CAVITY AND WAVEGUIDE DESIGN BY TRIANGULAR MESH CODE URMEL-T

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ABSTRACT

Computer codes are increasingly essential for the design of RF accelerating cavities and waveguides. The code URMEL-T uses a triangular mesh and the FIT discretization method [1], [2], i.e. a finite difference method. Going beyond the capability of the most highly developed codes, URMEL-T deals with cavities of cylindrical or translationally invariant symmetry and waveguides - each with arbitrary dielectric and/or permeable material insertions. The theory underlying the code transforms Maxwell's equations via difference equations into a linear algebraic eigenvalue problem which has to be solved for only a small number of lowest eigenvalues. The solution provides fields, eigenfrequencies (propagation constants for waveguides) and the usual quality factors, voltages and other shunt impedance-related quantities.

INTRODUCTION

Cylindrical Cavities

In cavities with cylindrical symmetry the electromagnetic fields are periodic in the azimuthal variable (φ) with period 2π. This fact and the harmonic time dependence of the fields allow their description by a Fourier series:

$$\vec{F}(r,\varphi,z,t) = \sum_{m=0}^{\infty} Re \{\{ \underline{F}_r(r,z)\vec{e}_r + \underline{F}_\varphi(r,z)\vec{e}_\varphi + \underline{F}_z(r,z)\vec{e}_z\} e^{im\varphi} e^{i\omega t}\}, \qquad (1)$$

with the complex magnitude $\underline{\vec{F}} = \underline{\vec{E}}$ or $\underline{\vec{H}}$ and the unit vectors $\vec{e}_r$, $\vec{e}_\varphi$, $\vec{e}_z$ in r,φ,z direction. Furthermore the materials are assumed loss free, i.e. ϵ, μ are real and the conductivity is equal to zero. So we may write

$$\vec{E} = \sqrt{Z_0}\sin\omega t\vec{E}', \quad \vec{H} = \sqrt{Y_0}\cos\omega t\vec{H}', \qquad (2)$$

with $Z_0 = \sqrt{\mu_0/\epsilon_0}$, $Y_0 = \sqrt{\epsilon_0/\mu_0}$, $c = 1/\sqrt{\mu_0\epsilon_0}$.

Then, with $k = \omega/c$, Maxwell's equations are given by :

$$\begin{aligned} curl\,\vec{H}' &= \epsilon_r k\vec{E}', \\ curl\,\vec{E}' &= \mu_r k\vec{H}'. \end{aligned} \qquad (3)$$

The azimuthal dependence $e^{im\varphi}$ leads to several groups of modes:

$m = 0$: "TE-" or "H-"modes with $\vec{E} = (0, E_\varphi, 0)$

$m = 0$: "TM-" or "E-"modes with $\vec{H} = (0, H_\varphi, 0)$ (accelerating)

$m > 0$: these so-called deflecting or transverse modes are excited by off-axis particles

Waveguides

In waveguides which do not change their characteristics in the z-direction (cartesian coordinate) waves with pure exponential dependence on z are proper solutions of Maxwell's equations. Consequently we may write for waves travelling in the negative z-direction

$$\vec{F}(x,y,z,t) = Re\{\{ \underline{F}_x(x,y)\vec{e}_x + \underline{F}_y(x,y)\vec{e}_y + \underline{F}_z(x,y)\vec{e}_z\} e^{i\beta z} e^{i\omega t}\}, \qquad (4)$$

with the complex magnitude $\underline{\vec{F}} = \underline{\vec{E}}$ or $\underline{\vec{H}}$ and the cartesian unit vectors $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$. With the substitution of equation (2) we again obtain equation (3) for Maxwell's equations.

DISCRETIZATION

Because of the cylindrical symmetry of the cavity (respectively the z-independence of the waveguide shape) the azimuthal dependence (or the z-dependence) of the fields can be taken out of the numerical computation (compare [1], [2],[3]) and a two dimensional grid is sufficient.

The basic ideas of the FIT method have been transferred to a triangular mesh. This mesh has the advantage of approximating well the cavity or waveguide geometry even for elliptical or circular structures with relatively coarse grids.

