CUMULATIVE BEAM BREAKUP WITH A DISTRIBUTION OF DEFLECTING MODE FREQUENCIES*

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Introduction

The theory of beam breakup[1-5] has been worked out for identical uncoupled cavities and for a constant beam current. The difference equations have been solved exactly for a coasting beam, and expressions have been obtained for the steady state solution where the input beam displacement is constant or modulated at an arbitrary frequency.[5] In addition, an approximate result was obtained for the transient by means of a saddle point approximation. These results were shown to be in excellent agreement with numerical simulations for parameters appropriate to a 30 cavity 1300 MHz standing wave rf linear accelerator structure with a 2.5 Mev, 6.5 amp. coasting beam.[5]

The dominant feature of the transient result is amplification of the transverse displacement corresponding to the real exponent

$$e_t \cong \frac{3\sqrt{3}}{4}\left(\frac{RL}{\gamma}\right)^{1/3} N^{2/3} M^{1/3}, \qquad (1)$$

where M is the bunch number, N is the cavity number and RL/γ is a parameter proportional to the current and the ratio $Z_\perp/Q$ for the cavities, as defined in Reference 5. For an accelerated beam, this takes the form

$$e_t \cong \frac{3^{3/2}}{2^{5/3}}\left(\frac{eZ_\perp' T^2}{QW'}\right)^{1/3} (Izt)^{1/3}, \qquad (2)$$

where z is the accelerator length, t is the pulse duration, W' is the rate of the energy gain per meter, and $Z_\perp' T^2$ is the transverse shunt impedance per meter of cavity, including the transit time effect. Equation (2) corresponds to the expression obtained at SLAC for a traveling wave linac. Experimental studies[1,2] of the dependence of starting current I on pulse duration and accelerator length give approximate confirmation of the form of Eq. (2), with the conclusion that beam breakup occurs when e_t is between 15 and 20, corresponding to an amplification of some stimulus by a factor of order 10^7 to 10^8. Efforts to identify the initial noise stimulus were inconclusive.

It is clear that the large amplification in Eq. (1) or (2) depends on coherent oscillations of all the cavities. In the present work we explore the modification in e_t caused by a distribution in the frequency of the deflecting mode from cavity to cavity.

Analysis

The analysis of beam breakup with fluctuating parameters is carried out in detail elsewhere.[6] We start with the usual difference equations and obtain the displacement as a power series in the parameter RL/γ. This series is summed approximately for the parameter range

$$1 \ll j \ll N \ll M, \qquad (3)$$

where j is the power of RL/γ for which the summand is a maximum. The result for the maximum displacement as a function of N and M is

$$\frac{\xi_{envelope}}{\xi_o} \cong \frac{f^{1/3}}{M^{5/6}\sqrt{6\pi}}$$

$$\exp\left(-\frac{M\omega\tau}{2Q} + \frac{3\sqrt{3}}{4} f^{2/3} M^{1/3} - \frac{\sqrt{3}}{2}\varepsilon^2\left(\frac{\omega\tau}{Q}\right)^2 M^{5/3} f^{-2/3}\right) \qquad (4)$$

where

$$f = N(RL/\gamma)^{1/2}. \qquad (5)$$

The parameter

$$\varepsilon = Q(\Delta\omega)_{rms}/\omega \qquad (6)$$

which is related to the rms spread in the deflecting mode frequency of the cavities, is assumed to be small compared to unity, although the result appears to be valid for larger ε, since the displacement in Eq. (4) goes rapidly to zero as ε increases, as expected.

Comparison with Simulations

Simulations have been performed using the difference equations for the parameters of Reference 5, namely $RL/\gamma = 2.88 \times 10^{-3}$, $N = 30$, $f = 1.61$, $Q = 1000$, $\omega\tau/2\pi = 24/13$, for a Gaussian distribution of deflecting mode frequency. Typical results for two different random number seeds are shown in Figure 1 for $\varepsilon = Q(\Delta\omega)_{rms}/\omega = 1$.

Figure 1

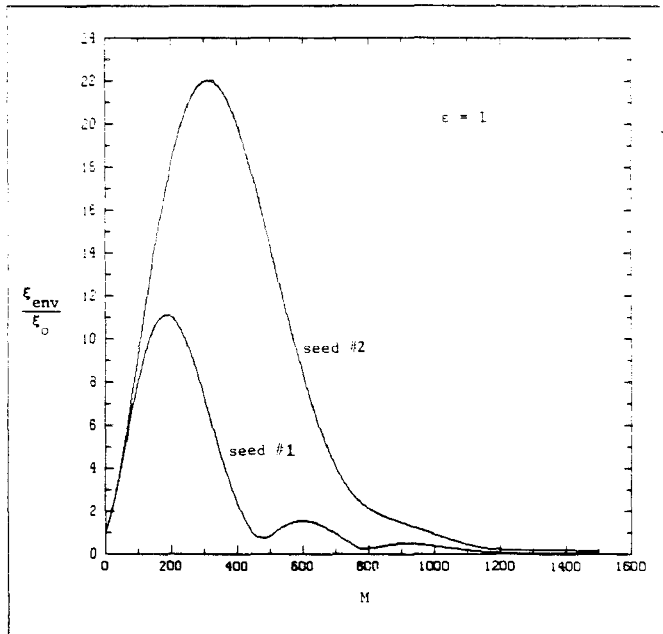

Fig. 1 ξ_{env}/ξ_o for $\varepsilon = Q(\Delta\omega)_{rms}/\omega = 1$ for two different seeds.

