FREQUENCY SCALING OF LINEAR SUPER-COLLIDERS*

A. Mondelli, D. Chernin, A. Drobot
Science Applications International Corporation
1710 Goodridge Drive, McLean, VA 22012

M. Reiser and V. Granatstein
University of Maryland
College Park, MD 20742

Introduction

The development of electron-positron linear colliders in the TeV energy range will be facilitated by the development of high-power rf sources at frequencies above 2856 MHz. Present S-band technology, represented by the SLC[1], would require a length in excess of 50 km per linac to accelerate particles to energies above 1 TeV. By raising the rf driving frequency, the rf breakdown limit is increased, thereby allowing the length of the accelerators to be reduced. Currently available rf power sources set the realizable gradient limit in an rf linac at frequencies above S-band[2,3].

This paper presents a model for the frequency scaling of linear colliders, with luminosity scaled in proportion to the square of the center-of-mass energy. Since wakefield effects are the dominant deleterious effect, a separate single-bunch simulation model is described which calculates the evolution of the beam bunch with specified wakefields, including the effects of using programmed phase positioning and Landau damping. The results presented here have been obtained for a SLAC structure, scaled in proportion to wavelength.

Frequency Scaling Model

The beam parameters at the interaction point of the collider determine its usefulness for experiments. There are eleven such parameters: the bunch size (σ_x, σ_y, σ_z), the number of particles per bunch (N), the number of bunches per rf pulse (b), the rf repetition frequency (f), the particle energy (γ), the total luminosity (L_T), the beamstrahlung energy loss (δ_{BS}), the disruption parameter (D), and the average beam power ($\bar{P}_b$). These parameters must satisfy the four defining relationships,

$$L_T = N^2 f b H_D/(4\pi\sigma_y\sigma_x) \qquad (1)$$

$$D = 2r_e N\sigma_z/[\gamma\sigma_y(\sigma_x + \sigma_y)] \qquad (2)$$

$$\delta_{BS} = (2/3)r_e^3[N^2\gamma/(\sigma_x\sigma_y\sigma_z)]FH_D H_\delta \qquad (3)$$

$$\bar{P}_b = bNf\gamma mc^2 \qquad (4)$$

where r_e is the classical electron radius, H_D is the pinch enhancement factor, H_δ is the quantum correction factor for beamstrahlung, and F is the correction to beamstrahlung for flat beams[2].

If seven of the quantities are specified, the remaining four typically can be computed. There are 330 such model "scenarios", some of which are nonsense since they overspecify one or more of the parameters. For example, $\bar{P}_b$ cannot be freely specified along with N, γ, b, and f. Each of the meaningful combinations can provide the basis for a scaling study. For the present study the parameter set (L_T, γ, b, δ_{BS}, σ_x, σ_y, σ_z) is specified, and the model computes N, f, D, and $\bar{P}_b$ to satisfy Eqs. (1) - (4). An iterative numerical procedure is used to evaluate H_δ, thereby self-consistently including the transition between classical and quantum beamstrahlung.[2]

The interaction-point model is independent of the details of the acceleration process. Having solved this model, an rf linac model is then utilized to specify the required accelerator parameters. Starting with the average beam power, for example, the linac model works backwards through the energy flow chain to obtain the average ac and rf power required[2], $\bar{P}_{ac} = \bar{P}_{rf}/\eta_{rf} = \bar{P}_b/(\eta_{rf}\eta_s\eta_{bT})$. The efficiency for each stage is either specified or calculated in the model. The ratio of the beam energy to the rf energy stored in the accelerating structure is the beam efficiency, η_{bT}, which is computed from the longitudinal wakefields, as described below. The ratio of the energy stored in the structure to the incident rf energy is the structure efficiency, η_s, which is specified in terms of the attenuation factor, $\tau = \omega t_p/2Q$, where ω is the rf frequency, t_p is the fill time, and Q is the cavity quality factor. Using τ = 0.59, η_s = 0.59 is obtained. The rf efficiency, η_{rf}, is the ratio of the average rf power incident at the accelerator to the average ac power, and is assumed to be η_{rf} = 0.34.[2] The parameters η_s, τ and η_{rf}, have been assumed to be independent of frequency.