Allocation of the field components to the grid

As a main idea of the FIT method two dual grids are used - one for the electric and one for the magnetic field components. Different components associated with one mesh point are allocated at different locations of the grid. The characteristic and the advantage of this allocation is the preservation of the interrelation between the integrals over areas and the line integrals in Maxwell's equations and the continuity at material boundaries. Field components on the triangular mesh are denoted by F, the ones on the dual mesh by $\tilde{F}$

For the triangular mesh we have to distinguish two cases:

1. If all triangles of the mesh inside the cavity have angles less than or equal to $\pi/2$ we choose as the dual mesh lines the perpendicular bisectors of the sides (dual mesh $\mathbf{G_M}$).

2. If the automatic mesh generator cannot avoid triangles with an angle over $\pi/2$ the centres of mass are taken as the dual mesh points (dual mesh $\mathbf{G_S}$). It should be noted that the field component $\tilde{F}$ in this dual mesh is in general not perpendicular to the field component F on the triangular mesh.

The triangular mesh (denoted as **G**) contains two kinds of triangles alternating in the rows: one has the vertex on top while the other one is standing upright. One of each kind is associated to each mesh point k.

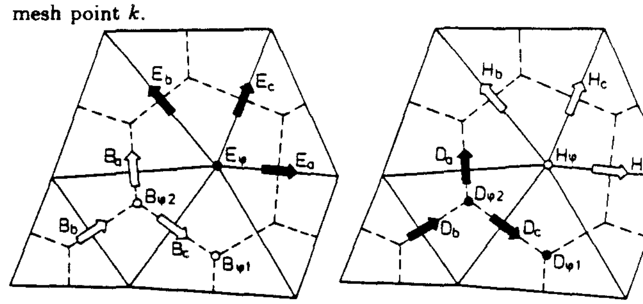

Figure 1 : Allocation of the field components

Allowed material properties in the different grids

As mentioned above, the permittivity and the permeability shall be real. For $\mathbf{G_M}$ as the dual mesh materials may be inserted with μ_r and ϵ_r varying from triangle to triangle and either E or H could be chosen for F. Only continuous components occur at the triangle boundaries, i.e. tangential E and normal B (respectively tangential H and normal D).

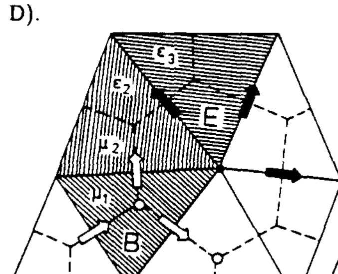

Figure 2 : E on **G**, B on $\mathbf{G_M}$, μ_r, ϵ_r varying

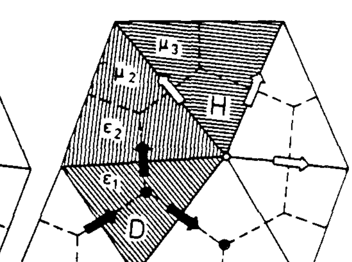

Figure 3 : H on **G**, D on $\mathbf{G_M}$, μ_r, ϵ_r varying

If only the dual mesh $\mathbf{G_S}$ can be used we have to place a restriction to assure the continuity of the field components: Only insertions with constant μ_r but varying ϵ_r, or constant ϵ_r but varying μ_r are allowed and F will be taken as shown in figures 4 and 5 so that all components are continuous. In figure 4 it is seen that (continuous) tangential E occurs at the triangle interfaces, and so ϵ_r can vary from triangle to triangle. However since on the dual mesh the magnetic field component is not normal to the interfaces, μ_r cannot vary from triangle to triangle . Similarly, in figure 5 it can be seen that, with H on the mesh G, μ_r can vary whereas ϵ_r cannot. The constants μ_0 and ϵ_0 shown in figure 4 and 5 stand here in fact for constant μ_r or constant ϵ_r all over the cavity or waveguide.

