PRESENT OPTICS OPTIONS FOR TeV COLLIDERS*

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ABSTRACT

A practical approach for implementing TeV collider optics with high luminosities $\mathcal{L} \approx 10^{33}$ $[cm^2\ s]^{-1}$ but without large pinch effects is given using current alternatives. Characteristics are considered that constrain the optics and the types and orders of magnets required. A modified linac FoDo cell based on permanent magnet hybrid quadrupoles is discussed. Similarly, a demagnifying, permanent magnet telescopic system that allows variation of beta, eta and energy is suggested for the final focus. The basic cell for low emittance damping rings can also be constructed solely from permanent magnets. Small diameter, low permeability, high field permanent magnets have proven useful for injection and extraction lines and are also compatible with the large particle detectors near the interaction regions as well as with exotic experiments for production and use of secondary beams or for multi-bunch coalescing schemes for control of longitudinal bunch distribution. An 8–10 GeV prototype cell and final focus experiment is proposed to verify and study such systems as well as do some interesting physics tests. One example, which could be used with the PEP storage ring, would convert an external electron beam into a photon beam to avoid beamstrahlung effects – a major problem for high energy e$^\pm$ colliders.

Collision Constraints

Given an acceptably small phase space volume populated by the number of particles required to provide a given luminosity, $\mathcal{L}$, one can specify the requirements and tolerances of the component systems based on a few dominant processes. Because there is no known fundamental limitation on emittances at the SLC level or lower by at least an order of magnitude or more, a normalized, "invariant", transverse emittance of

$$\epsilon_n \equiv \gamma\sigma\sigma' = \gamma\frac{\sigma^2}{\beta} \leq 3\times10^{-6} \text{ rad m} \tag{1}$$

will be assumed[1,2] with σ the rms beam radius, γ the energy in mass units and β the focusing function of the system. The luminosity is then

$$\mathcal{L} = H\frac{N_+N_-}{4\pi\sigma_x^*\sigma_y^*}fn = H\frac{N_+N_-\gamma}{4\pi\epsilon_n(\beta_x^*\beta_y^*)^{1/2}}fn \tag{2}$$

where H is a factor relating to the effective size and overlap of the interacting bunches, β^* the beta at the interaction region, f the accelerator pulse repetition rate, n the number of bunches per pulse and N the number of particles per bunch.

Such simple scaling relations are disarming e.g. since $\eta^*=\eta'^*=0$, one is tempted to use a linear calculation for σ^* or possibly assume it results solely from geometric aberrations - neither of which is valid[3]. Nevertheless, because of various collective effects, it should be simpler and cheaper to reduce β^* down to values comparable to the longitudinal beam size $2\sigma_z$ than reduce ϵ further as long as this can be done without significantly increasing β elsewhere or otherwise driving higher order aberrations. Since SLC is expected to reach $\beta^* \approx 4$ mm before higher order aberrations limit it, a value of $\beta^* = 1$ mm should be possible here because of the lower emittances assumed as long as comparable energy spreads can be maintained. For TeV colliders with round beams this gives

$$\begin{aligned}\mathcal{L} &= 1.3\times10^{31}E(TeV)Hfn \\ &\xrightarrow{f=120} 1.5\times10^{33}HnE(TeV) \quad [cm^2\ s]^{-1}\end{aligned} \tag{3}$$

for equal bunches with N=5 $\times$ 10^{10}. With fast damping, resonant extraction rings in the energy range 1–5 GeV it should be possible to increase both f and n with the assumed emittance. Increasing n by an order of magnitude[2] is important for energy efficiency and especially for bunch energy spread since it allows N to be reduced. Such tradeoffs or whether one uses flat or round beams are not the questions here but rather how to get low emittance beams from a damping ring to the interaction region i.e. the optics and how to realize them at reasonable costs.

Magnet Constraints

The problem of producing, preserving and colliding bunches with very low emittance and short length in a stable way for long periods of time is formidable considering the many practical questions of accelerator and magnet misalignment, the various sources of beam jitter and ground vibration as well as the associated questions of magnetic and mechanical hysteresis. Permanent magnets appear preferable over other current alternatives on most of these points quite apart from the main design criteria of multipole strength, harmonic quality and cost. Development work over the past few years at SLAC, in collaboration with Vacuumschmelze Inc., has been directed at such questions with good results[4]. Figures 1–2 compare such magnets to conventional electromagnets, showing how a library of 5 easy-axis orientations has been used to make all of the low order multipoles for a variety of applications[5,6].

