PROGRESS ON PLASMA ACCELERATORS*

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Abstract

Several plasma accelerator concepts are reviewed, with emphasis on the Plasma Beat Wave Accelerator (PBWA) and the Plasma Wake Field Accelerator (PWFA). Various accelerator physics issues regarding these schemes are discussed, and numerical examples on laboratory scale experiments are given. The efficiency of plasma accelerators is then revealed with suggestions on improvements. Sources that cause emittance growth are discussed briefly.

1. Introduction

The acceleration gradient attainable from the currently existing high energy accelerators is of order 10 MeV/m. To apply the present technology to future ultra-high energy accelerators, the sizes are necessarily be enormous. In recent years, various novel ideas on future accelerators have been proposed,[1,2] among them plasma accelerators promise to provide very high gradients.

Plasmas are known to exhibit oscillations where electrons and ions execute periodic motions. For fully ionized plasmas with density perturbation n_1 due to charge separation during oscillations, there will be an induced instantaneous longitudinal electrostatic field $\vec{E}$, *i.e.* $\nabla \cdot \vec{E} \sim k_p E \sim 4\pi e n_1$, where $k_p = \omega_p/c$ is the plasma wave number. Since the maximal possible density perturbation is $n_1^{\max} \sim n_0$, the maximal acceleration gradient provided by the $\vec{E}$ field is $eE^{\max} \sim 4\pi e^2 n_0/(\omega_p/c) \sim \sqrt{n_0}$ eV/cm. For a laboratory plasma of density $n_0 = 10^{18}$ cm^{-3}, we have $eE^{\max} \sim 100$ GeV/m. This is more than three orders of magnitude better than that of the conventional accelerators.

To have an effective acceleration of relativistic particles, it is necessary that the plasma wave phase velocity be close to c, the speed of the high energy injected beam. To achieve such a plasma oscillation several ideas have been suggested. In terms of the nature of the driving sources, there are basically two types of plasma accelerators, namely, the laser beam driven and the electron beam driven plasma accelerators. The Plasma Beat Wave Accelerator (PBWA),[3] which exemplifies the first type, employs two laser beams beating at the plasma frequency ω_p, while the Plasma Wake Field Accelerator[4,5] (PWFA) replaces the laser beams by a bunched relativistic electron beam. Other concepts like the Plasma Fiber Accelerator[6] and the "Surfatron"[7] are variants of the PBWA aiming at improving its deficiency in different ways, while the Plasma Grating Accelerator[8] replaces the beating lasers by a polarized laser beam side-injected on a plasma where static ion ripples are prepared by an acoustic wave.

In this paper we review primarily the concepts of PBWA and PWFA, which are by far the most developed plasma accelerator schemes. Other ideas mentioned above will only be discussed auxiliary to these schemes. Following mainly Ref. 9, we first review different ways to excite fast waves in a plasma in Section 2. We then investigate various accelerator physics issues associated with particle acceleration in these plasma waves in Section 3. Numerical examples are given in Section 4 which serve to illustrate possibilities in design. Next we discuss the efficiency in both PBWA and PWFA in Section 5, and provide suggestions that would make both highly efficient. In Section 6 we reveal the question of emittance growth due to Coulomb scatterings and the imperfection of the driving beam. Finally, experimental efforts in testing either the PBWA or the PWFA are briefly reviewed.

2. Ways to Excite Fast Waves in a Plasma

2.1. *Beating Lasers*

It is well known that a plane electromagnetic wave cannot cause any net drift of a charged particle along its direction of propagation. An originally stationary charged particle experiencing such a EM wave would execute a "figure 8" closed orbit motion. In the case of two beating EM waves the amplitudes of the EM fields vary along the direction of propagation. Accordingly the force due to the magnetic field does not balance with that due to the electric field, and the charged particle would drift in the longitudinal direction. This net force is called the ponderomotive force.

A plasma can be driven resonantly by the beating lasers through the ponderomotive force if the frequency and wave number differences between the two lasers match with those of the plasma wave, *i.e.* $\omega_p = \omega_1 - \omega_2$ and $k_p = k_1 - k_2$, where ω_i and k_i are the frequency and wave number of the two lasers, and ω_p the plasma frequency, $\omega_p \equiv [4\pi e^2 n_0/m]^{1/2}$. The force that excites the plasma is most easily calculated from a Hamiltonian which has been averaged over the fast oscillations of the laser frequency. This leaves only the beating effect of the two laser frequencies. Assuming that $\omega_p \ll \omega_1, \omega_2 \cong \omega$, the averaged Hamiltonian can be written as

$$\langle H \rangle = \frac{\vec{p}^2}{2m} + \frac{e^2 E_0^2(r)}{4m\omega^2}\left[1 + \cos(k_p z - \omega_p t)\right] . \tag{2.1}$$

The last term is the ponderomotive potential due to a beating laser with a finite cross section. The divergence of the ponderomotive force can be derived from the above Hamiltonian if the radial profile of the laser $E_0^2(r)$ is specified, *i.e.* $\nabla \cdot \vec{F} = -\nabla^2 \langle H \rangle$.

Assume that the radial dependence of the ponderomotive potential is given by

$$E_0^2(r) = 2E_0^2 \begin{cases} K_2(k_p a)\, I_0(k_p r) + \dfrac{1}{2} - \dfrac{2}{(k_p a)^2} - \dfrac{r^2}{2a^2} & r < a \\ I_2(k_p a)\; K_0(k_p r) & r > a \end{cases} , \tag{2.2}$$

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where K_n and I_n are modified Bessel functions. This radial profile is parabolic near the origin but falls off exponentially for $r > a$. It is chosen to yield a simple parabolic dependence in the ponderomotive force expression and to simplify the discussions on the transverse behaviors of the PBWA in the following sections.