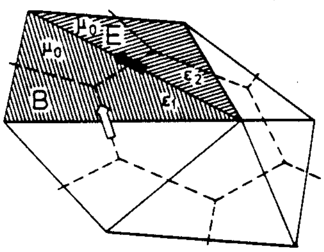

Figure 4 : E on G, B on $\mathbf{G_S}$, μ_r constant, ϵ_r varying

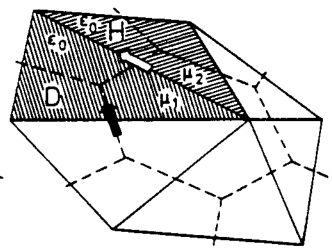

Figure 5 : H on G, D on $\mathbf{G_S}$, μ_r varying, ϵ_r constant

Deflecting modes, $m > 0$

Maxwell's equations (3) in integral form are solved for a chosen $m > 0$ in the following way, which is described in more detail in [4]:

- They are discretized on the triangular mesh G with $\mathbf{G_S}$ (respectively $\mathbf{G_M}$) for $(\vec{F}', \vec{F}') = (\vec{E}', \vec{B}')$ or $(\vec{H}', \vec{D}')$. The boundary conditions are included in the discretization.
- In the resulting equations all $\vec{F}'$-components and the azimuthal $\vec{F}'$-component F_φ are eliminated by substitution leading to difference equations connecting each field component F_a, F_b, F_c with ten neighbours.

The equations which are given for each mesh point n correspond in their matrix representation to a linear algebraic eigenvalue problem. Thus an eigenvalue problem

$$A\vec{x} = k^2\vec{x} \tag{5}$$

remains to be solved. Its eigenvalues are the squared wave numbers of the resonant frequencies ($k = \omega/c$) and the eigenvectors

$$\vec{x} = (F_{a,1}, \ldots, F_{a,N}, F_{b,1}, \ldots, F_{b,N}, F_{c,1}, \ldots, F_{c,N})^T \tag{6}$$

give the corresponding electric and magnetic fields (N = number of mesh points). Here an advantage of the FIT method becomes obvious: This method solves a linear problem and does not need an estimation of the frequency sought. The 3N x 3N matrix of the eigenvalue problem is sparse with only a few off-diagonals.

Monopole modes, $m = 0$

Here we have to distinguish the TE0- and TM0-modes. Maxwell's equations are solved for m=0 as follows:

- They are discretized on the triangular mesh with $(\vec{F}', \vec{F}') = (\vec{E}', \vec{B}')$ for TM-modes and $(\vec{F}', \vec{F}') = (\vec{H}', \vec{D}')$ for TE-modes.
- In the resulting equations all $\vec{F}'$-components are eliminated . $\vec{F}_{\varphi 1}$ and $\vec{F}_{\varphi 2}$ are the only non-zero $\vec{F}'$-components. In this case the difference equations connect each field component $\vec{F}_{\varphi 1}$ and $\vec{F}_{\varphi 2}$ with three neighbours.

As for $m > 0$ a linear eigenvalue problem (5) is to be solved. The eigenvectors are now

$$\vec{x} = (\vec{F}_{\varphi 1,1}, \ldots, \vec{F}_{\varphi 1,N}, \vec{F}_{\varphi 2,1}, \ldots, \vec{F}_{\varphi 2,N})^T. \tag{7}$$

The 2N x 2N - matrix A is sparse, can be made symmetric and has only three off-diagonals.

Waveguides and cavities of translational invariance

The difference equations for the waveguide problem are similar to those for the deflecting modes. Instead of a certain azimuthal dependence we have the longitudinal dependence

$$\vec{E}(x, y, \Delta z) = \vec{E}_0(x, y)e^{i\beta\Delta z} \doteq \vec{E}_0(x, y)(1 + i\beta\Delta z) \tag{8}$$

with the propagation constant β. The same is true for cavities that do not change their geometry in z-direction for $0 \leq z \leq L$ where L is the longitudinal length of the cavity.