A. Permanent Magnets

Figure 2 gives the optical strength, S_N, for the multipoles in Fig. 1 in terms of their equivalent pole-tip field and radius:

$$S_N(M) \equiv -\frac{1}{(N-1)!}\frac{\partial^{N-1}B_y}{\partial x^{N-1}} = \frac{B_{PT}}{R^{N-1}} \propto MJ_rF_b(R,N)\ . \tag{4}$$

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The multipole strength is directly proportional to the number of blocks, their remanent magnetization and a function that depends on the size, shape and location of the blocks. The field distribution, in terms of S_N, follows directly from Fig. 1:

$$\vec{B}_N(\vec{z}) \ = B_{PT} \ (\frac{r}{R})^{N-1} \ e^{-i[\frac{\pi}{2}+(N-1)\theta]} \qquad r \leq R \qquad (5)$$

where $\vec{z} = re^{i\theta} = x + iy$. One can readily show that the maximum strength occurs when $M \rightarrow \infty$ and the blocks completely fill the space between two concentric cylinders with radii $R_i \leq r \leq R_o$ in which case:

$$S_N \ = \ \frac{J_r}{R_i^{N-1}} \frac{N}{N-1} \Big[1 - (\frac{R_i}{R_o})^{N-1}\Big] \longrightarrow \ \frac{J_r}{R_i^{N-1}} \frac{N}{N-1} \qquad (6)$$

and $S_{N=1} = J_r \, ln(\frac{R_o}{R_i})$. Equation (6) was derived by Blewett as early as 1965[7]. One can understand the slopes and magnitudes in Fig. 2 from this expression which is shown as the dashed lines for $h = R_o - R_i = 5$ cm which is consistent with current manufacturing capabilities. The limit $R_i/R_o \rightarrow 0$ is designated S_N^{max} in Fig. 2 and is relevant for magnet radii $R_i \lesssim 1$ cm i.e. radii small compared to practical block heights, h.

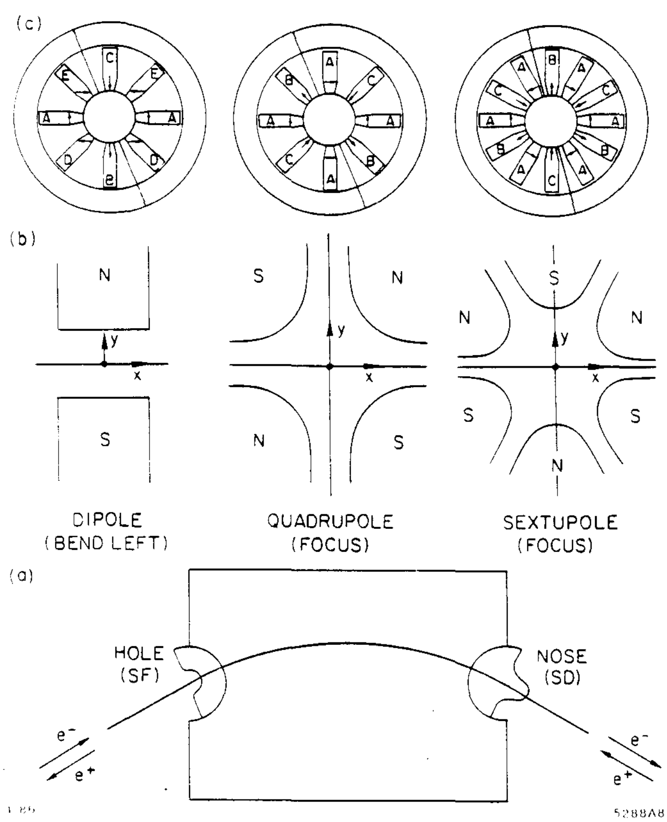

Fig. 1: Some different ways of obtaining dipole, quadrupole and sextupole fields using: (a) "combined function" systems with rotatable end shims; (b) conventional, iron-dominated electromagnets and (c) permanent magnets. The magnetic midplane is defined by y=0 and polarities are all positive with respect to one another except as noted by SD.