To find the plasma response to the ponderomotive force and the electric field associated with the plasma wave, we work with the linearized, nonrelativistic fluid equations in a plasma,

$$\frac{\partial n_1}{\partial t} + n_0 (\nabla \cdot \vec{v}_1) = 0 \ , \qquad \frac{\partial \vec{v}_1}{\partial t} = \frac{e\vec{\mathcal{E}}_1}{m} + \frac{\vec{F}_{\rm ext}}{m} = \frac{\vec{F}}{m} \ , \qquad (2.3)$$

and solve for the perturbed plasma density n_1. $\vec{\mathcal{E}}_1$ is the electric field due to n_1, *i.e.* $\nabla\vec{\mathcal{E}}_1 = 4\pi e n_1$, and $\vec{F}_{\rm ext}$ is the external force due to the driving beam.

In our approach the plasma perturbation grows linearly in time during which the laser pulse extents. For a laser pulse duration τ the maximum perturbation can be shown[9] to be

$$n_1(r) = \frac{\omega_p \tau E_0^2 k_p^2}{32\pi m\omega^2}\left(1 - \frac{r^2}{a^2}\right) \qquad r < a \ . \qquad (2.4)$$

Beyond the linear regime the growth saturates due to various effects. One effect is the relativistic frequency detuning[10] where the plasma frequency shifts to $\omega_p/\gamma^{3/2}$ when the electrons in the plasma becomes relativistic. In this case the laser drive is off resonance, and the perturbation saturates at a maximum value $n_1^{\rm max}$,[11]

$$\left(\frac{n_1}{n_1^{\rm max}}\right)_{\rm sat.} = \left[\frac{16}{3}\left(\frac{eE_1^{\rm laser}}{m\omega_1 c}\right)\left(\frac{eE_2^{\rm laser}}{m\omega_2 c}\right)\right]^{1/3} \equiv \left[\frac{16}{3}\,\alpha_1\alpha_2\right]^{1/3} . \qquad (2.5)$$

Another degradation comes from strong couplings between the primary beat wave and the larger k secondary electrostatic modes. It is found experimentally[12] that under some conditions this effect saturates the beat wave amplitude well below that expected from relativistic detuning. In this paper we will only consider the situation where $\alpha_i \ll 1$ so that Eq. (2.4) is valid.

The longitudinal and transverse electric fields for $r < a$ can be found by solving the Poisson's equation, $\nabla^2\phi_1 = -4\pi e n_1$, and are given by[9]

$$\begin{aligned} \mathcal{E}_z &= -\frac{\omega_p\tau k_p e E_0^2}{4\omega^2 m}\left\{K_2(k_p a)\, I_0(k_p r) + \frac{1}{2}\left(1 - \frac{r^2}{a^2}\right) - \frac{2}{(k_p a)^2}\right\} \\ &\quad \times \cos(k_p z - \omega_p t) \ , \\ \mathcal{E}_r &= -\frac{\omega_p\tau k_p e E_0^2}{4\omega^2 m}\left\{K_2(k_p a)\, I_1(k_p r) - \frac{r}{k_p a^2}\right\}\sin(k_p z - \omega_p t) \ . \end{aligned} \qquad (2.6)$$

2.2. *Relativistic Electron Bunch*

For the case of a relativistic electron driving bunch, the situation is very similar. We only need to replace the gradient of the driving force by $\nabla\cdot\vec{F} = 4\pi e^2(n_1 + n_b)$, where n_b is the driving bunch density. Consider the following density profile $n_b = \sigma(r)\delta(z - v_b t)$. To compare with the PBWA we use a parabolic distribution given by

$$\sigma(r) = \begin{cases} \dfrac{2N}{\pi a^2}(1 - r^2/a^2) & r < a \\ 0 & r > a \end{cases} \ , \qquad (2.7)$$

where N is the total number of particles in the driving bunch.

The electric fields for the two schemes turn out to be remarkably similar, except that the coefficient in Eq. (2.6) is replaced by $16eN/a^2$ for the case of PWFA.

For reasons which we will discuss later, the transverse size of the driven beam must be somewhat smaller than the transverse size of the laser beams or the driving electron beam. In addition if $k_p a \gg 1$, then the electric fields for both schemes are of the following form:

$$\left.\begin{aligned} \mathcal{E}_z &\simeq -A\left(1 - \frac{r^2}{a^2}\right)\cos(k_p z - \omega_p t) \\ \mathcal{E}_r &\simeq 2A\frac{r}{k_p a^2}\sin(k_p z - \omega_p t) \end{aligned}\right\} \quad r \ll a \qquad (2.8)$$

where $A = \omega_p\tau k_p e E_0^2/8\omega^2 m$ for PBWA, and $A = 8eN/a^2$ for PWFA. Other than different coefficients, the forces that the driven electrons experience share the same physical characteristics in both schemes. To be specific there is a longitudinal force $e\mathcal{E}_z$ that either accelerates or decelerates the driven bunch of electrons, and there is a transverse force $e\mathcal{E}_r$ shifted in phase which either will focus or defocus the driven bunch. It is clear that we have both acceleration and focusing over 1/4 of the plasma wavelength.