In order to check the range of validity of Eq. (4), we plot

$$w(M) = \ln\left(\frac{\mathcal{E}_{env.}}{\mathcal{E}_o}\right) + \frac{M\omega\tau}{2Q} - \frac{3\sqrt{3}}{4}\,\varepsilon^{2/3}M^{1/3} + \frac{5}{6}\ln M \quad (7)$$

against $M^{5/3}$. The plots are shown in Fig. 2 for values of ε from 0 to 10 for the two random number seeds of Fig. 1. It is clear that the linear relation is confirmed over a range which depends somewhat on the random number set used, but which clearly includes the region in which $\mathcal{E}_{env}$ reaches is first maximum.

Figure 2(a)

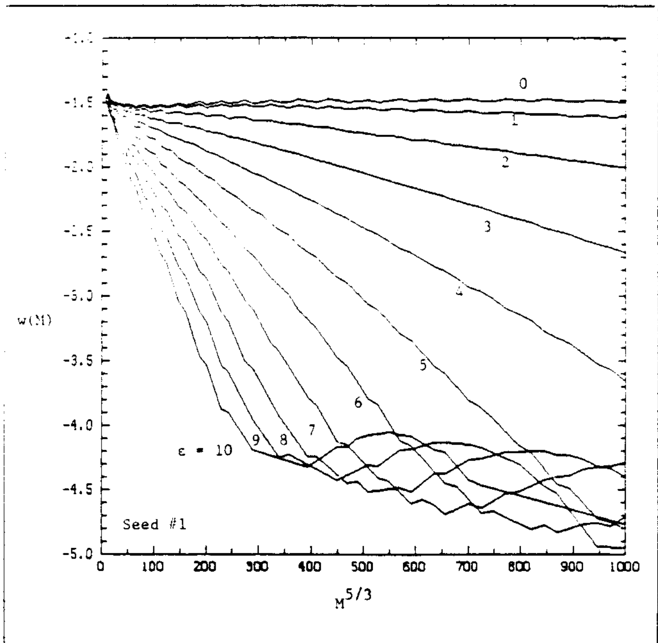

Figure 2(b)

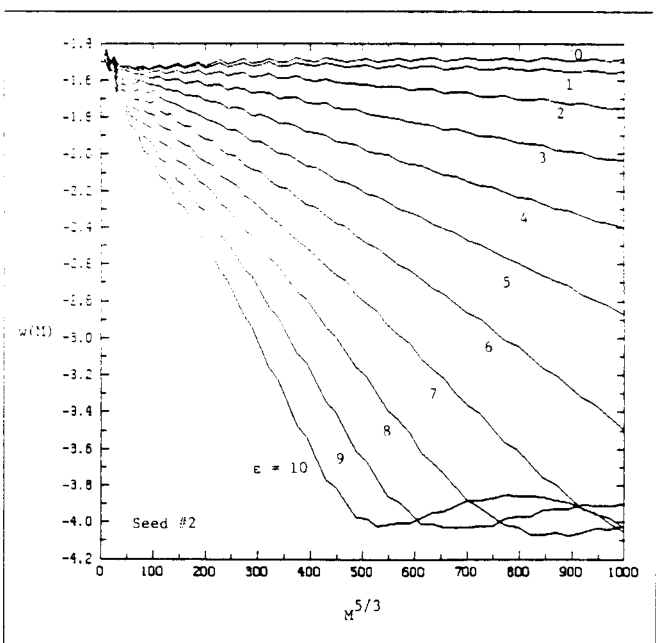

Fig. 2 ω(M) vs $M^{5/3}$ for ε = 0,1,2 •••,10 for two different seeds.

In Table I we list the slope and intercept found from straight line fits to the curves in Fig. 2. The approximate proportionality of the slope to ε^2, as predicted by Eq. (4) for small ε, appears to hold, even for values of ε^2 as high as 100.