The longitudinal and transverse wakefields of the accelerating structure are required by the scaling model. The present study has utilized the known SLAC monopole (W_z[V/C-m]) and dipole ($W_\perp$[V/C-m^2]) δ-function wakefields which are scaled in frequency[3]. The transverse wakefields are used in a single-bunch simulation model, to compute emittance growth and Landau damping, as described in the next section. The longitudinal wakefields are used to compute the average accelerating gradient, the energy spread, and the beam efficiency.

The longitudinal wakefield alters the effective gradient experienced by a particle at phase θ within the bunch, relative to the crest of the rf wave, to $E(\theta)$ given by

$$E(\theta) = E_0\cos(\theta) - NeW_{zT}(\theta), \qquad (5)$$

where E_0 is the peak gradient, and $W_{zT}(\theta)$ is the integrated longitudinal wakefield[3] at phase θ.

The single-bunch beam efficiency is defined as the ratio of the kinetic energy gained by the bunch to the rf energy stored in the accelerating structure,

$$\eta_b = NeE_a/w_s \qquad (6)$$

where $w_s=E_0^2/(4k_1)$ is the rf energy stored per unit length, k_1 is the loss factor for the fundamental accelerating mode, and E_a is the average accelerating gradient acting on the bunch.

In the study described here, the rf sources are assumed to just compensate for the energy sag between bunches.[3] The total beam efficiency for b bunches is then given by

$$\eta_{bT} = b\eta_b/[1+(b-1)\eta_b] \ . \qquad (7)$$

The over-all efficiency of the rf linacs is given by $\eta_{TOT}=\eta_{bT}\eta_s\eta_{rf}=\eta_{bT}/5$, which is typically $\leq 5\%$.

During acceleration an energy spread develops within the bunch because of the finite extent of the bunch on the rf wave and because of the effects of the longitudinal wakefields. These two effects can be made to counteract each other by placing the bunch ahead of the crest of the wave. Then, the head of the bunch will ride lower on the rf wave than the tail, but the tail will experience the decelerating wakefield due to the head. The energy spread is defined here as three times the standard deviation, σ_γ, of the energy distribution.

A model scenario for a 1 TeV collider can be examined assuming a total luminosity, $L_T=10^{37}$ $m^{-2}s^{-1}$, and $\delta_{BS}=30\%$ energy loss to beamstrahlung. The beams are assumed round with $\sigma_x = \sigma_y = 10^{-7}$m. The longitudinal bunch size is assumed to be $\sigma_z=\sigma_{z0}(\omega_S/\omega)$, where $\omega_S=2\pi(2856$ MHz) is the SLAC operating frequency. The peak accelerating gradient is assumed to scale with frequency as $E_0=(50MV/m)(\omega/\omega_S)$. With these parameters, the model has been numerically computed for various values of σ_{z0}. Figure 1 shows the resulting curves of average ac power per linac vs. frequency. For $\sigma_{z0} \geq 1$mm, the curves all display a broad minimum in the frequency range 4-15 GHz, and show only small variations in the power required over the entire frequency range. With σ_{z0}=1 mm, the average ac power is approximately 360 MW/linac ± 10% over the entire frequency interval studied. Bunches with σ_{z0}>1mm suffer large emittance growth because they are long enough to span the peak of the transverse dipole wakefield. The results below, therefore, correspond to σ_{z0} = 1 mm.