2.3. *Side-Injected Polarized Laser*

Another way of exciting fast plasma wave, as proposed in the Plasma Grating Accelerator,[8] employs side-injected laser which is polarized along the direction of a static density ripple in the plasma, $n(z) = n_0 + n_1(z) = n_0 + \delta n \sin k_r z$, where k_r is the wave number of the density ripple. Such a ripple might be produced by an ion acoustic wave or by ionizing a grating. The laser field wiggles the electrons in the ripple by an amount $\delta z = (eE_0/m\omega^2)\cos\omega_0 t$, while the ions are too massive to respond. This produces a longitudinal electric field disturbance given by the Poisson equation:

$$\frac{\partial E_z}{\partial z} = 4\pi e\delta n = 4\pi e\,\frac{\partial n}{\partial z}\,\delta z \simeq \frac{\omega_p^2}{\omega_0^2}\,\frac{\delta n}{n_0}\,E_0 k_r \cos k_r z \cos\omega_0 t \ . \qquad (2.9)$$

Though interesting, this idea needs to be further studied. In the following, we will only discuss further the first two driving mechanisms that apply to the PBWA and the PWFA, respectively.

3. Acceleration in Plasma Waves

In this section we consider the PBWA and the PWFA as accelerating systems and discuss some dynamic aspects of an accelerated electron bunch. In order to make optimum use of the laser beam it is necessary to match the Rayleigh length R to the acceleration section. Here we choose the section length L to be twice of R. This in turn determines the diffraction limited spot size, $a^2 = R\lambda/\pi = L\lambda/2\pi$. On the other hand, a relativistic electron beam is much less divergent, and is given by a radius b. According to Eq. (2.8) the acceleration gradient $e\mathcal{E}_z$ has a parabolic dependence on r, which induces an energy spread among particles at different radii after being accelerated for some distance. For high energy physics purposes, the final energy spread has to be limited to a small percentage. This can be insured if $b \ll a$. The accelerated beam is injected behind the driving beams on axis at a proper phase such that it is both accelerated and focused. The structure for the PWFA is basically the same other than that the driving electron bunch is essentially divergenceless in radius.

3.1. The Betatron Oscillation

While accelerating, the driven beam will in general slip over the phase of the plasma wave. If this phase slippage is slow, which is the case in both schemes, then we can calculate the transverse focusing effects as if the beam were at a fixed phase on the wave.

The differential equation governing the transverse "betatron" oscillations of a highly relativistic particle is

$$\frac{d^2r}{dz^2} = e\,\frac{\mathcal{E}_r}{\gamma mc^2}\,, \tag{3.1}$$

where γmc^2 is the particle's instantaneous relativistic mass. Thus, for small radius from Eq. (2.8), we find the beta function to be

$$\beta = \left[\frac{k_p a^2 \gamma mc^2}{eA\sin\phi}\right]^{1/2}, \tag{3.2}$$

where ϕ is the phase at which the particle locates.

3.2. Energy Spread

From Eq. (2.8) it is evident that for a driving beam with finite transverse size, the longitudinal field varies transversely. Since the field varies parabolically in the transverse direction, the average energy gain is reduced slightly and an energy spread is induced. If we assume that the beam is already very relativistic, then the average gain in energy relative to that for a particle on the axis of the plasma wave is

$$\Delta E_{\rm ave} = \Delta E\left(1 - \frac{2}{3}\left(\frac{b}{a}\right)^2\right)\,. \tag{3.3}$$

The corresponding energy spread induced in one stage for the model we have chosen is

$$\left[\frac{\delta(\Delta E)}{\Delta E}\right]_{\rm rms} = \frac{\sqrt{2}}{3}\left(\frac{b}{a}\right)^2\,. \tag{3.4}$$

3.3. Phase Slippage

For both accelerator schemes the phase velocity of the plasma wave is not equal to the velocity of the driven bunch. This means that the driven bunch will slip in phase along the plasma wave as it is accelerated. For the PBWA we maximize $\mathcal{E}_z$ for a given L by optimizing the phase shift δ. If we choose a laser frequency ω, an acceleration length L, and a phase slippage δ for speed of light particles; then the plasma frequency is given by[13]

$$\omega_p = \left(\frac{2\delta c\omega^2}{L}\right)^{1/3}\,. \tag{3.5}$$

On the other hand, the acceleration gradient that the driven bunch sees varies along L due to the phase slippage. If the total phase slippage over the entire acceleration length is δ, then the average acceleration gradient is related to the ideal gradient by a phase slippage form factor $\sin\delta/\delta$, that is

$$e\mathcal{E}_z^{ave} = \alpha mc\omega_p\frac{\sin\delta}{\delta}\,, \tag{3.6}$$

where $\alpha \equiv n_1/n_0$. Here the phase has been allowed to slip from the top of the cosine down one side so that the bunch is always in a focusing region. The average acceleration gradient can be maximized for a given L if

$$\delta \simeq \frac{5\pi}{16} \quad \text{and} \quad \frac{\sin\delta}{\delta} \simeq 0.85 \quad \text{PBWA}\,. \tag{3.7}$$

For the PWFA we consider only relativistic driving and driven bunches. In addition we require that the final energy of the driving bunch after the distance L is still relativistic. In this case we can calculate the phase slippage along the plasma wave since the plasma wave phase velocity is equal to the velocity of the driving bunch. Following Ref. 5 we integrate the relative velocity along the length L to obtain

$$\delta \simeq \frac{\pi L}{\lambda_p}\left[(\gamma_{1i}\gamma_{1f})^{-1} - (\gamma_{2i}\gamma_{2f})^{-1}\right] \quad \text{PWFA}\,. \tag{3.8}$$

Since in an actual high energy accelerator the second term would be quite small, one can neglect it when using Eq. (3.8).

