Proceedings of the 1986 International Linac Conference, Stanford, California, USA

BEAM BREAKUP IN A MULTI-SECTION RECIRCULATING LINAC*

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Introduction

It has long been recognized that recirculating a beam through a linac cavity in order to provide more efficient acceleration can also lead to an instability in which the transverse displacement on successive recirculations can excite modes which further deflect the initial beam. The effect is of particular concern for superconducting rf cavities where the high Q (or order 10^9) implies low starting currents for the instability. Previous work[1,2] has addressed this effect by calculating the beam trajectory in a single cavity, and its effect on excitation of unwanted modes. In this paper we extend the analysis of Gluckstern, Cooper and Channel[3] to the case of recirculation of a cw beam, and show how to compute the starting current for a multi-cavity structure with several recirculations. Each of the cavities is assumed to provide a simple impulse to the beam proportional to the transverse displacement in that cavity.

Analysis for a Coasting Beam

The difference equations for transverse displacement and momentum on the p^{th} traversal ($p-1^{th}$ recirculation), denoted by the two component vector $u_p(n,M)$ for the M^{th} bunch at the entrace to the n^{th} cavity, can be generalized from that for a single traversal to obtain

$$u_p(n+1,M) = P\, u_p(n,M) + \sum_{r=1}^{J} PG\, \hat{g} \sum_{k=1}^{M-1} u_r(n, M + (p-r)\, M_o - k)\, s_k\, (\omega\tau). \quad (1)$$

The notation is mostly that of Reference 3. In addition

$$P = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \quad \text{is the 2x2 transport matrix between cavity impulses} \quad (2)$$

$$G = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad (3)$$

$$\hat{g} = R/\gamma \quad \text{(proportional to current, transverse impedance, etc.)} \quad (4)$$

M_o = number of bunches in one recirculation (5)

$$s_k = e^{-k\frac{\omega\tau}{2Q}} \sin(k\omega\tau) \quad (6)$$

J = total number of passes (J-1 recirculations) (7)

We can obtain a steady state solution to Eq. (1) of the form

$$u_p(n,M) \equiv e^{i\Omega(M + pM_o)}\, v_p(n), \quad (8)$$

where Ω is the mode "frequency" of this solution. This leads to

$$v_p(n+1) = P\left[v_p(n) + gG \sum_{r=1}^{J} v_r(n)\right], \quad (9)$$

where

$$g = \hat{g} \sum_{k=1}^{\infty} s_k\, e^{-i\Omega k} = \frac{(R/2\gamma)\sin\omega\tau}{\cos(\Omega - i\omega\tau/2Q) - \cos\omega\tau}\,. \quad (10)$$

Our task is to solve Eq. (9), including the recirculation transport, for Ω as a function of current (R). Stability requires that all modes have

$$\mathrm{Im}\Omega > 0\,. \quad (11)$$

Clearly the starting current (R) is that for which $\mathrm{Im}\Omega = 0$ is first obtained as the current is increased from 0.

It is possible to solve Eq. (9) if P and g are independent of n and γ. The solution is

$$v_p(n+1) = P^n\, v_p(1) + \left[\frac{(P + Jg\, PG)^n - P^n}{J}\right] \sum_{r=1}^{J} v_r(1), \quad (12)$$

where the order of factors is important since P and G do not commute.

If we denote by L the transport matrix from the entrance to cavity N+1 on the p^{th} pass (N stands for the total number of cavities and N+1 is the entrance to a fictitious cavity) to the entrance to the 1st cavity on the $(p+1)^{st}$ pass, we have, taking Eq. (8) into account,

$$v_{p+1}(1) = e^{-i\Omega M_o}\, Lv_p(N+1). \quad (13)$$

This then allows us to write

$$v_{p+1}(1) = e^{-i\Omega M_o}\left[T\, v_p(1) + S \sum_{r=1}^{J} v_r(1)\right], \quad (14)$$

where the 1 pass recirculation matrix T, including phase shifts, is

$$T = LP^N \equiv e^{i\Omega M_o}\, \hat{T}\,, \quad (15)$$

and where

$$S = L\left[\frac{(P + Jg\, PG)^N - P^N}{J}\right] \equiv e^{i\Omega M_o}\, \hat{S}. \quad (16)$$

Once again, we can obtain a general solution to Eq. (14) if T and S are independent of the pass number. Specifically, one finds

$$(1-\hat{T})\, Wy = \sum_{r=1}^{J} \hat{T}^{r-1}\, \hat{S}\, v_1(1), \quad (17)$$

where

$$W = 1 - [J-1 + (J-2)\,\hat{T} + (J-3)\,\hat{T}^2 + \dots \hat{T}^{J-2}]\hat{S}, \quad (18)$$

and where the vectors x and y are defined by

$$v_p(1) = \hat{T}^{p-1} x + y \quad , \quad v_1(1) = x + y. \qquad (19)$$

It is clear that the normal mode solutions correspond to the solutions of Eq. (17) with the right side set equal to zero. The modes are therefore determined by the equation

$$\det W = 0, \qquad (20)$$

since the vanishing of det $(1 - \hat{T})$ would correspond to integral tune. The equivalent of Eq. (20) with varying S and T on each pass can also be written out explicitly. It is

$$\det \{1 - \hat{S}^1 - (\hat{S}^2 + \hat{T}^2\hat{S}^1) - (\hat{S}^3 + \hat{T}^3\hat{S}^2 + \hat{T}^3\hat{T}^2\hat{S}^1) + \cdots\} = 0, \qquad (21)$$

where the superscript identifies the pass number for which the value of $\hat{S}$ or $\hat{T}$ is calculated.