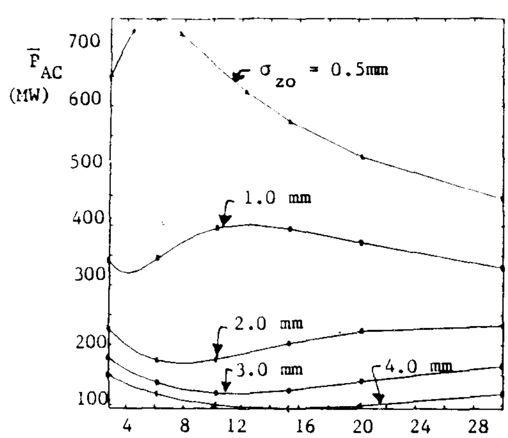

Figure 1. Average AC Power vs. Frequency for Various Bunch Lengths, σ_{zo}

When scaled to X-band (8-12 GHz), the length of each linac will be 7.6-4.8 km, and the number of rf feeds per linac will be in the range 6300-8000. The number of particles per bunch (N) is nearly independent of frequency, while the repetition frequency (f) increases monotonically with frequency. N is approximately 1.39 × 10^{10} and f lies in the range 280-450 Hz.

The total efficiency of the accelerator for these parameters is 2-3%, and the mimimum energy spread is $3\sigma_\gamma/\gamma$=0.5% at θ_0=3-5°. At this small phase advance, the average accelerating gradient is essentially the peak gradient. The peak rf power required to generate this gradient is 325 MW/feed.

The final focus[4] will require a normalized emittance $\leq 10^{-5}$ rad-m, which will require improvements in the damping rings over SLC, as well the control of emittance growth due to transverse wakefields, as described in the next section.

Single Bunch Wakefield Effects

Using a simple model we have investigated wakefield effects for a specific TeV linac design, using values for bunch size and charge, accelerator operating frequency, gradient and length obtained from the scaling model described above.

A bunch is divided into n slices of equal thickness, numbered from the front of the bunch, i = 1,...n. The distance of slice i from the head of the bunch is denoted ξ_i; the charge of slice i is Q_i. Neither ξ_i nor Q_i is allowed to change in the model. The centroid X_i and energy $mc^2\gamma_i$ of slice i satisfy

$$X_i'' + \frac{\gamma_i'}{\gamma_i} X_i' + \frac{k_o^2}{\gamma_i} X_i = - \frac{e}{m\gamma_i c^2} \sum_{j=1}^{i-1} Q_j W_\perp(\xi_i - \xi_j) X_j \qquad (8)$$

$$\gamma_i' = - \frac{e}{mc^2} \left[E_o \cos(\psi_o - \frac{\omega\xi_i}{c}) \ , \right. - \left. \sum_{j=1}^{i} Q_j W_z(\xi_i - \xi_j) \right] \qquad (9)$$

where a prime denotes a derivative with respect to distance z along the accelerator. ψ_o is the phase location of the bunch head on the accelerating wave and k_o^2 represents the external focusing, in the smooth approximation. Both ψ_o and k_o^2 are specified functions of z.

In addition to the centroid quantities X_i and γ_i, internal degrees of freedom of slice i are follcwed by tracking the variances $\sigma_{xx,i}$, $\sigma_{x'x',i}$, and $\sigma_{xx',i}$. Wakefields do not act directly on these variances and the normalized emittance of each slice is conserved.

One may express the normalized emittance of a K-V distribution as

$$\epsilon_N^2 = 16\ \bar{\gamma}_i^2\ [\sigma_{xx}\sigma_{x'x'} - \sigma_{xx'}^2\ ] \qquad (10)$$

where $\sigma_{xx} = \overline{\sigma_{xx,i}} + \overline{x_i^2} - \overline{x_i}^2$, etc. and an overbar means a (charge weighted) average over slices. We have taken (10) as a definition for the general case.

We have considered a single Gaussian bunch with σ_z = 0.37mm, truncated in the forward direction at $4\sigma_z$ and in the backward direction at $2\sigma_z$, of total charge -2.2nC traveling through a linac operated at 7.79GHz with a peak gradient of 135 MeV/m over its length of 7.57 km. The bunch length is chosen so that it does not extend beyond the peak of the delta function transverse wake. The bunch is divided into 16 slices, each initially upright and displaced 11μ from the axis due to injection error with an initial energy of 1.21 GeV (which is the location of the damping rings in SLC). The initial radius of each slice is 110μ and the initial normalized emittance of each slice, and the bunch, is $1.\times10^{-5}$ rad-m. The wake potentials used are those of a SLAC structure, scaled appropriately to operation at 7.79 GHz.[3] The focusing in our example has been chosen so that a phase shift of 8.1°/m is maintained up to an energy of 22GeV, beyond which the quadrupole strengths are assumed fixed at their maximal values; these maximal values are taken to be 5× those currently in use at SLAC.[6] The betatron phase shift gradually decreases to 1.2°/m at the end of the accelerator.