From the above discussion we see that in PBWA the phase slippage is a non-negligible effect that influence the performance of the scheme. To avoid a large δ, two ideas have been introduced. The Surfatron[7] employs a transverse external magnetic field which forces the accelerated particles to "surf" around plasma waves. With proper arrangements, the beam could in principle lock into a fixed phase. The Plasma Fiber Accelerator,[6] on the other hand, tries to increase the phase velocity of the beat wave to near c. This is achieved by creating a duct structure in the plasma, in which the density is low inside and high outside such that the EM wave is evanescent, enabling the plasma beat wave phase velocity to be equal to any prescribed velocity within the channel.

3.4. The Transverse Size

At the beginning of this section we defined our system with a diffraction limited laser spot size at the waist, $a^2 = R\lambda/\pi = L\lambda/2\pi$. In terms of a chosen phase slippage δ and eliminating the section length L, we can verify that

$$a^2 = \frac{2\delta\, c^2\,\omega}{\omega_p^3} \quad \text{PBWA}\,. \tag{3.9}$$

Notice that the effect of Rayleigh diffraction diminishes when the laser beam reaches the threshold power,[14]

$$W_c \simeq \frac{2\pi}{3}\, n_0\, mc^2\left(\frac{c}{\omega_p}\right)^2\left(\frac{\omega}{\omega_p}\right)^2\,. \tag{3.10}$$

When above the threshold, the laser beam would exhibit relativistic self-focusing effect during propagation through the plasma. This occurs because the electrons near laser beam axis are the ones being driven the hardest, which acquire higher relativistic masses and therefore result in higher laser group velocities. From computer simulations[15] it is found that the laser beam will focus down to a radius $\sim c/\omega_p$ asymptotically. One obvious merit for invoking the relativistic self-focusing effect in PBWA is to extend the Rayleigh length substantially. But there are shortcomings. It is not clear whether a system can be properly arranged such that only self-focusing, and no filamentation[15] occurs to the laser beam. In addition, the strong radial gradient of a focused beam provides a radial ponderomotive force which blows plasma out of the channel in a time as fast as one ion plasma period, necessitating the use of laser pulses shorter than this. In turn this imposes a stringent constraint on the necessary laser power. To avoid these subtleties we still confine ourselves in a Rayleigh diffraction dominated regime for the remaining discussions.

For the PWFA since we would like to fix the number of particles N_1 in the driving bunch, the transverse size is determined by the desired accelerating field,

$$a = \left[\frac{8r_e N_1 mc^2}{e\mathcal{E}_z}\right]^{1/2} \quad \text{PWFA}\,, \tag{3.11}$$

where r_e is the classical electron radius.

3.5. The Energy Requirement

In the PBWA the laser power for the beam profile given in Eq. (2.2) is $W = \pi a^2 E_0^2 c/16\pi$. If we assume that we have a laser pulse length τ, the energy necessary to drive the plasma wave density to αn_0 is[13]

$$W\tau = \frac{\alpha\delta m^2 c^5}{e^2\omega_p}\left(\frac{\omega}{\omega_p}\right)^3 \quad \text{PBWA}\,. \tag{3.12}$$

where Eq. (3.9) has been used to eliminate a^2. On the other hand, the total energy in the driving bunch of N_1 particles for the PWFA is simply given by

$$W\tau = N_1 E_1 \quad \text{PWFA}\,. \tag{3.13}$$

4. Numerical Examples

Now we come to specific examples of both schemes. Our approach is to choose a set of parameters that we fix from the beginning. The remaining parameters can then be calculated in terms of those chosen ones using the formulas derived previously. To make meaningful examples we employ only those lasers and electron beams that are presently available. Therefore we need to fix the laser frequency ω by choosing a particular laser source. If we further fix the section length L, the phase slippage determines the plasma frequency ω_p. This means that $\mathcal{E}_z$ is a derivable quantity in PBWA.

To keep the dimensions to a laboratory scale, we select the acceleration lengths to be 10 cm and 100 cm. These two lengths are then combined with two different laser frequencies, the Nd: Glass laser and the CO_2 laser, to form four sets of sample calculations. For the PBWA the trapping parameter $\alpha \equiv n_1/n_0$ is chosen to be 0.25, which is approximately the saturation value,[11] and the phase slippage is taken to be the optimum value given in the previous section. Finally, we assume that the laser pulse length and the growth time for the plasma wave τ is about 159 cycles ($\omega_p\tau = 1000$).

Since the PWFA is not so restrictive in its design, we can now set the parameters to match some of those for the PBWA. In particular we use the same acceleration gradient and the same a/λ_p. The number of particles in the driving bunch is taken from the present number in the SLC and the bunch length is assumed to be somewhat less than the plasma wavelength. The initial and final energies of the driving bunch are selected so that the final energy of the bunch tail is 90% of its initial energy. As we can see from Tables 1 and 2, the phase slippage for the PWFA is much smaller than that for the PBWA. All parameters except the efficiency and the energy in the driving beam turn out to be quite comparable. In particular note from the values of β that the focusing for both schemes is quite strong. The energy required for the driving bunch is consistently higher for the PBWA; however, because it is less efficient in these examples, the number of particles N_2 which can be driven is comparable to that in the PWFA.