Each of the matrix elements in Eq. (20) or (21) is expected, because of Eq. (16), to be a polynomial in the beam current (g or R) of order N. Thus, Eq. (20) or (21) is expected to be an algebraic equation of order 2N, barring cancellations, for R in terms of Ω. Since the onset of instability occurs when both Ω and R are real, the starting current will be the root with the smallest real value of R for real Ω. If one resorts to numerical determination of the roots, one needs to sweep over real values of Ω until one finds roots for which R is real.

Simple Cases

Before exploring methods for computation in the most general case, we will consider cases with one or two cavities and several recirculations.

One Cavity (N=1)

In this case, Eq. (16) leads to

$$\hat{T}^j = e^{-i\Omega M_j} L_j P_j \quad , \quad \hat{S}^j = g_j \hat{T}^j G, \qquad (22)$$

where we now permit M_o, L, P and g to be different for each pass. Since

$$\hat{S}^j_{11} = g_j \hat{T}^j_{12} \; , \; \hat{S}^j_{12} = 0 \; , \; \hat{S}^j_{21} = g_j \hat{T}^j_{22} \; , \; \hat{S}^j_{22} = 0, \qquad (23)$$

we find from Eqs. (10) and (21)

$$\frac{\cos(\Omega - i\omega\tau/2Q) - \cos \omega\tau}{\sin \omega\tau} = \frac{R}{2} K, \qquad (24)$$

where

$$K \equiv \frac{1}{\gamma_1} [(\hat{T}^1)_{12} + (\hat{T}^2\hat{T}^1)_{12} + (\hat{T}^3\hat{T}^2\hat{T}^1)_{12} + \cdots] + \frac{1}{\gamma_2} [(\hat{T}^2)_{12} + (\hat{T}^3\hat{T}^2)_{12} + \cdots] + \frac{1}{\gamma_3} [(\hat{T}^3)_{12} + \cdots] + \cdots. \qquad (25)$$

For $R|K| << 1$ one can write an approximate solution for Eq. (24) as

$$\Omega = 2m\pi \pm (\omega\tau - \frac{RK}{2}) + \frac{i\omega\tau}{2Q} . \qquad (26)$$

The stability condition, $\mathrm{Im}\Omega > 0$, then becomes

$$\pm \frac{R}{2} \mathrm{Im}K = \mp \frac{1}{\gamma_1} [(L^1)_{12} \sin(\Omega M_1) + (T^2T^1)_{12} \sin(\Omega(M_1 + M_2)) + \cdots] \mp \frac{1}{\gamma_2} [(T^2)_{12} \sin(\Omega M_2) + (T^3T^2)_{12} \sin(\Omega(M_2 + M_3)) + \cdots] \mp \cdots < \frac{\omega\tau}{RQ} . \qquad (27)$$

Stability can thus be guaranteed if

$$\frac{1}{\gamma_1} [|(T^1)_{12}| + |(T^2T^1)_{12}| + \cdots] + \frac{1}{\gamma_2} [|(T^2)_{12}| + \cdots] + \cdots < \frac{\omega\tau}{RQ} . \qquad (28)$$

Equation (27) is in agreement with Vetter's result[2] for single cavity recirculation.

One Cavity with Storage Ring ($N=1$, $J\to\infty$)

In this case we take S and T to be constant and write $S = gTG$. In the limit $J \to \infty$, the product gJ is finite and therefore $g \to 0$. This permits us to write Eq. (17) as

$$\det [(1 - \hat{T}) W] \to \det [e^{-i\Omega M_o} - T - gJTG] = 0 \qquad (29)$$

whose solution is

$$\cos(\Omega M_o) - \cos\mu = g \frac{J\beta}{2} \sin\mu = \frac{(JR\beta/4\gamma) \sin \omega\tau}{\cos(\Omega - i\omega\tau/2Q) - \cos \omega\tau} \qquad (30)$$

using the Courant Snyder representation of T,

$$2 \cos \mu = T_{11} + T_{22} \quad , \quad T_{12} = \beta \sin \mu. \qquad (31)$$

Equation (29) agrees with earlier results on coupled bunch modes in circular accelerators.[4,5]

Two Cavities, Two Passes (N=2, J=2)