Table I

Straight Line Fit of w(M) vs $M^{5/3}$

	seed #1			seed #2		
ε	Intercept	Slope	Slope/ε^2	Intercept	Slope	Slope/ε^2
0	-1.5	0		-1.5	0	
1	-1.5	1.4×10^{-4}	1.4×10^{-4}	-1.5	$.6\times10^{-9}$	$.6\times10^{-4}$
2	-1.5	5×10^{-4}	1.6	-1.5	2.5	.62
3	-1.5	11	1.2	-1.5	5.0	.56
4	-1.5	18	1.1	-1.5	8.5	.53
5	-1.5	28	1.1	-1.5	3.5	.52
6	-1.5	35	1.0	-1.5	18	.50
7	-1.4	51	1.0	-1.5	25	.51
8	-1.4	68	1.1	-1.5	32	.50
9	-1.4	84	1.0	-1.5	40	.49
10	-1.4	110	1.1	-1.5	47	.47

Furthermore, the prediction for the intercept is -1.30 according to Eq. (4) and for the rms value of the slope is $.85 \times 10^{-4}$, in reasonable agreement with the values obtained from the two seeds in Table I. (The discrepancy in the intercept is consistent with the saddle point approximation and the variation in slope with the different seeds for a sample of 30 cavities.)

It is a simple matter to calculate the value of M at which the displacement in Eq. (4) reaches its maximum. It occurs at

$$\frac{M_{max}}{M_o} \cong \frac{M_\varepsilon}{M_o} - \frac{2.5}{p_o\,\sqrt{1+10\,\varepsilon^2}}\,, \qquad (8)$$

where

$$\frac{M_\varepsilon}{M_o} = \left(\frac{1+\sqrt{1+10\varepsilon^2}}{2}\right)^{-3/2},$$

$$M_o = \left(\frac{3}{4}\right)^{3/4}\left(\frac{Q}{\omega\tau}\right)^{3/2} f, \qquad (9)$$

$$p_o = M_o\,\frac{\omega\tau}{Q}\,.$$

The maximum value of the exponent in Eq. (4) is given by

$$\frac{p_\varepsilon}{p_o} \cong \frac{M_\varepsilon}{M_o}\left(\frac{2+3\,\sqrt{1+10\varepsilon^2}}{5}\right) \qquad (10)$$

where

$$p_o \cong .80\left(\frac{Q}{\omega\tau}\right)^{1/2} f. \qquad (11)$$

For $10\varepsilon^2 \gg 1$, this becomes

$$p_\varepsilon \cong \frac{6}{5}\left(\frac{2}{5}\right)^{1/4}\frac{p_o}{\sqrt{\varepsilon}} \cong \frac{.80\ f}{\sqrt{1.1}\ \tau(\Delta\omega)_{rms}}\,, \qquad (12)$$

corresponding to an equivalent "Q"

$$Q_{eq} \cong \frac{\omega}{1.1(\Delta\omega)_{rms}} \quad . \qquad (13)$$

In Fig. 3 we summarize the change in location and peak value of the displacement for different values of ε. The peak moves very rapidly to lower values of M as ε increases. The corresponding reduction in the logarithm of the peak displacement is less rapid.

Figure 3

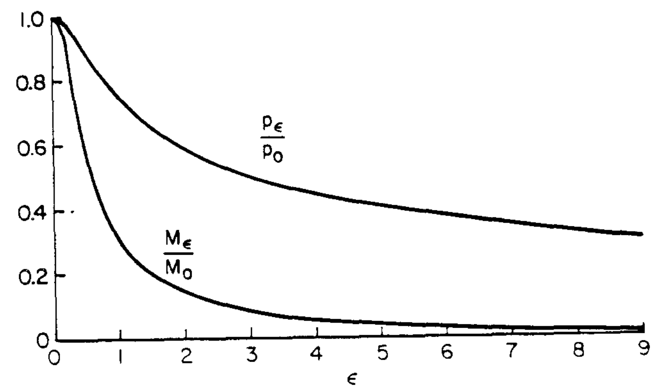

Fig. 3 Reduction in parameters controlling M_{max} and the maximum exponent as a function of ε.

Discussion and Conclusions

Analytic results have been obtained for transient beam breakup with a distribution of deflecting mode frequencies in the cavities. The validity of Eq. (4) has been confirmed by simulations even for values of ε an order of magnitude larger than required for the analysis.

For a linac consisting of a large number of cavities, it is expected that construction tolerances will cause the deflecting mode frequency to vary by at least a few parts in 10^4, even with tuning of the accelerating mode. For Q in the range 5,000 - 10,000, the corresponding value of ε will be at least 1 or 2. According to Fig. 3, the maximum value of the exponent will be reduced by an order of 30% or more. If the above numbers are applied to the original operation of SLAC, it is possible that the breakup exponent observed at SLAC would correspond to a growth of order 10^5 to 10^6 (e^{11} to e^{14}), instead of the 10^7 to 10^8 obtained by ignoring the variation in deflecting mode frequency.

The use of superconducting rf with Q of order 10^9 creates a concern related to beam breakup effects[7], even if the deflecting modes are damped by non-superconducting loads. Clearly the presence of a distribution of deflecting mode frequencies can reduce the seriousness of beam breakup.

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References

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6. R.L. Gluckstern and R.K. Cooper (to be published).

7. This concern has been addressed for CEBAF by J.J. Bisognano and G. Krafft in paper TH3-9 at this conference.