Table 1. Plasma Beat Wave Accelerator

Chosen Parameters	Values			
ω [sec^{-1}]	Nd:Glass 1.78×10^{15}		CO_2 1.78×10^{14}	
L [cm]	10	100	10	100
α	0.25	0.25	0.25	0.25
δ [rad]	$5\pi/16$	$5\pi/16$	$5\pi/16$	$5\pi/16$
$\sin\delta/\delta$	0.85	0.85	0.85	0.85
$\omega_p\tau$	1000	1000	1000	1000
Derived Parameters				
ω_p [10^{13} sec^{-1}]	2.65	1.23	.571	.265
n_0 [10^{16} cm^{-3}]	21.7	4.67	1.00	0.22
$e\mathcal{E}_z$ [GeV/m]	9.38	4.36	2.00	0.94
a [mm]	0.13	0.41	0.41	1.30
a/λ_p	1.82	2.70	1.25	1.82
β [$\sqrt{\gamma/\sin\phi}$ mm]	0.18	0.57	0.57	1.80
N_2 [10^{10}]	$1.95\eta_2$	$9.04\eta_2$	$4.19\eta_2$	$1.95\eta_2$
$W\tau$ [J]	23.9	515.4	11.1	239.2

Table 2. Plasma Wake Field Accelerator

Chosen Parameters	Values			
L [cm]	10	100	10	100
$e\mathcal{E}_z$ [GeV/m]	9.38	4.36	2.00	0.94
N_1	5×10^{10}	5×10^{10}	5×10^{10}	5×10^{10}
E_1 [GeV]	1.04	4.84	0.22	1.04
a/λ_p	1.82	2.70	1.25	1.82
Derived Parameters				
a [mm]	0.25	0.36	0.54	0.78
δ[10^{-3} rad]	5.5	2.5	42	18
ω_p [10^{13} sec^{-1}]	1.37	1.41	.439	.438
n_0 [10^{16} cm^{-3}]	5.90	6.18	.606	.604
α	0.38	0.17	0.25	0.11
β [$\sqrt{\gamma/\sin\phi}$ mm]	0.28	0.59	0.73	1.52
N_2 [10^{10}]	$2.25\eta_2$	$2.25\eta_2$	$2.25\eta_2$	$2.25\eta_2$
$W\tau = N_1E_1$ [J]	8.33	38.8	1.76	8.33

5. The Efficiency

5.1. The Efficiency with Untailored Driving Beams

The overall efficiency of the plasma accelerators can be divided into three parts. The first part is the efficiency of conversion of 'wall plug' energy to either laser energy or electron beam energy. These two efficiencies may be quite different, however, we will not discuss them here. The second efficiency is the conversion of either laser or electron beam energy to plasma energy. The third efficiency is that for conversion of the plasma energy to the driven electron beam. The efficiency of the transfer of energy from the laser to the plasma has been calculated for the PBWA model we have chosen.[13] For a general phase shift δ the ratio of the plasma energy to the laser energy is given by

$$\eta_1 = \frac{P.E.}{W\tau} = \frac{\alpha\delta}{4} \quad \text{PBWA}\,. \tag{5.1}$$

If laser depletion is included in the analysis, this number will be reduced slightly.

The efficiency of the transfer of energy from an electron beam to the plasma is quite different. In this case one must consider the beam loading effects. If we could treat the bunch as a macro-particle, then for a very relativistic driving bunch we could extract nearly all of its energy before it's velocity changed enough to yield a phase slip. However, due to beam loading this is not possible since the leading edge of the driving bunch loses essentially no energy to the plasma while the trailing edge loses twice as much as that calculated for a point-like particle. Thus, for very short bunches in the model that we considered in the previous sections, we can only extract about 1/2 of the energy

$$\eta_1 = \frac{1}{2} \quad \text{PWFA} . \tag{5.2}$$

The final efficiency to calculate is that from the plasma to the trailing bunch. This efficiency is the same for both cases provided that the characteristics of the plasma wave are the same. The total acceleration gradient experienced by a bunch with N_2 particles in a plasma wave is

$$G \equiv \frac{dE_2}{dz} = e\mathcal{E}_z f - 4e^2\frac{N_2}{b^2} \quad , \tag{5.3}$$

where f is the phase slippage form factor. The second 'beam loading' term is due to the plasma wake induced by the trailing bunch. The efficiency is given by the total energy gained by the bunch divided by the plasma energy,

$$\eta_2 = N_2 GL\left(\frac{\mathcal{E}_z^2}{8\pi}\frac{\pi a^2}{2}\ L\right)^{-1} \quad . \tag{5.4}$$

This efficiency has a maximum when $N_2 = f\mathcal{E}_z b^2/8e$, and the value is given by

$$\eta_2^{\max} = f^2\ \frac{b^2}{a^2} \quad . \tag{5.5}$$

For the PWFA f can be taken to be essentially unity while for the PBWA f is given by Eq. (3.7). If we require that the induced energy spread due to the transverse variation of the longitudinal field is say 1%, then the maximum efficiency of transfer of plasma energy to the electron beam is about one order of magnitude smaller than the SLAC structure with a 1 mm Gaussian bunch,[16] which is $\eta_2^{\max} \simeq 0.3$. Even the $\eta_2^{\max}$ cannot be realized since it would require full beam loading and thus yield 100% energy spread between the head and the tail of the accelerated bunch.

5.2. *Asymmetric Driving Bunches in PWFA*

The various efficiencies discussed above seem to be rather low. But there are ways for improving them. In this section we discuss the improvements on η_1. Next section is discussions on η_2.

For the η_1 in the PBWA, since the laser is not depleted too much, it might be possible to reuse the laser beam after a suitable amplification. This would yield a very high repetition rate. The laser self-focusing mechanism may be another possibility. But as we discussed earlier, this effect may have other complications. Both ideas need to be further studied.

