BEAM DYNAMICS IN LONG TRANSIT TIME LINAC CAVITIES FOR HEAVY ION BOOSTERS

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Abstract

A new accurate and relatively fast formalism has been derived for beam dynamics through long accelerating elements. This work was initiated by the development of heavy ion booster cavities, in particular of helix type which present no exact symmetry.

Introduction

The early linacs [1] made use of the so called Panofsky equation :

$$\Delta W = qV_0 T\cos\phi \qquad (1)$$

with T, transit time factor

$$T = \sin(kg/2)/(kg/2) \text{ where } k = \omega/v \qquad (2)$$

ω angular rf frenquency, v velocity of the particle. Such an expression of T was later improved [2] by multiplying it by :

$$I_0(\kappa r)/I_0(\kappa a) \text{ with } \kappa^2 = k^2 - (\omega/c)^2$$

r distance of the trajectory to the axis and correcting g into g_c (see fig. 1)

$$g_c = g + 0.85\rho_c \qquad (3)$$

(quite accurate empirical formula).

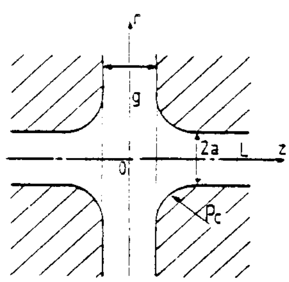

Fig. 1. Classical gap.

In the 60's more elaborate theories [3] were derived giving a general definition of T and justifying the use of a "thin lens" method for the treatment of a gap.

The field distribution on the axis of a cavity is obtained for a given cell geometry from computer codes [2] which simultaneously can give the value of integrals like :

$$V = 2\int_0^L E_z(z)\,dz \quad , \quad VT(k_s) = 2\int_0^L E_z(z)\cos k_s z\,dz$$

$$VT'_k(k_s) = -2\int_0^L zE_z(z)\sin k_s z\,dz \;, \;\ldots \qquad (4)$$

$$VS(k_s) = 2\int_0^L E_z(z)\sin k_s z\,dz \;,\; VS'_k(k_s) = \int_0^L zE_z(z)\cos k_s z\,dz \qquad (5)$$

$$\text{with } k_s = \omega/v_s \qquad (6)$$

v_s being the nominal velocity of the beam when crossing the gap. The actual values of T are then obtained from Taylor's expansion around that velocity with the help of the derivatives given above.

Longitudinal and transverse "thin lens" equations are now given by :

$$\Delta W = qVT(k)I_0(xr)\cos\varphi + qV\frac{\partial}{\partial k}[T(k)xI_1(xr)]\,r'\sin\varphi$$

$$\Delta\varphi = \frac{qV}{2W}k\frac{\partial}{\partial k}[T(k)I_0(xr)]\,\sin\varphi - \frac{qV}{2W}k\,\frac{\partial^2}{\partial k^2}[T(k)xI_1(xr)]r'\cos\varphi \qquad (7)$$

$$\Delta r' = -\frac{qV}{2W}T(k)\frac{x}{k}I_1(xr)\sin\varphi + \frac{qV}{2W}\{\frac{\partial}{\partial k}[T(k)\frac{x^2}{k}I'_1(xr)] - T(k)I_0(xr)\}r'\cos\varphi$$

$$\Delta r = -\frac{qV}{2W}\,\frac{\partial}{\partial k}[T(k)\frac{x}{k}I_1(xr)]\,\cos\varphi - \frac{qV}{2W}\{\frac{\partial^2}{\partial k^2}[T(k)\frac{x^2}{k}I'_1(xr)]\,\sin\varphi - \frac{\partial}{\partial k}[T(k)I_0(xr)]\,\sin\varphi\}r' \qquad (7)$$

equations which refer to a trajectory which crosses the mid gap plane at velocity v (different from v_s), phase ϕ, distance r from the axis and slope r'.

The term $\Delta\phi$ gives to the first two equations the Hamiltonian character, since they derive from the generating function :

$$H = -qVT(k)I_0(xr)\sin\varphi + qV\,\frac{\partial}{\partial k}[T(k)xI_1(xr)]r'\cos\varphi \qquad (8)$$

Liouville's theorem is then satisfied to first order in qV/W. It is also satisfied in rr' plane.

One difficulty of this formalism is that mid gap values are needed. These ones are obtained from another set of intrinsic equations, making use of the S coefficients ; for instance one can take, forgetting r' contribution (mg mid gap, i thin lens input).

$$W_{mg} - W_i = qVI_0(xr)[T(k)\cos\varphi - S(k)\sin\varphi]/2 \qquad (9)$$

Such expressions, derived from eq. (4) and (5) assume the symmetry of the gaps with respect to to their mid plane.

