ACCELERATION OF ELECTRONS BY THE WAKE FIELD OF PROTON BUNCHES†

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In this paper we re-present and discuss more details of a novel idea to accelerate low-intensity bunches of electrons (or positrons) by the wake field of intense proton bunches travelling along the axis of a cylindrical rf structure. Accelerating gradients in excess of 100 MeV/m and large "transformer ratios", which allow for acceleration of electrons to energies in the TeV range, are calculated. A possible application of the method is an electron-positron linear collider with luminosity of 10^{33} cm^{-2} s^{-1}. The relatively low cost and power consumption of the method is emphasized.

Introduction

Recently a considerable amount of attention has been given to the conceptual design of a linear collider for electrons and positrons in the TeV energy range and luminosity of 10^{33} cm^{-2} sec^{-1}. A list of possible parameters for the collider is given in Table 1. It is obvious that the large luminosity can be obtained mainly because of the assumed beam dimensions at the collision point. These dimensions are considerably smaller than those obtained with present technology.

A crucial parameter in the design of the linear collider is the power in the beam to be accelerated, given by the number of particles per bunch, the final energy and the repetition rate. It is important to keep this number reasonably small, possibly around few MWatts, which corresponds to the example given in Table 1. In fact new methods of acceleration are expected to have relatively low efficiency, and the total power required depends on the acceleration efficiency. This fact is so important and overriding that it is the main reason why large luminosities have been proposed with very small beam dimensions.

In order for the linear collider to become a practical reality in the near future, two major considerations are to be taken into account: cost and power efficiency. They will eventually provide the selection for the method to be used. For instance an efficiency of 0.1 % is already too low because it could easily correspond to a total power level of few GWatts.

Linear accelerating structures made of a sequence of metallic rf cavities are certainly among the most promising candidates, since they can support accelerating field gradients in excess of 100 MeV/m. The major problem with these devices is that they have to be operated in the very high frequency range, 30 GHz or more, and it is not easy to find adequate, efficient power sources.

Several ideas have recently been discussed, which involve the use of one or more beams of electrons, tightly bunched, at low energy and high intensity. These beams provide energy to an rf structure which is then used to power the main accelerating linear system where the principal bunches of either electrons or positrons are to be accelerated.[1] It is also possible to stimulate radiation from the beam travelling through wiggler magnets, and to use the radiation at the proper frequency to power the main linear accelerator.[2] In a way, all these methods are reminiscent of the conventional Klystrons; the power of a low-energy, high-intensity electron beam is converted in electro-magnetic power at higher frequency to energize sections of otherwise conventional linear accelerators.

In the past we have proposed a similar concept[3,4] which we wish to expand in this paper, and to correct few misunderstandings at that time. It has been suggested that it is possible to use the same rf structure for both the driver beam and the beam of particles to be accelerated (Beam Transformers),[5] rather then have them travel in separated devices. We concur with this approach, but in our case the primary beam is made of protons, for reasons explained below. We have called the device: the WAKEATRON.

The WAKEATRON

The WAKEATRON is intended to be a linear collider for electrons and positrons with energy of 1 TeV and luminosity of 10^{33} cm^{-2} s^{-1} as shown in Table 1. In this device, electrons and positrons are accelerated on the wake field of intense and relatively long proton bunches. All the beam bunches involved travel along the axis of an rf structure as shown in Fig. 1. The rf structure is made of a sequence of cavities having a gap g, the outer radius b and the inner radius, the region where the beams travel, equal to a.

A possible lay-out of the WAKEATRON is given in Fig. 2. It is made of two parts which are identical to each other but arranged symmetrically around the crossing point where the two beams collide. One part is to accelerate electrons and the other positrons. Each part is made of a proton source which generates tight bunches in a conventional way. There is an electron beam source at one side and a positron beam source at the other. The acceleration of electrons and positrons takes place in the two sections of the WAKEATRON itself which are identical to each other. One proton bunch is extracted from each side and injected into its respective sections of rf structure immediately followed by either an electron or a positron bunch. This will occur at some repetition rate at which all sources are to be adjusted to.

With reference to Fig.1, the driver bunch, made of protons, creates a wake field so that each proton will lose an amount U of energy per unit length. This loss is a function of the location of the proton within the bunch. Because protons have a heavier mass than electrons, and because they lose different amount of energy, they move with respect to each other in a process we have called "mixing". Therefore if the mixing frequency is large enough, one is justified to take an average energy loss $\bar{U}$ per unit length, which is the same for all particles. This average value depends on the rms length of the bunch (σ) and on the geometry of the rf cavities (a,b and g).