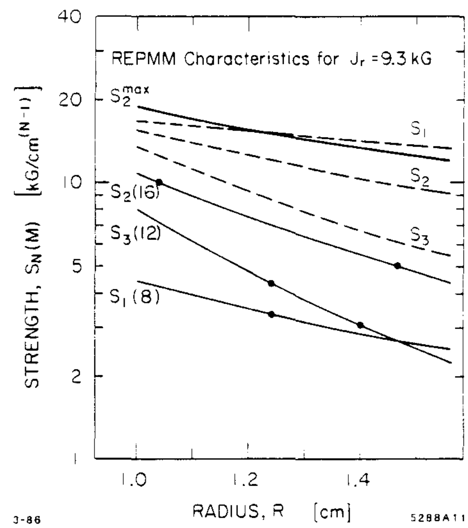

Fig. 2: Strengths for the multipoles shown in Fig's. 1 and 3 for blocks with J_r = 9.3kG. The 8-block quad in Fig. 1 has strength $2S_2(8) = S_2(16)$. The dots represent magnets made for the SLC damping rings, their injection and extraction lines and the final focus.

Dipoles with $S_1 = B_{PT} \geq 2$ T for $R \leq 1$ cm can be made using newer materials such as *Nd-Fe-B* and this number is also expected to grow significantly in the future. The value $J_r = 9.3$ kG corresponds to the $SmCo_5$ blocks which we have used for several years so this can be scaled up to 11–12 kG. High quality quadrupoles with gradients $S_2 \geq 1$ T/cm are also good design benchmarks for radii $R \geq 1$ cm i.e. for quads that can encapsulate X-band or smaller cavities rather than fit inside them. While all of this depends on the allowable radius, the trend toward lower emittance beams clearly favors rare earth permanent magnet multipoles for many systems of the next collider e.g. the full damping ring of bends, quads and sextupoles[5]; the injection and extraction lines of damping rings[6] with esoteric elements such as septa; the linac lattice quads in new configurations; the final focus optics[8] and even the beam containment optics for the next generation of high power klystrons[9].

B. Permanent Magnet Hybrids

There are many kinds of PM hybrids possible although one usually thinks in terms of a single magnet which uses PM material to drive what is otherwise a conventional, iron-dominated magnet e.g. Halbach and coworkers[10] have developed a variable strength quadrupole which uses an interesting hybrid mechanical flux "shunt." Such terminology also refers to combinations of different types of magnets[8] in either a combined or separated function sense such as combinations of conventional and PM magnets(Fig. 1).

From the limit of Eq. (6), the correspondence between B_{PT} and J_r implies good steel is necessary to match the strengths of pure REPMM's. The growing use of $Nd_2Fe_{14}B$ should reduce material costs and increase J_r sufficiently that one might question using soft iron hybrids especially considering the costs of high quality steel such as vanadium permendur. Three reasons favoring such a choice are **tunability**, **quality** and better **stability** in some environments e.g. FoDo cells for linacs and damping rings.

While there have been a variety of techniques suggested for tuning pure PM magnets, very little has actually been done. Besides ref's. 8 & 10, there is also the method of Gluckstern and Holsinger[11]. The various methods depend greatly on whether it is possible to use steel or not since soft steel rods or screws have been used in a number of ways as tuning shunts.

Questions of quality are also considerably simplified by using iron. Good iron poles for small bore magnets can be used in a variety of ways. Tunable multipoles can be made like conventional multipoles[10] or in a way that improves quality[12]. A 12–pole design would provide "look–a–like" dipoles, quadrupoles and sextupoles whose dipole field can be described as better than a dipole's and whose quadrupole field is better than a quadrupole's. Strength and quality are usually the opposing poles of magnet design. In high energy physics, strength problems have been a major motivation for developing superconducting technology so one usually gives away strength for acceptable quality by enlarging the bore. High field permanent magnets operate well into the third quadrant of the B–H curve which presently implies significant nonlinearities so it is simpler to use good quality steel poles and give away some strength. However, look–a–like multipoles can be driven with PM material longitudinally which should improve both strength and uniformity.