This formalism proved to be very accurate (better than 1 %) ; it is currently used in computer codes [4]. An estimate of the residual error could be obtained [3] from the second order term of the perturbation theory which is the basis of the treatment [5]. Unfortunately, the expression resulting from the use of Fourier integrals required the use of approximations only valid for short gaps as normally used, with g of the order of $\beta\lambda/4$ ($kg \sim \pi/2$) and a field E_z on the axis extending over less than $\beta\lambda/2$ (transit time < π).

New accelerating devices

Apart from the incidental use of proton linacs to accelerate α's, deuterons or other particles at half velocity, heavy ion accelerators often use structures in a wide range of velocities to cope with different particles and different energies.

While GSI use reentrant cavities, recent superconducting machines adopt spiral ring or split ring resonators with two or three gaps ; short helices λ or $\lambda/2$ long, with no gap and possibly no exact symmetry (fig. 2 and 3) constitute another class.

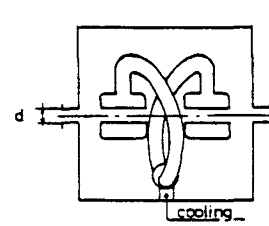

Fig. 2. Split ring resonator.

The treatment of acceleration through such devices introduces several differences with respect to the one described :

1) each cavity is to be used not only around a nominal v_s (or k_s) but in a wide range (factor 2 or 3 for instance),

2) even if multigap cavities can be treated as a succession of individual gaps, helices are not relevant to that method (in addition to an E_0 type field there is in a helix some amount of H_0 component ; this one is not accelerating and is not considered here),

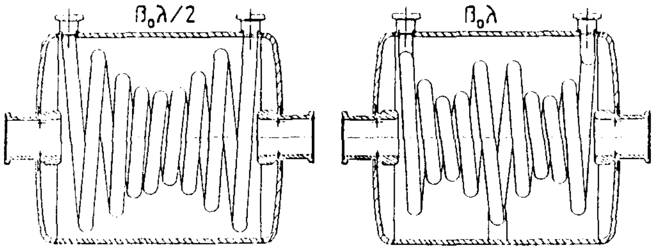

Fig. 3. Helix resonators.

3) symmetry is not necessarily present as shown by experimental measurements on helices.

4) transit time through such devices when treated as a whole is much longer than π and can go up to 4π or $5\,\pi$.

Field distribution in long cavities used in a wide range of velocities

In a cavity where the axial E_z field is obtained either from computer or experimentally, one can express it in the form of Fourier series satisfying all the continuity conditions of the EM field. When there is a symmetry in z, the mid point can be taken as origin and the expression has the form :

$$E_z(z) = A_0 + \sum_{j=1}^{N} A_j \cos j\pi \frac{z}{L_0} \qquad (10)$$

with $2L_0$, total lenght of the cavity. Usually, less than 10 coefficients are enough to obtain a good accuracy.

Such a type of Fourier series expression can of course be obtained for any period length $2L \geq 2L_0$. If L is then chosen such that $kL = n\pi$ with n small integer, one can compute.

$$[V]T(k)=2\int_0^{L_0} E_z(z)\cos kz\, dz$$

$$= \frac{A_0}{k}\sin k\frac{L_0}{2} + \frac{1}{2}\sum_{j=1}^{N} A_j\left[\frac{\sin(kL_0+j\pi)}{k + j\pi/L_0} + \frac{\sin(kL_0-j\pi)}{k - j\pi/L_0}\right] \qquad (11)$$

and consider $\boxed{V}$ T (k) as the n^{th} component of a new Fourier series representing the field (for classical single gaps one would have $n = 1$) ; $\boxed{V}$ is an arbitrary normalization factor, **T being dimensionless.** Fig. 4 shows typical values of $\boxed{V}$ T(k) for a 3 gap system.

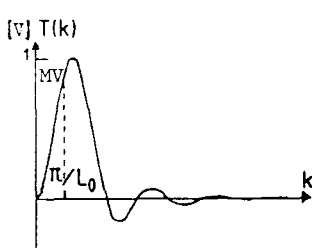

Fig. 4. $\boxed{V}$ T(k) for a λ Helix or 3 gap cavity.

One can compute similarly $\boxed{V}$ T'(k) ... $\boxed{V}$ S(k) ... : $\boxed{V}$ S(k) refers to an antisymmetrical field where E_z is the image of the actual field for $z < 0$ (for the computation of mid cavity values).

Accurate second order terms

With the Fourier series expression, the second order terms of the perturbation method which is the basis of the derivation $\boxed{5}$ take the form of series instead of integrals. Since, with a small n (particle Fourier component), only a few terms of the Fourier spectrum are of large amplitude and since in the second order correction the square of the term index j appears in the denominator of the various terms of the series, this correction, in particular for energy, is quite tractable in a computer code [(a)].