The wake field behind the proton bunch has an oscillatory behaviour with positive and negative maxima. Very short bunches of electrons or positrons can be located to correspond with the accelerating peaks, where we can define an accelerating gradient U_{max}. This is larger than the average loss $\bar{U}$ per particle in the driver bunch. The "Transformer Ratio" is defined as the ratio $U_{max}/\bar{U}$. This ratio can be made larger than the factor of two suggested by an earlier theorem,[6] by providing driver bunches with a length comparable to the outer dimension of the rf cavities, and by requiring particles in the driver bunch to "mix" with each other. This justifies and explains our suggestion to use protons as the driving particles.

Table 2 gives a list of parameters for a desired source of intense, short proton bunches at high repetition rate. These parameters are rather close to those of hadron facilities (LAMPF II, TRIUMF II, AGS II, European Hadron Facility,...) that have been investigated and found to be feasible.

We can assume that protons can be decelerated from an initial value of 100 GeV down to 10 GeV, before they are disposed of. Thus each proton will lose an average total of 90 GeV. If the transformer ratio were 11, electrons and positrons can be accelerated to about 1 TeV.

Calculation of the Wake Fields

A point charge, travelling along the axis of the rf structure shown in Fig. 1, excites an infinite sequence of longitudinal modes, the n-th of which described by the wave number k_n and the amplitude w_n. A test particle following at a distance z receives an acceleration rate, that is an amount of energy gained per unit length, given by the wake function

$$w(z) = \sum_n w_n \cos(k_n z) \qquad (1)$$

If we take a bunch of finite length with N particles and linear density f(z), then the acceleration rate for a test particle at a distance z from the center of the bunch is

$$U(Z) = N\sum_n w_n \int_z^\infty f(z') \cos k_n(z'-z)\, dz' \qquad (2)$$

For a gaussian distribution with rms length σ and a particle following at a distance $z >> \sigma$

$$U(z) = N \sum_n w_n e^{-k_n^2\sigma^2/2} \cos(k_n z) \qquad (3)$$

We can also calculate the average energy loss $\bar{U}$ per particle in the driver bunch, that is the average of U(z) given by eq. (2) over the distribution function f(z). We have

$$\bar{U} = \frac{N}{2} \sum_n w_n e^{-k_n^2\sigma^2} \qquad (4)$$

The transformer ratio is then

$$r = 2\,\frac{\sum_n w_n e^{-k_n^2\sigma^2/2} \cos(k_n z)}{\sum_n w_n e^{-k_n^2\sigma^2}} \qquad (5)$$

which is a function of the location of the test particle. In the limit of zero bunch length

$$r = 2\,\frac{\sum_n w_n \cos(k_n z)}{\sum_n w_n} \qquad (6)$$

It is not possible then for this ratio to reach value in excess of 2.

In the limit of long bunches, which is the case we are interested here, it is possible to retain only the lowest mode in the summations at the r.h. side of eq.s (3 and 4). In this case we have

$$U(z) = Nw_o\, e^{-k_o^2\sigma^2/2} \cos(k_o z) \qquad (7)$$

of which the maximum value is

$$U_{max} = Nw_o\, e^{-k_o^2\sigma^2/2} \qquad (8)$$

which we assume to correspond to the location of the test particle. Also

$$\bar{U} = \frac{N}{2} w_o e^{-k_o^2\sigma^2} \qquad (9)$$

and the transformer ratio is

$$r = U_{max}/\bar{U} = 2e^{k_o^2\sigma^2/2} \qquad (10)$$

In the approximation of long bunches, it is then possible to obtain large transformer ratios. Of course the larger is the ratio the lower is the accelerating gradient, and one is forced to a compromise.

We can estimate the wake amplitude w_o and wave number k_o for the fundamental resonating mode of the cavities shown in Fig. 1. In the case a,g << b, we have

$$w_o = \frac{4e^2}{\pi\varepsilon_o b^2} \qquad (11)$$

and

$$k_o = \frac{2\pi}{2.61b} \qquad (12)$$

We have tested our equations with several runs with TBCI,[7] and we have indeed found agreement between the code results and our single mode approximation model. It seems the model breaks down for $\sigma/b < 0.5$.

Mixing

The energy gain or loss depends on the particle position within the bunch. Particles will acquire different momenta and, because of their heavy mass, will exchange position with respect to each other. We have called this: "mixing". A high rate of mixing is required so that in average, that is over a "mixing period", all the particles lose the same amount of energy. It is because of this effect that large transformer ratios, as calculated before,can be justified.