Again Maxwell's equations are written in integral form and discretized in full analogy to the case of deflecting modes with $(\vec{F}', \vec{F}') = (\vec{E}', \vec{B}')$. Then all $\vec{B}'$- components and the longitudinal $\vec{E}'$-component E_z are eliminated . The resulting linear algebraic eigenvalue problem has the squared propagation constants for a given frequency ω as eigenvalues.

This option of URMEL-T renders it possible to compute the functional relationship between the frequency and the propagation constant for e.g. dielectric loaded waveguides.

EXAMPLES

We will present calculations for several realistic cavities and waveguides with and without material insertions.

Dielectric loaded cavity

The computed frequency shift caused by a Teflon cylinder (dielectric constant $\epsilon_r = 2$) inserted in a DORIS-cavity shows a good agreement with measured data [5]:

Table I : Frequency shift by a small Teflon cylinder inserted in a DORIS-cavity (N=968)

mode	calculated original frequency	measured original frequency	calculated frequency shift	measured frequency shift
TM010	498.406 MHz	498.488 MHz	6.356 MHz	6.614 MHz
TM011	739.302 MHz	745.667 MHz	7.419 MHz	8.399 MHz
TM110	775.286 MHz	775.870 MHz	4.606 MHz	5.980 MHz

The reasonable difference found is due to the slightly modified cavity shape taken for the URMEL-T calculation where the flanges are left out.

Cavity loaded with a ferrite ring

As example for permeable material insertions we choose a cavity with an inserted ring made of ferrite with $\mu_r = 1.5$ and $\epsilon_r = 14.5$. The figures show field-plots of the lowest dipole modes.

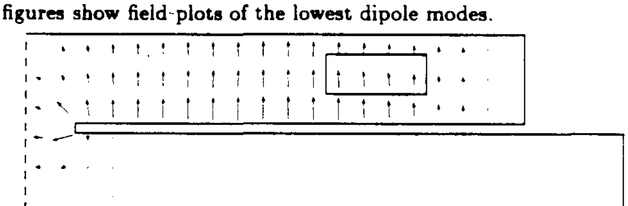

Figure 6 : Electric field at $\varphi = 0$ for the mode 1-EE-1 with f=417.17 MHz.

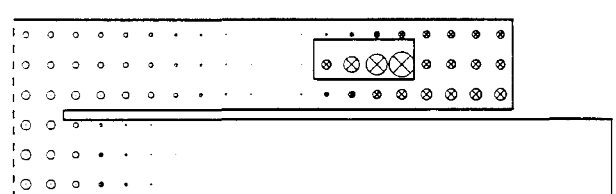

Figure 7 : Magnetic field at $\varphi = 0$ for the mode 1-EE-1.

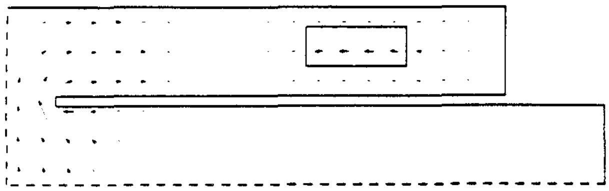

Figure 8 : Magnetic field at $\varphi = \pi/2$ for the mode 1-EE-2 with f=599.10 MHz.

Waveguide

Figure 9 shows a cut through the dielectric waveguide together with a triangular mesh. The corresponding transverse fields for the fundamental mode with frequency 3 GHz are plotted in figure 10.

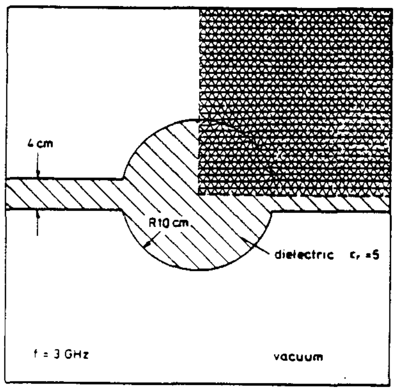

Figure 9 : Dielectric waveguide with mesh for the part which is essential for a run of URMEL-T