For ΔW, for instance, it writes :

$$\Delta W_{cor} = \frac{q^2[V]^2 n\sin 2\varphi}{8W\pi}\Big[2k\frac{\partial T(k)}{\partial k}\sum_{j=0,\neq n}^{\infty}\frac{T(\frac{j}{n}k)}{j^2-n^2} - 2T(k)\sum_{j=0,\neq n}^{\infty}\frac{T(\frac{j}{n})(j^2+n^2)}{(j^2-n^2)^2}$$

$$+\sum_{j=0,\neq n}^{\infty}\frac{T(\frac{|2n-j|}{n}k)T(\frac{j}{n}k)}{(j-n)^2} + \sum_{j=0}^{\infty}\frac{T(\frac{2n+j}{n})T(\frac{j}{n}k)}{(j+n)^2} - \frac{T(k)^2}{4n^2} \qquad (12)$$

$$-\frac{\partial T(k)}{\partial k}T(k)\frac{k}{2n^2} + \frac{\partial^2 T(k)}{\partial k^2}T(k)\frac{k^2}{2n^2}\Big]$$

This correction results in very accurate values. In practice, even for very long accelerating structures, this second order correction has only to be applied to ΔW since it turns out to be extremely small for $\Delta\phi (\lesssim 0.1)$ and since Δr, $\Delta r'$ corrections would be meaningless, being much smaller than what results from any practical misalignment.

Small additional ΔW and $\Delta\phi$ terms insure Liouville up to 2nd order.

Asymmetry of field distribution

An easy extension of the previous formalism is the treatment of an antisymmetrical structure like a two gap cavity.

The addition of some amount of the other parity to either symmetrical or antisymmetrical field gives the possibility, by about doubling the computing time, to treat an accelerating cavity having some asymmetry. Such a treatment would of course still make use of mid cavity values in a "thin lens" formalism.

Treatment using input values

The extra complication resulting from an asymmetry and the excellent quality of second order corrections led to the idea of treating the full accelerating cavity (2 or 3 gaps or helix) as the second half of a double length one (sum of symmetrical and antisymmetrical fields, see fig 5), using the mid cavity calculation formalism with the input values (second order corrections have of course to include T and S terms, equal together, plus cross TS = ST terms).

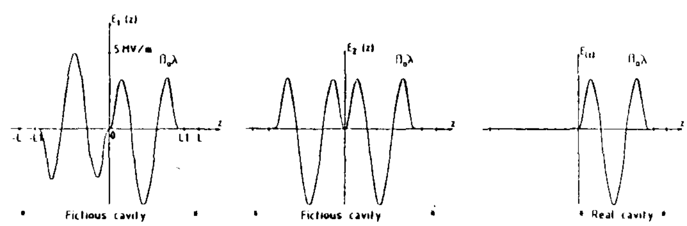

Fig. 5. Double cavity method. Field decomposition.

This method has been tested for the heavy ion Saclay booster. Taking an extreme case of very slow particles badly accelerated in a λ helix, fig. 6a shows as function of input phase :

- the acceleration obtained from a 64 step integration routine using predictor-corrector method, accurate but lengthy (32 steps did not preserve Liouville's theorem, 128 steps still give a slightly different result). (see Fig.7 which shows the evolution of v, r' and r along the cavity).
- the remaining error when using only first order formalism.
- and when using 1st and 2nd order terms.

Input values are directly used ; one can observe the non sinusoidal variation of the energy gain and the

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(a) It appeared that a good method to check the exactness of the correction terms computation is to change the value of n : the results must remain constant.

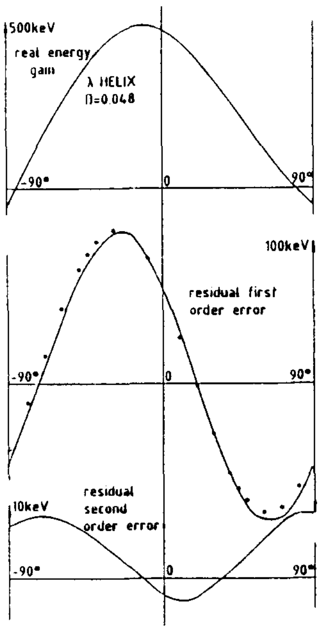

Fig. 6a

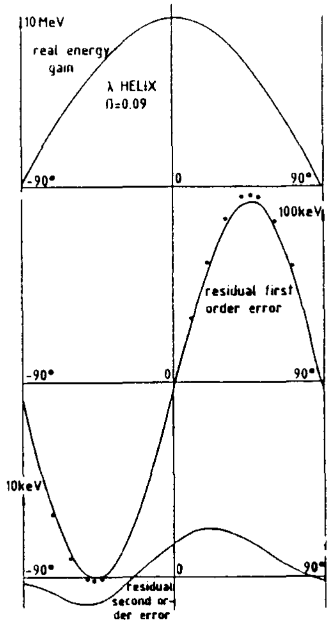

Fig. 6b

Fig. 6. Accuracy when using 1st and 2nd order computations : Energy gain as function of input phase ; 6a) very low velocity particles, 6b) average velocity.

small 2nd order residual error, of the same order as the numerical one.