The η_1 in the PWFA can be largely improved if one properly tailors the longitudinal charge distribution of the driving bunch.[17] Calculations show that the optimal efficiency η_1 for a given number of driving particles occurs when the bunch current is composed of the following two components: (1) a delta-function "precursor," and (2) a linear current ramp with length $\omega_p\tau$ following immediately after the precursor. The number of particles in the two components are supposed to have the ratio $2/(\omega_p\tau)^2$. With this arrangement, the energy extraction efficiency is[18]

$$\eta_1 = \frac{1+(\omega_p\tau)^2}{2+(\omega_p\tau)^2} \quad \text{PWFA} . \tag{5.6}$$

When $\omega_p\tau \gg 1$, η_1 approaches 100%. For more realistic charge distributions, for example a half-Gaussian profile or a "door step" profile,[17] η_1 would only be slightly degraded.

Actually, there is a more important motivation for shaping the driving bunch current in the PWFA, which is to increase the "transformer ratio" R, defined as the ratio of the maximum accelerating electric field behind the driving bunch E_m^+, to the maximum retarding electric field within the bunch E_m^-. If a monoenergetic driving bunch excites a wake field, and if within distance L the particle in the bunch that experiences the maximum retarding field E_m^-, which would stop the earliest, loses energy $\Delta\gamma mc^2 = eLE_m^-$, then the maximum possible energy gain for a test charge behind the bunch will be $R\Delta\gamma mc^2$ in the same distance.

For symmetric bunches R cannot be larger than 2, and is generally around 1. This means that it takes tremendous number of stages to boost GeV beams to TeV energies. However, for the optimal charge distribution discussed above, the transformer ratio is $R = \sqrt{1+(\omega_p\tau)^2}$. When the bunch length occupies many plasma wavelengths, *i.e.* $\omega_p\tau \gg 1$, R can be very large.

The difficulty involved in this idea is that the tail of the long bunch would generally be pinched by the transverse wake field excited by its head, which would quickly disrup the bunch if the wake field is too strong. This is indeed the case for a parabolic radial distribution discussed in the previous sections. However, if one also tailors the radial distributin such that it is constant in radius, and for $k_pa \gg 1$, the wake field is essentially one dimensional and the transverse field is exponentially suppressed as we will see in the next section.

5.3. *Improving the Energy Absorption Efficiency*

Now we turn to the improvement of the energy absorption efficiency η_2. There are recently two ideas suggested by Ruth and Chen[9] and van der Meer,[19] separately. The former suggests a modification of the driving beam's transverse profile such that it be independent of r, with a large cross sectional radius compared to the plasma wavelength (*i.e.* $k_pa \gg 1$). The second suggestion does not invoke modifying the transverse profile, but requires the opposite extreme that $k_pa \ll 1$. We will review them briefly in the following.

Consider in the PWFA that the driving bunch profile to be $\sigma(r) = N/\pi a^2$ for $r < a$. It can be straightforwardly verified that for both k_pa and $k_p r$ large,

$$\mathcal{E}_z \simeq -\frac{4eN}{a^2}\left(1-\frac{1}{2}\left(\frac{a}{r}\right)^{1/2}e^{-k_p(a-r)}\right)\cos\left(k_pz-\omega_pt\right) \quad . \tag{5.7}$$

We see that the field is quite constant for $r < a$ and drops exponentially to 1/2 its value at $r = a$. Therefore the field is essentially constant until $r \simeq a$.

If we consider a trailing beam with full width b, the full energy spread induced by the transverse variation of $\mathcal{E}_z$ is

$$\left[\frac{\delta(\Delta E)}{\Delta E}\right]_{\text{full}} = \frac{1}{2}\left(\frac{a}{b}\right)^{1/2} e^{-k_p(a-b)} . \qquad (5.8)$$

In this case the efficiency is again given by $\eta_2^{\max} = (b/a)^2$. As an example, let us consider the case in Table 2 column 1. To maintain the acceleration field of 9.38 GeV/m, we increase the number of particle in the driving beam to $N_1 = 1 \times 10^{11}$ while fix $a = 0.25$ mm. Furthermore we increase the plasma density by a factor of 3 such that $k_p a = 9.6$. If we restrict the full energy spread to about 1%, the trailing beam radius b can be increase to $b/a \simeq 0.80$, which yields an efficiency $\eta_2^{\max} \simeq 0.64$.

Similar arguments apply to the PBWA. Assume that, instead of a Gaussian distribution, the laser beam radial profile is modified into the following:

$$E_0^2(r) = E_0^2 \begin{cases} 1 - k_p a\, K_1(k_p a)\, I_0(k_p r), & r < a \\ k_p a\, I_1(k_p a)\, K_0(k_p r), & r > a \end{cases} . \qquad (5.9)$$

When both $k_p a$ and $k_p r \gg 1$,

$$E_0^2(r) \simeq E_0^2 \left[1 - \frac{1}{2}\left(\frac{a}{r}\right)^{1/2} e^{-k_p(a-r)}\right] , \quad r < a . \qquad (5.10)$$

This is the same radial behavior as in Eq. (5.7). By applying the same method given in Section 2, it can be shown that the corresponding $\mathcal{E}_z$ has the same form as in Eq. (5.7), except that the coefficient $4eN/a^2$ is replaced by $\omega_p r k_p e E_0^2/8\omega^2 m$. Recall that in the PBWA $\eta_2^{\max} = (\sin\delta/\delta)^2 (b/a)^2$, we find from the above example that $\eta_2^{\max} \simeq 0.46$.