Optics Constraints

In a symmetric FoDo lattice with equal focusing and defocusing strengths, the β's in the quads are directly proportional to the cell length L i.e.

$$\beta_{x,y} = L\,\frac{1 \pm sin(\phi/2)}{sin\phi} \tag{7}$$

$$cos\phi = (1 - \frac{L^2}{8F^2}) \tag{8}$$

where ϕ is the phase advance per cell and F is the focal length. While $\phi = 76°$ minimizes β for *fixed* L, the gain is negligible compared to 60° or 90° so $\frac{\lambda}{6}$ or $\frac{\lambda}{4}$ cells are often used. The focal length for such cases is

$$\frac{1}{F} = \frac{Gl}{B\rho} = \frac{4}{L}sin\frac{\phi}{2} = \frac{2\sqrt{2}}{L} \quad \text{or} \quad \frac{2}{L} \tag{9}$$

with G the gradient and l the effective length of the quadrupoles. Setting $l = L/2m$ for a packing fraction $1/m$ gives an L and β that scale as:

$$L = \left[8m(\frac{B\rho}{G})\sin\frac{\phi}{2}\right]^{1/2} \overset{\phi=\frac{\pi}{3}}{\longrightarrow} 36.5\left[m\frac{E(TeV)}{G(kG/cm)}\right]^{1/2} \quad [m] \ . \tag{10}$$

This expression provides several approaches depending on the application and the kinds of magnets one favors e.g. storage rings generally have L and m fixed with E and G variable but damping rings needn't vary any of these so that permanent magnets or their hybrids may actually be superior. In the case of PM FoDo cells for linacs, the simplest approach is to let everything vary except the phase and gradient and build magnets in a modular fashion[4]. L and β then scale as $\gamma^{\frac{1}{2}}$ and σ as $\gamma^{-\frac{1}{4}}$:

$$\frac{L}{L_\circ} = \left(\frac{m}{m_\circ}\right)^{\frac{1}{2}}\left(\frac{\gamma}{\gamma_\circ}\right)^{\frac{1}{2}} \ . \tag{11}$$

With a 1.2 GeV beam from a damping ring as for SLC, with $m_\circ = 2$ and $G = 10$ kG/cm one gets $L_\circ = 0.57$ m, $\beta_{max} = 0.98$ m and $\sigma_{max} = 35\ \mu$ for $\phi = 60°$ as shown in Table I.

Table I: Representative FoDo cases for $E = 1.21$ GeV, $m = 2$ and $G = 10$ kG/cm.

ϕ(Deg)	L/F	C	$L_\circ$(m)	β_{max}(m)	$\frac{\beta_{max}}{\beta_{min}}$	$\sigma_{max}(\mu)$
15.0°	0.52	18.7	0.29	1.27	1.30	40
22.5°	0.78	22.8	0.35	1.11	1.48	37
30.0°	1.04	26.3	0.41	1.03	1.70	36
45.0°	1.53	32.0	0.50	0.97	2.24	35
60.0°	2.00	36.5	0.57	0.98	3.00	35
90.0°	2.83	43.4	0.68	1.15	5.83	38

In this scenario one starts with a 60–90° phase shift to minimize wake effects and varies m to offset γ and then L. With the present SLAC gradient of $\gamma' = 17$ MeV/m and a $\frac{\lambda}{4}$ cell with L=25 m, one has $\phi'=2\pi/100$ m^{-1}. For an order of magnitude increase in accelerating gradient $\gamma' = 170$ MeV/m, one should decrease ϕ' accordingly i.e. $\lesssim 2\pi/10$ m^{-1} for the same tune which is only G=1.16 kG/cm. However, since wake effects from accelerator or magnet misalignments increase the effective size proportional[2] to $(\frac{\beta}{\gamma})(\frac{\Delta}{\sigma})$ for rms, transverse errors Δ one wants small β's at low energies. Since $\beta \approx 1$ m is more than an order of magnitude better than anywhere in the SLC lattice and since it should also be possible to improve Δ by another order of magnitude[13] with small PM quads, wake effects need not be a problem even for shorter wavelength accelerators.

