p_a\ (kW/m) \ = \frac{136.6}{s_g f^2} \quad . \tag{13}$$

The average power per section is

$$P_a\ (kW) \ = \ p_a L \ = \frac{541 v_g}{s_g f^{7/2}} \tag{14}$$

Using the above expressions we obtain the system parameters as a function of frequency listed in Table 1.

Table 1. Peak and average power vs frequency
$E_a = 21 MV/m$, $f_r = 180pps$, and $\eta_s = 0.581$

freq GHz	T_f μs	v_g $m/\mu s$	a cm	b cm	L m	P_p MW	p_a kW/m	P_a kW
1.0	3.96	3.66	3.31	5.95	14.5	296	14.6	221
2.86	0.82	3.66	1.16	2.09	3.00	36.4	1.79	5.36
10	.125	3.66	.331	.595	0.46	2.96	.146	.0668
10	.125	60.0	.858	1.50	7.50	138	.415	3.11
30	.024	60	.286	.500	1.44	15.3	.046	.066
30	.024	48	.286	.610	1.16	17.8	.054	.077

The rf to beam energy conversion efficiency is inversely proportional to p_a. Hence, it is clear from Table 1 that high frequencies are desirable. But, increasing the frequency decreases the aperture size and length. These can be remedied by increasing the group velocity which has the added advantage of increasing the section bandwidth and improving cooling and vacuum. The decrease in elastance and high peak to accelerating field ratio due to increased group velocity can be remedied by using nose cones and magnetic coupling.

Lines 4 and 5 of Table 1 are parameters for a large aperture disk-loaded structure. Its s_g decreased from 9.37 of the SLAC section, $a/\lambda = 0.111$ to 3.29 for $a/\lambda = 0.286$. The values in the last line in Table 1 are for the zig-zag structure shown in Fig. 5. Changing from a disk-loaded structure to a zig-zag structure further improves the cooling, vacuum, and mechanical construction of the section. At high group velocity the elastance of a zig-zag section is about the same as that of a DLWG: $s_g = 2.82$ for the zig-zag and $s_g = 3.29$ for the DLWG.

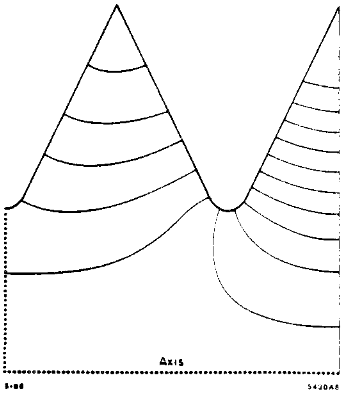

Fig. 5. Zig-Zag structure.

CONCLUSION

Expression for peak and average power were given in terms of parameters that are identical for both TW and SW sections. Expressions for beam induced gradient and output power when the section operates in the traveling wave tube mode can also be expressed in terms of s, T_o, and v_g for TW sections or s, T_o, and T_e for SW sections. These expressions and expressions for the case of beam loaded sections have been given in a companion paper.[1] After the fill time is determined from peak and average power trade-off (peak and AC power trade-off for superconducting sections) the section parameters and peak and average power requirements of lossy, lossless TW and SW sections operating in the single bunch or CW mode can be obtained from the given expressions.

REFERENCES

1. Z. D. Farkas, "New Formulation for Linear Accelerator Design", IEEE Transactions of Nuclear Science, NS-32, p. 2738, October 1985.
2. R. D. Miller "SW/TW Structures" this conference
3. Z. D. Farkas "Superconducting Traveling Wave Accelerators," SLAC/AP-29, April 1984
4. P. B. Wilson "High Energy Electron Linacs", February 1982, SLAC-PUB-2824