S. van der Meer suggests another way of improving η_2 in PWFA by taking $k_p a \ll 1$. In this case, for $k_p a \ll 1$ and $k_p r \ll 1$ we find

$$\mathcal{E}_z \simeq -2eNk_p^2 \left[\frac{3}{4} - C + \ln\frac{2}{k_p a} - \left(\frac{r}{a}\right)^2 + \frac{1}{4}\left(\frac{r}{a}\right)^4\right] , \qquad (5.11)$$

where $C = 0.577\ldots$ is the Euler's constant. The full energy spread for a trailing beam of radius b is therefore

$$\left[\frac{\delta(\Delta E)}{\Delta E}\right]_{\text{full}} \simeq \frac{(b/a)^2}{\frac{3}{4} - C + \ln\frac{2}{k_p a}} , \quad k_p a \ll 1 . \qquad (5.12)$$

On the other hand, it can be shown[9] that the maximum efficiency in this case is

$$\eta_2^{\max} \simeq \frac{\frac{3}{4} - C + \ln\frac{2}{k_p a}}{\frac{3}{4} - C + \ln\frac{2}{k_p b}} , \quad k_p a, k_p b \ll 1 . \qquad (5.13)$$

Again, keeping the full energy spread to about 1%, we find the transverse size $(b/a)^2 \simeq 0.032$. The maximum efficiency can then be calculated from Eq. (5.13) to be $\eta_2^{\max} \simeq 0.65$.

From the above discussions we see that the second efficiency η_2 can in principle be very good in both PWFA and PBWA from both schemes. However, if one wishes to incorporate an asymmetric current profile in PWFA such that the first efficiency η_1 and the transformer ratio R are also optimized, then the first scheme has an advantage over the second scheme. This is because the focusing field in the former case is exponentially suppressed relative to the longitudinal field, whereas the ratio of $\mathcal{E}_r/\mathcal{E}_z$ in the later case scales like $(1/k_p a)(r/a)$, where $k_p a \ll 1$, and the tail of the long asymmetric bunch would be strongly pinched by its head. Another problem with the strong focusing is that the accelerated particles which oscillate in the focusing field emit large synchrotron radiation. However, since the radiation is very sensitive to β, this problem may be solved with more careful design.

6. Emittance Growth in Plasma Accelerators

The emittance of the accelerated beam can be very small when either of the improvement schemes discussed above is employed. For example, in the case of a flat driving beam ($k_p a \gg 1$) we have weak focusing and

$$\beta = \left[\frac{\gamma}{\sin\phi}\, \frac{a^3\, e^{k_p a}}{(2\pi)^{1/2}(k_p a)^{3/2} N r_e}\right] . \qquad (6.1)$$

With the same numerical example discussed above, we find $\beta = 125$ m at 10 GeV when $\sin\phi = 0.1$. If we assume perfect matching, the emittance at 10 GeV is then $\epsilon = b^2/\beta \simeq 3.2 \times 10^{-10}$ m-rad, which yields an invariant emittance $\epsilon_n \simeq 6.4 \times 10^{-6}$. To compare with rms emittance one might divide by a factor of 5. This is coming close to interesting values for large linear colliders.

To this point the plasma accelerators, in particular the PWFA, seem to be able to provide not only high gradients, but also high efficiency and how emittance. The remaining question is whether the low emittance can be reasonably preserved during acceleration. There are inherent sources for emittance growth due to the nature of the plasma that cannot be removed. There are also sources derived from imperfection of the system, in particular the imperfrection of the driving beam. We discuss them briefly in the following.

6.1. *Inherent Errors*

Coulomb Scattering: It has been shown by Montague and Schnell[20] that the emittance growth due to Coulomb scatterings between the beam particles and the plasma ions can be expressed as

$$\frac{d\epsilon_n}{dz} = \frac{1}{\gamma}\, 2\pi r_e^2\, n_0 \beta \ln\left(\frac{\lambda_D}{r_c}\right) , \qquad (6.2)$$

where λ_D is the Debye length of the plasma and $r_c \simeq 0.7 \times 10^{-13}$ cm is the effective Coulomb radius. In the same example which we just discussed, at plasma thermal temperature $kT = 5$ eV, the growth of emittance due to this effect will be $\Delta\epsilon_n \simeq 5.3 \times 10^{-6}$ m-rad when the beam is accelerated from 10 GeV to 1 TeV. Unfortunately the value occurs to be the same order of mangitude as the designed emittance. This is, however, by no means the optimum design. By choosing the parameters more carefully, $\Delta\epsilon_n$ may be somewhat reduced relative to ϵ_n. But to be sure, the emittance growth due to Coulomb scatterings is a non-negligible effect in our weak focusing (or large β) approach.

6.2. *Driving Beam Imperfection*

Since the plasma accelerators assume staging techniques to accelerate the particles to TeV energy or above, while each stage requires a separate driving beam, any random error on the designed driving beam density distribution would cause emittance growth. In analysing these errors in the PWFA, any azimuthal perturbation in the driving beam can be decomposed into multipole moments. It can be shown that the electric field in the plasma generated by the m^{th} moment scales as $\mathcal{E}_m \propto 1/m^2$. Therefore the most severe effects are contributed by the lowest modes. Notice, however, that this picture pressumes a transversely rigid bunch. This is true only when the bunch is infinitely thin. In reality the driving bunch will always have a finite thickness, this is particularly true when the technique of improving η_l is invoked. With the help of the transverse self-focusing, the variations of the density distribution are expected to be smeared out along the course of travel, and this effect is hoped to be not too damaging to the preservation of designed emittance.