Fig. 6b shows similar results of the same cavity for faster particles (maximum energy gain).

Used in a Monte Carlo 6d simulation (using 100 particles) of the 48 cavity linac, fig. 8 shows the input and output longitudinal phase space plots and the r.m.s. emittance values : one can observe the emittance growth (the adjustment is not optimized).

Such a computation, including many intermediate outputs with r.m.s. computations is 5 to 10 times faster than the step by step routine which does not satisfy as well Liouville's theorem.

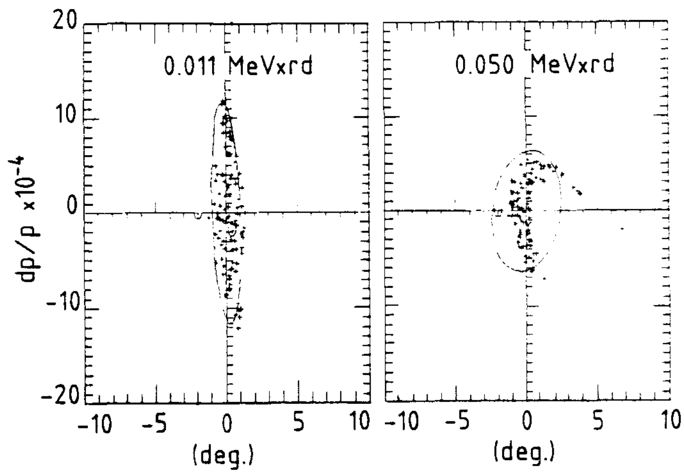

Fig. 8. Example of longitudinal emittance plots in input and output of the Saclay booster with canonical area values.

Conclusion

A new formalism for beam dynamics through long cavities with no symmetry required and making a direct use of input coordinates has been derived and tested ; it proved to be extremely satisfactory up to cavity lengths of a least 2.5 $\beta\lambda$. Used for helix structures it could be applied as well to 2 or 3 gaps structures of any kind. Detailed computations are given in ref. [6].

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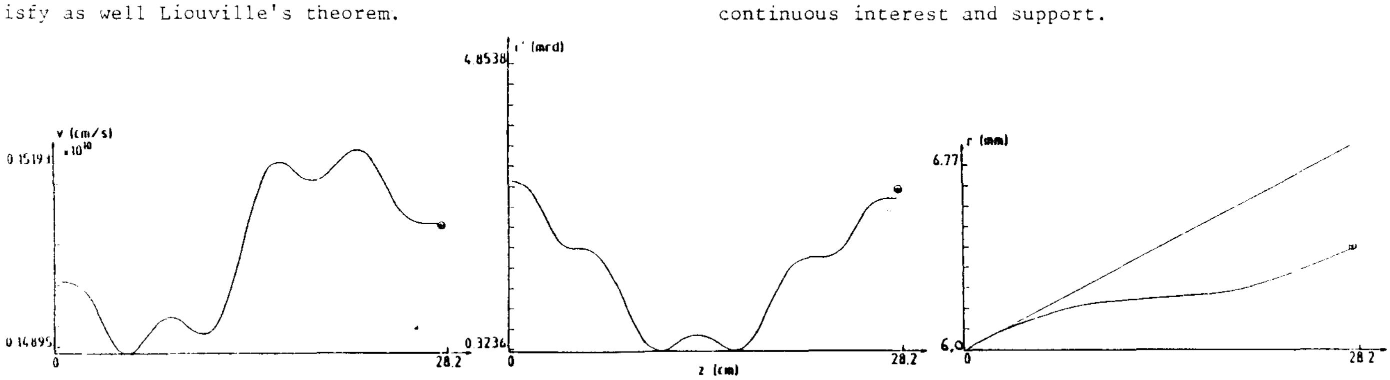

Fig. 7. Evolution of velocity, trajectory slope and displacement when crossing a λ helix cavity. The dot indicates the result of the direct computation (first approximation, β input = 0.048, r' input = 3 mrad, r input = 6 mm).

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