The complexity quickly escalates for two or more cavities. For two or more passes, the polynomial equation in R is of order N. Specifically, the equation for two cavities is

$$\frac{\cos(\Omega - i\omega\tau/2Q) - \cos \omega\tau}{R \cos \omega\tau} = \frac{K}{2} \qquad (32)$$

with

$$K = e^{-i\phi} (\frac{p_{CA}}{\gamma_A} + \frac{p_{DB}}{\gamma_B}) \pm [e^{-2i\phi}(\frac{p_{CA}}{\gamma_A} + \frac{p_{DB}}{\gamma_B})^2 - 4e^{-2i\phi} \frac{p_{BA}p_{DC}}{\gamma_A\gamma_B} + 4e^{-i\phi} \frac{p_{CB}}{\gamma_B} (\frac{p_{BA}}{\gamma_A} + \frac{p_{DC}}{\gamma_C})]^{1/2}, \qquad (33)$$

where $\phi = \Omega M_o$ is the phase shift in one recirculation, and where $p_{\beta\alpha}$ is the 12 element of the transport matrix from cavity α to cavity β. The cavity designations are A and B on the first pass, and C and D on the second pass. Once again, stability can be guaranteed if

$$|\mathrm{Im}\ K| < \frac{\omega\tau}{RQ} . \qquad (34)$$

Since the precise deflecting mode frequency is not known, and since M_0 is usually a large number, one needs to choose the value of ϕ which maximizes $|\mathrm{Im}(K)|$. If one assumes that the term in γ_A^{-1} dominates in Eq. (33), this suggests choosing parameters such that $p_{CA} = 0$. However, the optimum choice of parameters will depend on the values of all the parameters in Eq. (33).

For J > 2, the order of the equation for R is still N, but with many new parameters. Obviously some systematic approach is necessary for a situation like that at CEBAF[6] where N=400, J=4.

General Approach

If we consider the vector w(n) to have 2J components, each pair of which is $v_p(n)$ for p = 1,2, ··· J, Eq. (9) can be written as

$$w(n+1) = A_n\, w(n), \qquad (35)$$

where A_n is a 2Jx2J matrix whose elements are explicitly given in Eq. (9). Clearly

$$w(N+1) = B\, w(1)\ ,\ B = A_N A_{N-1} \cdots A_1 . \qquad (36)$$

Returning to the two component vector form, Eq. (36) can be written as

$$v_p(N+1) = \sum_{r=1}^{J} B_{pr}\, v_r(1), \qquad (37)$$

where each B_{pr} is a 2x2 submatrix of Eq. (36). Using Eq. (13) we have

$$v_{p+1}(1) - \sum_{r=2}^{J} L_p e^{-i\Omega M_p} B_{pr}\, v_r(1)$$

$$= L_p e^{-i\Omega M_p} B_{p1}\, v_1(1)\quad ,\quad p = 1,2, \cdots J-1. \qquad (38)$$

Clearly the normal modes Ω are the roots of the (2J-2)x(2J-2) determinant of the left side of Eq. (38), i.e.

$$\begin{vmatrix} (e^{i\Omega M_1} - L_1 B_{12}) & -L_1 B_{13} & -L_1 B_{14} \\ -L_2 B_{22} & (e^{i\Omega M_2} - L_2 B_{23}) & -L_2 B_{24} \\ -L_3 B_{32} & -L_3 B_{33} & (e^{i\Omega M_3} - L_3 B_{34}) \end{vmatrix} = 0. \qquad (39)$$

This analysis also permits the use of different deflecting mode frequencies for each cavity.[7]

This approach to determining the starting current is now being implemented for the CEBAF design[5] in order to explore the optimum choice of transport parameters.

Simulations

In a separate paper at this conference[8] Bisognano and Krafft report on numerical simulations for the CEBAF design. Specifically they solve Eq. (1) numerically and look for solutions which grow with increasing M. The maximum value of current for which the displacement remains bounded is the starting current for the instability.

In order to obtain more accurate results, and to explore the dependence of the starting current on the many adjustable transport parameters, the method described in the preceding section is now being implemented.

Summary

The analysis of Gluckstern, Cooper and Channell has been extended to a multi-cavity recirculating linac with J passes, and an equation derived for Ω, the "frequency" of the "modes" of beam bunch oscillation. The results obtained for 1 cavity agree with those previously obtained.[2] Numerical work applicable to CEBAF is under way.[8]

References

* Work supported by the Department of Energy.

1. H. Herminghaus and H. Euteneuer, Nuclear Instr. and Meth. 163, 299 (1979).

2. A.M. Vetter, Stanford Technical Note HEPL TN-80-2 (July 1980).

3. Gluckstern, Cooper, and Channell, Particle Accelerators 16, 125 (1985).

4. T. Suzuki and K. Yokoya, Nucl. Instr. and Meth. 203, 45 (1982).

5. R.L. Gluckstern and T. Suzuki, KEK Report 83-29 (1984).

6. Conceptual Design Report of the Continuous Electron Beam Accelerator Facility (CEBAF), February 1986, Newport News, VA.

7. Varying deflecting mode frequency is analyzed for a multi-cavity linac in paper TH3-40 at this conference by R.L. Gluckstern, R.K. Cooper and F. Neri.

8. J.J. Bisognano and G. Krafft, paper TH3-9 at this conference.