If the bunch length is comparable to the wavelength of the mode excited, there will be locations within the bunch that are "fixed", that is they correspond to zero energy gain or loss. It is possible to calculate the motion of the particles in the proximity of the fixed points, and calculate the mixing angular frequency Ω in the limit of small displacements. This is given by

$$\Omega = c \sqrt{\frac{Nw_o}{\gamma^2 E} f(\bar{z})} \qquad (13)$$

where $f(\bar{z})$ is the linear density at the location $\bar{z}$ of the fixed point. For a gaussian distribution with centre at z = 0

$$f(z) = \frac{\exp(-z^2/2\sigma^2)}{\sqrt{2\pi}\ \sigma} \qquad (14)$$

We expect the shape of the driver bunch to change during the deceleration. Its average energy E, which appears also in the expression (13) for the mixing frequency, will continuously decrease. Therefore the mixing rate, the transformer ratio and the rate of energy loss and gain will also vary. We have verified this with computer codes that simulate both the driver as well as the test beam motion. To compensate for the changing bunch shapes, it may be possible to restore the initial rates by modifying the dimensions of the following rf cavities accordingly.

A Conceptual Design

To achieve large acceleration gradients, one wants cavities with small outer radius b, as one can see from Eq. (11). We propose the following dimensions:

$$a = 1 \text{ mm}$$
$$b = 4 \text{ mm}$$
$$g = 2 \text{ mm}$$

If we take 3×10^{11} protons in a bunch as proposed in Table 2, then the maximum accelerating gradient is

$$Nw_o = 432 \quad \text{MeV/m}$$

For large transformer ratios one has to adjust the bunch length accordingly. A list of possible combinations is given in Table 3. The bunch length is usually small and this, because of the large number of particles involved, is a matter of concern. For instance a transformer ratio of 10 which would allow a final electron beam energy close to 1 TeV, requires $\sigma = 3$ mm.

We report in Table 2 the performance of two proton sources. The first is the one we require for the WAKEATRON driver, and the second is the proposed European Hadron Facility (EHF). It is possible to start from the latter to which then one adds an extra ring to accelerate protons from 30 to 100 GeV. A possible way to shorten the bunches is to adjust the transition energy of the last ring to about the beam energy at extraction.

The power efficiency of the proton source is the ratio of the final, average power in the proton beam to the total power required to operate the facility. This efficiency is already about 10 %, a relatively large figure, for the EHF and we expect an even larger number for the WAKEATRON driver. This is an interesting and useful feature: the larger the proton beam current the more efficient is the source. Indeed, one can reach a situation where most of the power involved is in the beam and in the rf system that provides acceleration, usually in a one-to-one ratio. The power of the magnet system does not depend on the beam current and will become only a relatively small fraction of the total.

The cost of the source can also be estimated. It is relatively modest compared to larger enterprises like, for instance, the SSC, the 20 × 20 TEV^2 proton-proton super collider.

Table 3 summerizes our results, based on the single mode approximation. For a given transformer ratio, we have estimated the required rms bunch length, the maximum accelerating gradient U_{max} and the average loss rate $\bar{U}$ per particle in the driver bunch. Assuming that protons are decelerated from 100 down to 10 GeV, we have also calculated the total energy gain ΔE_{final} for beams of electrons and positrons as well the distance L that all the beams have to travel, which then gives roughly the length of the WAKEATRON. We show the average power P_e of the electron (positron) beam for 5×10^9 particles in a bunch at the repetition rate of 5×10^3 bunches per second as shown in Table 1. The energy transfer efficiency from one beam to the other is given by the ratio P_e/P_p, where P_p is the average power in the proton beam which we have taken to be 24 MW, as also shown in Table 2. The transfer efficiency is quite large and useful. This can be thought as the equivalent of the conventional klystron efficiency, and it is actually the energy recovery efficiency of the proton beam. The overall efficiency is obtained by multiplying the transfer efficiency with the efficiency to operate the proton source, which, as also shown in Table 2, we assume to be 24 %. The overall efficiency is given on the last column of Table 3. For a transformer ratio of 11, it is an interesting and useful 4 %.