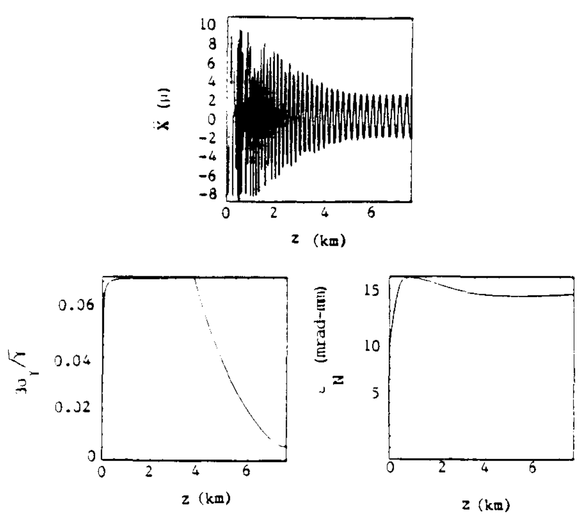

Figure 2: Centroid position, energy spread and emittance of a Gaussian bunch versus distance along a 1 TeV linac.

Figure 2 shows plots of X_i, $3\sigma_\gamma/\gamma$, and ϵ_N versus z for a case employing a (non-optimized) implementation of Landau damping.[5] During the first half of the accelerator the bunch is placed at -20° with respect to the rf crest, thereby generating a large energy spread, and, consequently, a large spread in betatron wavelengths. This spread renders the transverse wakes less effective since leading slices no longer drive trailing slices exactly at their resonant betatron frequency. When the energy spread gets very large one can, after a while, actually see the emittance decrease for a time as the slices oscillate in nearly opposite phase and the wakefields of a leading slice oppose the motion of a trailing slice. The result is that centroid displacement and emittance growth are reduced.

During the second half of the acceleration the bunch is shifted to 26.53°, giving the correct average gradient in the acceleration field to minimize the energy spread. The final result is that the final beam emittance is only a factor of ≈ 1.45 greater than the initial emittance while the energy spread, which had been as large as 7.2% during acceleration, achieves a final value very close to that for the case without Landau damping, namely 0.55%. This final beam quality has been achieved at a cost of 7% in final energy. When Landau damping is not used, the emittance is found to grow by a factor of 18 from its initial value.

Acknowledgements

It is a pleasure to thank J. Lawson, P. Wilson, and K. Bane for many stimulating discussions about the work presented here. This work has been supported by the U.S. Department of Energy under contract number DE-AC05-85ER40216.

References

1. SLAC Linear Collider Design Report, SLAC-229 (1980).

2. A.A. Mondelli, D.P. Chernin, A.T. Drobot, M. Reiser, and V. Granatstein, IEEE Trans. Nucl. Sci. NS-32, 3157 (1985).

3. P.B. Wilson, SLAC-PUB-3674 (1985); Physics of High-Energy Particle Accelerators (AIP Conf. Proc. No. 87, 1981), pp. 452-563.

4. R. Stiening in Physics of High Energy Particle Accelerators (AIP Conf. Proc. No. 105, 1983), pp. 353ff.

5. V. Balakin, et al., Proceedings of the 12th Int'l. Conf. on High Energy Accelerators, Fermilab (1983); K. Bane, IEEE Trans. Nucl. Sci. NS-32, 2389 (1985).

6. B. Richter, "Large Linear Collider" presented to Woods Hole Panel, May 20, 1983.