The aspect ratio of the beam in the quads scales as the square root of the β's which depends only on phase advance:

$$\frac{\beta_{max}}{\beta_{min}} = \frac{1 + \sin(\phi/2)}{1 - \sin(\phi/2)} \ . \tag{12}$$

This can be made more symmetrical by going to a higher quad multiplicity in the cell or using a small phase advance i.e. short cells and/or long focal lengths which should reduce higher order effects. While ideal, bend-free FoDo cells have no purely chromatic aberrations, e.g. $\eta(s) = 0$, they do have higher order, *mixed* chromatic terms which may need correction for high enough energy spreads. These are proportional

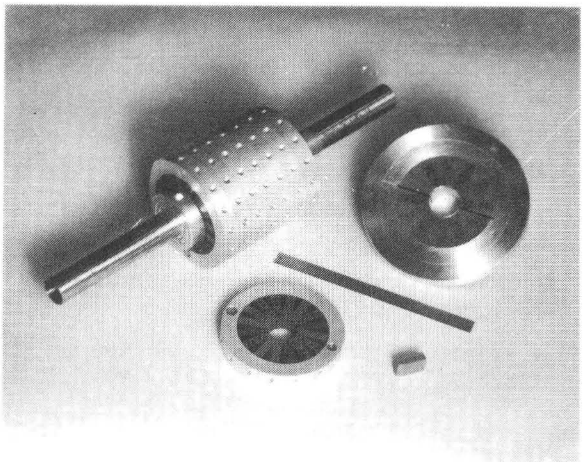

Fig. 3: Photograph of a one-layer and multi-layer quad prototype for the SLC final focus and a one-layer, split-ring sextupole for the SLC damping rings made from blocks of nominal dimensions $l \times w \times h = 1.27 \times 0.64 \times 2.10$ cm. l is the effective length of the magnets. The sextupole halves are held together with a symmetric ring for magnetic measurements.

to $(L/F)^i x^j x'^k \delta^l$ so one wants $\frac{L}{F}$ small. This needn't produce wake effects for small enough β_{max}.

For fixed optics, one should keep L/F constant e.g. use a fixed phase advance ϕ. We get another scaling relation by letting either $l(m)$ or G vary e.g. specifying G in terms of beam size for conventional or PM magnets large compared to their bore $G \propto B_{PT}/\sigma \propto J_r/\sigma$ gives an L and β that scale as $(m^2\gamma)^{\frac{1}{3}}$ i.e.

$$\frac{L}{L_\circ} = \left(\frac{J_\circ}{J}\right)^{\frac{2}{3}} \left(\frac{m}{m_\circ}\right)^{\frac{2}{3}} \left(\frac{\gamma}{\gamma_\circ}\right)^{\frac{1}{3}} \quad (13)$$

This is better than Eq.(11) because we have stronger quads e.g. $R_{PT} = 30\sigma \approx 1$ mm gives $G \approx$100–240 kG/cm depending on the PM material and $L_\circ = 6$ cm and $\beta = 0.25$ m at 1.21 GeV. However, to implement this with current technology, we need to go to higher multiplicity cells.

An intuitive example of such a cell which justifies previous statements is the symmetric quadruplet FoDoDoF. Using a thin lens approximation as before, the phase advance per cell of total length L is

$$cos\phi = (1 - \frac{1}{4}\frac{L^2}{F^2} + \frac{1}{2^9}\frac{L^4}{F^4}). \quad (14)$$

Without going into detail, some results for comparison to those of Table I are given in Table II. Notice that while the cell length and β are somewhat larger, the magnet length is about a factor of two smaller for the same gradient so a more integral structure should not be difficult with practical gradients of $G =$ 10–20 kG/cm for S-band accelerating structures or $G =$ 30–60 kG/cm with X-band. However, one needs to reconsider the basic design of the accelerating cavity to determine the maximum allowable packing fraction m which will probably be somewhat larger than assumed here.

Table II: Some FoDoDoF cases for $E = 1.21$ GeV, $m = 2$ and $G = 10$ kG/cm.