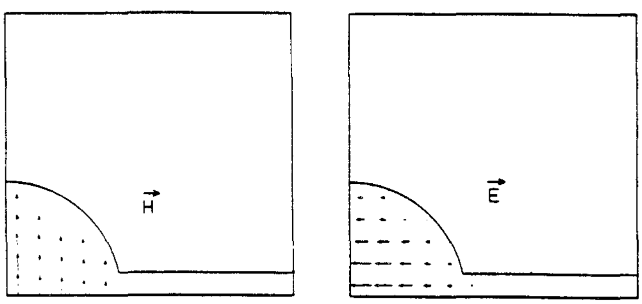

Figure 10: Field maps for the fundamental wave

A multi-cell cavity

To simulate the influence of the beam pipe it is usual to tune the last cell of a multi-cell cavity. The triangular mesh is able to follow even very small deviations between the radii of the middle cells and the outer cell while a rectangular mesh needs a very fine mesh which causes a steeply increasing number of mesh points in rectangular grids the finer the radial deviation is.

As illustration the tuned 1GHz DESY nine-cell superconducting cavity is shown. The radius of the middle cells is 139.595 mm while the last cell has a radius of 138.345 mm. We show the cavity together with the triangular mesh of 1960 points and contours of $r \cdot H_\varphi = const$ of the π mode. These lines show the direction of the electric field. Their density is proportional to $r \cdot \vec{E}$.

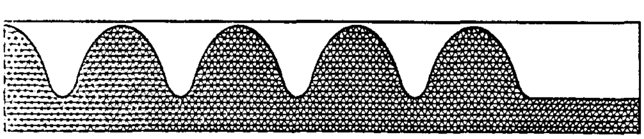

Figure 11: Tuned DESY nine cell cavity with mesh used

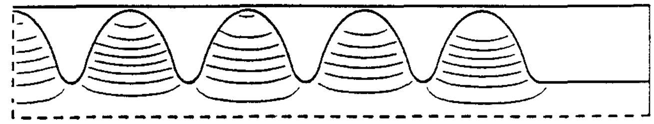

Figure 12: Contours of constant $r \cdot H_\varphi$ for the π-mode of the tuned nine cell cavity. Measurements [6] with a slightly different cavity gave 1000.1MHz as frequency, URMEL-T calculated 1007.5 MHz.

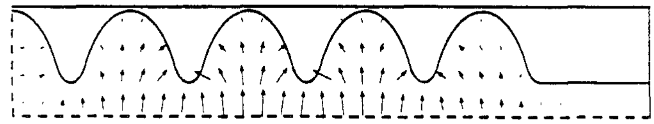

Figure 13 : Arrow plot of the electric field for the "TE111-$2\pi/9$"-like mode. The measurements gave 1271.2 MHz as frequency, URMEL-T calculated 1289.8 MHz.

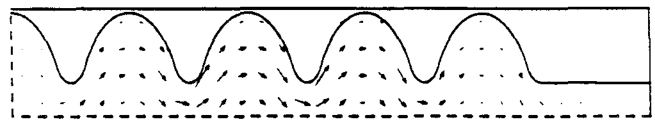

Figure 14 : Arrow plot of the magnetic field for the "TE111-$2\pi/9$"-like mode.

SUMMARY

The computer code URMEL-T enlarges the two dimensional scope of application of the FIT discretization method in two directions:

- First it allows the calculation of resonant modes (including the TE0-modes) in cylindrically or translationally symmetric cavities with dielectric and/or permeable insertions as well as the calculation of propagation constants in waveguides. Herewith URMEL-T offers a new feature in the domain of computational evaluation of RF-fields.

- Second URMEL-T is well suited to structures with elliptical or circular parts in their geometry, and for tuning multi-cell cavities. This is based on the properties of a triangular mesh combined with the powerful FIT method. The latter ensures that the solutions fulfill all Maxwell's equations, i.e. they are physically significant.

So URMEL-T presents a widely useful extension of the program group for the solution of Maxwell's equations.

ACKNOWLEDGEMENT

The authors wish to thank Richard K. Cooper from the Los Alamos National Laboratory for many useful suggestions and for careful reading of the manuscript.

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