7. Experimental Progress

The first experimental verification of the PBWA was performed at UCLA.[21] The 9.6 μm and 10.6 μm lines of a CO_2 laser were used to resonantly drive a plasma of density 10^{17} cm^{-3}. The conclusive evidence for the existence of high phase velocity plasma wave excited by the beating of the two laser lines comes from ruby laser Thomson scattering. The scattering angle is adjusted to k match to the fast plasma wave. By moving the fiber optics which collects the scattered light on a shot-to-shot basis, the k spectrum of the plasma wave has been obtained. Figure 1(a) shows that $\Delta k = k_p$. In addition, Fig. 1(b) shows the frequency shift of the ruby light from the stray position to be exactly $\Delta\omega = \omega_p$. These measurements unambiguously identify the wave as being the plasma beat wave. From Δk and the plasma density perturbation measured, it was inferred that a longitudinal electric field between 1 GeV/m and 3 GeV/m was achieved.

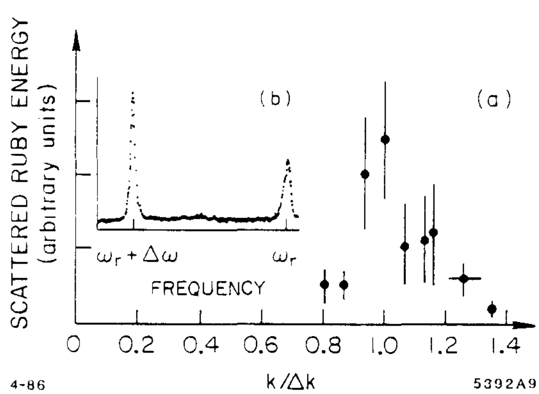

Fig. 1. (a) The k spectrum, and (b) the frequency shift, from the UCLA experiment.

At the mean time, an experiment on the PWFA is currently in progress at Argonne National Laboratory under the Wisconsin-Argonne-Fermilab collaboration. Other experimental efforts on either of the two schemes are being pursued or considered at Quebéc, Canada, Rutherford Appleton Laboratory, England, and at CERN, Switzerland.

8. Summary

The physical mechanisms of generating fast plasma waves are reviewed, and the accelerator schemes employing beating lasers (PBWA) and relativistic electron bunches (PWFA) are discussed. Gradients of order 1 GeV/m are theoretically predicted and experimentally verified. In addition, it is shown that the efficiency in these schemes, in particular in the PWFA, can in principle be very high if one can properly tailor the longitudinal and transverse density profiles of the driving bunch. The emittance in these schemes is found to be close to the interesting values for large linear colliders. This low emittance is potentially jeopardized by various inherent noise effects in the plasma and the imperfections in the driving beam. Further studies are needed for a more complete estimate of all these effects.

References

1. *Laser Acceleration of Particles*, Los Alamos, 1983, ed. P.J. Channell, AIP Conference Proceedings No. 91.
2. *Laser Acceleration of Particles*, Malibu, 1985, ed. C. Joshi and T. Katsouleas, AIP Conference Proceedings No. 130.
3. T. Tajima and J.M. Dawson, Phys. Rev. Lett. 43, 267 (1979).
4. P. Chen, R.W. Huff and J.M. Dawson, UCLA Report No. PPG-802, 1984, and Bull. Am. Phys. Soc. 29, 1355 (1984); P. Chen, J.M. Dawson, R.W. Huff and T. Katsouleas, Phys. Rev. Lett. 54, 693 (1985).
5. R.D. Ruth, A.W. Chao, P.L. Morton and P.B. Wilson, Part. Accel. 17, 171 (1985).
6. T. Tajima, *Proceedings of International Accelerator Conference*, Fermilab, 1983, pp. 470.
7. T. Katsouleas and J.M. Dawson, Phys. Rev. Lett. 51, 392 (1983).
8. T. Katsouleas, J.M. Dawson, D. Sultana and Y.T. Yan, IEEE Trans. Nucl. Sci. NS-32, 3554 (1985).
9. P. Chen and R.D. Ruth, ibid. Ref. 2; R.D. Ruth and P. Chen, in *SLAC Summer Institute on Particle Physics*, Stanford, California, 1985.
10. C.M. Tang, P. Sprangle and R. Sudan, Appl. Phys. Lett. 45, 375 (1984).
11. M.N. Rosenbluth and C.S. Liu, Phys. Rev. Lett. 29, 701 (1972).
12. C. Darrow, D. Umstadter, T. Katsouleas, W.B. Mori, C.E. Clayton and C. Joshi, UCLA preprint, April, 1986. Also submitted to Phys. Rev. Letts.
13. R.D. Ruth and A.W. Chao, ibid. Ref. 1.
14. C.E. Max and J. Arons, Phys. Rev. Lett. 33, 209 (1974).
15. C. Joshi, W.B. Mori, T. Katsouleas, J.M. Dawson, J.M. Kindel and D.W. Forslund, Nature 311, 525 (1984).
16. P.W. Wilson, in *Physics of High Energy Particle Accelerators*, ed. R.A. Carrigan, F.R. Huson and M. Month, AIP Conference Proceedings No. 87, Am. Inst. of Phys., 1982.
17. P. Chen and J.M. Dawson, ibid. Ref. 2; K.L.F. Bane, P. Chen and P.B. Wilson, IEEE Trans. Nucl. Sci. NS-32, 3524 (1985).
18. P. Chen, J.J. Su, J.M. Dawson, K.L.F. Bane and P.B. Wilson, Phys. Rev. Lett. 56, 1252 (1986).
19. S. van der Meer, CERN/PS/85-65 (1985).
20. B.W. Montague and W. Schnell, ibid. Ref. 2.
21. C. Joshi, C.E. Clayton, C. Darrow and D. Umstadter, ibid. Ref. 2.