ϕ(Deg)	L/F	$L_\circ$(m)	β_{max}(m)	$\frac{\beta_{max}}{\beta_{min}}$	$\sigma_{max}(\mu)$
15.0°	0.368	0.345	1.54	1.20	44
22.5°	0.552	0.422	1.24	1.32	40
30.0°	0.735	0.487	1.13	1.51	38
45.0°	1.088	0.593	1.03	1.77	36
60.0°	1.426	0.678	1.00	2.17	35

CONCLUSIONS

It is clear that one wants high gradient quads with good alignment tolerances and stability i.e some kind of PM hybrids. To get these it is important to go to shorter wavelength accelerating structures. Both of these possibilities are consistent with low emittance, high energy beams and can be integrated into a high quality optical system with high accelerating and focusing gradients in what is essentially a single monolithic structure. One approach would be to load the disk structure with PM hybrid steel quadrupoles which are longitudinally coupled by PM material between the alternating gradient quads analogous to the way some wigglers are now made. Such systems could be very strong, tunable and with minimal chromatic aberrations. Such cells allow special insertions for matching, diagnostics and correction every $2n\pi$ of phase advance without introducing second order geometric aberrations[14] which can propagate. The periodicity of such insertions would depend on a number of things but should simplify and reduce diagnostic and control costs.

The basic PM hybrid being suggested here[12] is also applicable to a damping ring FoDo cell and other systems including long undulators for production of coherent radiation. The use of such radiation for high energy physics[15] implies the possibility of a very interesting low energy(8–10 GeV) prototype for a TeV collider which could both test an important number of new accelerator subsystems and also provide some fundamental physics while providing a significant upgrade for the PEP storage ring facility.

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REFERENCES

1. B. Richter, Very High Energy Colliders, IEEE Trans. Nucl. Sci., Vol. NS-32(1985)3828 and W. K. H. Panofsky, Beam Dynamics Parameterization, SLAC-AAS-Note-13(1986).

2. P.B. Wilson, Linear Accelerators for TeV Colliders, SLAC-PUB-3674 and 2^{nd} Intl. Workshop on Laser Accelerators, Los Angeles Calif., Jan. 1985.

3. K.L. Brown and J.E. Spencer, Non-Linear Optics for the Final Focus of the Single-Pass-Collider, SLAC-PUB-2678 and IEEE Trans. Nucl. Sci., Vol. NS-28(1981)2568.

4. M. Baltay et al., Techniques for Making and Measuring Permanent Magnet Multipoles, SLAC-PUB-3988(1986).

5. J. E. Spencer, Some Uses of REPMM's in Storage Rings and Colliders, IEEE Trans. Nucl. Sci., Vol. NS-32(1985)3666.

6. A. Hutton, J. Spencer and M. Woodley, Injection and Extraction Optics with REPMM's.

7. K. Halbach, Design of Permanent Multipole Magnets with Oriented Rare Earth Cobalt, NIM **169**(1980)1.

8. J.E. Spencer, Some Optics Alternatives for the FFS, SLAC-CN-264(1984).

9. W.B. Herrmannsfeldt, Sturrock Focusing for Hollow Beams, SLAC-AAS-Note-18(1986).

10. K. Halbach, B. Feinberg, M. I. Green, R. MacGill, J. Milburn and J. Tanabe, Hybrid Rare Earth Quadrupole Drift Tube Magnets, IEEE Trans. Nucl. Sci., Vol. NS-32(1985)3643.

11. R.L. Gluckstern and R.F. Holsinger, Adjustable Strength Quadrupoles, IEEE Trans. Nucl. Sci., Vol. NS-30(1983)3326.

12. H.D. Ferguson, J.E. Spencer and K. Halbach, A General Ion-Optical Correction Element, NIM **134**(1976)409.

13. J.E. Spencer, PM Sextupole Protocol and Tolerances for the Damping Rings, SLAC-CN-309(1985).

14. K.L. Brown, A Second-Order Magnetic Optical Achromat, SLAC-PUB-2257.

15. J.E. Spencer, Higher Luminosities via Alternative IncidentChannels, SLAC-PUB-3645 (1985).