We have chosen the energy of the proton beam large enough so that it is possible to accelerate electrons to 1 TeV with a low transformer ratio of 10-12. Also we expect that the length of the proton bunches decreases with the energy, and one requires short bunches, as we have seen. On the other hand, the mixing frequency decreases with the beam energy; and if one wants to take advantage of the mixing process, then he is forced to a compromise. We have calculated the mixing period T at the initial energy of 100 GeV for an rms bunch length of 3 mm and the parameters described above. We have found a reasonable 3 μsec, which corresponds to a travel length of about 1 km. Preliminary results of computer simulations of the particle motion show that indeed this mixing period is adequate.

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Table 1. Electron-Positron Linear Collider

Energy per Beam	1	TeV
No. of Particles / Bunch	5×10^9	
Frequency of Encounter	5	kHz
Normalized Rms Emittance, σ^2/β^*	1×10^{-6}	m-rad/γ
β^*	5	mm
Rms Beam Spot Size, σ	500	A°
Rms Bunch Length	2	mm
Disruption Parameter, D	3	
Luminosity Enhancement Factor, H	3	
Energy Spread from Beamstrahlung, $\Delta E/E$	4	%
Luminosity	1×10^{33}	$cm^{-2}\ sec^{-1}$
Beam Power	2×4	MWatt

Table 2. Parameters for Proton Sources

	WAKEATRON Driver		HEF	
Energy	100	GeV	30	GeV
Average Current	240	μAmp	100	μAmp
No. of Protons / Bunch	3.0×10^{11}		2.8×10^{11}	
Bunch Extraction Rate	5.0	kHz	2.25	kHz
Rms Bunch Length	0.3	cm	20.	cm
Beam Average Power	24	MW	3	MW
Total Power for Source	100	MW	30	MW
Power Efficiency	24	%	10	%
Cost of the Facility	400	M$	300	M$

Table 3. WAKEATRON Performance versus Transformer Ratio

r	σ/b	U_{max} MeV/m	$\bar{U}$ MeV/m	ΔE_{final} TeV	L km	P_e MW	P_e/P_p %	P_e/P_{total} %
2	0.000	432.0	216.00	0.18	0.4	0.72	3.	0.7
5	0.562	172.8	34.56	0.45	2.6	1.80	7.5	1.8
8	0.691	108.0	13.50	0.72	6.7	2.88	12.	2.9
10	0.745	86.4	8.64	0.90	10.4	3.60	15.	3.6
13	0.803	66.5	5.11	1.17	17.6	4.68	19.5	4.7
16	0.847	54.0	3.38	1.44	26.7	5.76	24.	5.8
20	0.891	43.2	2.16	1.80	41.7	7.20	30.	7.2
25	0.933	34.6	1.38	2.25	65.0	9.00	37.5	9.0
30	0.966	28.8	0.96	2.70	93.8	10.80	45.	10.8
36.35	1.000	23.8	0.65	3.27	137.4	13.08	54.5	13.1

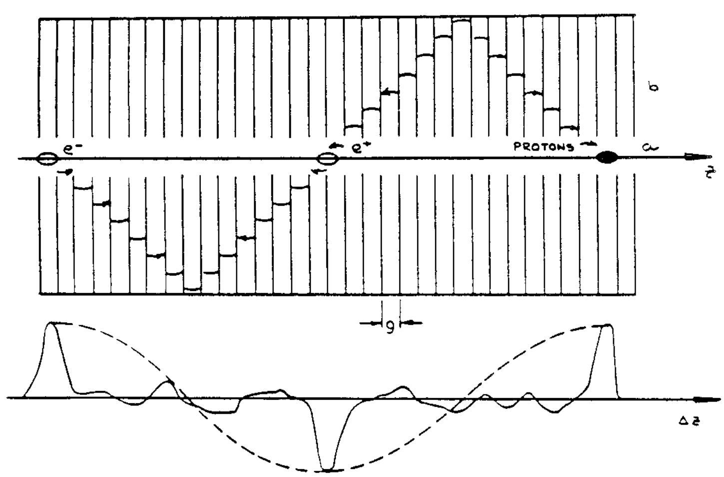

Fig. 1 Section of the WAKEATRON

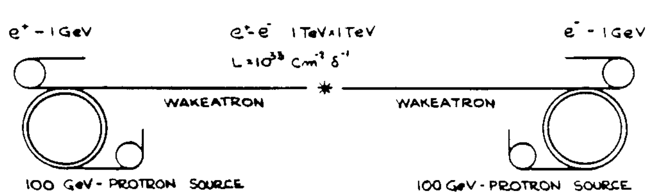

Fig. 2. The Linear Collider based on the principle of the WAKEATRON