RFQ SCALING-LAW IMPLICATIONS AND EXAMPLES*

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Summary

We demonstrate the utility of the RFQ scaling laws that have been previously derived.[1,2] These laws are relations between accelerator parameters (electric field, rf frequency, etc.) and beam parameters (current, energy, emittance, etc.) that act as guides for designing radio-frequency quadrupoles (RFQs) by showing the various tradeoffs involved in making RFQ designs. These scaling laws give a unique family of curves, at any given synchronous particle phase, that relates the beam current, emittance, particle mass, and space-charge tune depression with the RFQ frequency and maximum vane-tip electric field when assuming equipartitioning and equal longitudinal and transverse tune depressions. These scaling curves are valid at any point in any given RFQ where there is a bunched and equipartitioned beam. We show several examples for designing RFQs, examine the performance characteristics of an existing device, and study various RFQ performance limitations required by the scaling laws.

Introduction

We present four parametric scaling-law functions (C_1 to C_4) that (within given constraints) depend only on the zero-current transverse phase advance per period (σ_{T0}).[1,2] Given a value for σ_{T0}, values for C_1 to C_4 are determined. These functions contain the following parameters: beam current I, total transverse normalized emittance ϵ_T, particle charge/mass ratio Q/M_0, negative ratio of the transverse (longitudinal) space-charge to external focusing forces $\mu_{T(L)}$, RFQ rf wavelength λ, and vane-tip electric field at quadrupole symmetry E_0. The functional relationships $\overline{C}_i(I,Q/M_0,\epsilon_T,\lambda,E_0,\mu) = C_i(\sigma_{T0})$ are represented in this paper by the figures that were generated by the full set of equations found in Refs. 1 and 2.

We make a number of interesting observations resulting from these relationships, including the minimum rf frequency required for a given beam brightness and the RFQ parameter regime, including the minimum injection energy, required for a given beam current. A major intent of this paper is to make it relatively easy to determine the RFQ parameters for a given beam requirement. (It is interesting to note that the scaling behavior seen by Walter Lysenko,[3] in his simple model presented at this conference, has the same qualitative behavior seen here.)

Derivation[2]

The RFQ[4,5] is a device that provides transverse focusing, longitudinal sinusoidal bunching, and acceleration of beam particles. The transverse particle motion in the RFQ is approximately described by the Mathieu equation, and the longitudinal motion by a harmonic oscillator. Linear space-charge defocusing terms are calculated from the electric field components for a uniformly charged ellipsoid.[6] (Wangler[7] has argued that nonuniform charge-density ion beams rapidly evolve to a uniform charge-density distribution. This argument indicates that we are using a realistic space-charge-force model.)

We obtain relationships between I, ϵ_T and, $\sigma_{T0(L0)}$ [transverse (longitudinal) zero-current phase advances generated by the RFQ in one rf period (two RFQ cells)], which are related to the beam parameters through the space-charge parameters $\mu_{T(L)}$ [$\sigma_{T(L)}$ is the transverse (longitudinal) space-charge depressed tune, and $\sigma_{T(L)} = \sigma_{T0(L0)}\sqrt{1-\mu_{T(L)}}$]. We study two cases: The first case assumes equipartitioning (implying that $\epsilon_T\sigma_T = \epsilon_L\sigma_L$)[8,9] and equal transverse and longitudinal tune depressions ($\mu_L = \mu_T$). For the second case, the ratio μ_L/μ_T can be any fixed quantity, but we require that $\sqrt{(1-\mu_T)/(1-\mu_L)} = \sqrt{\epsilon_T\sigma_T/\epsilon_L\sigma_L} = \epsilon_T\sigma_{T0}/\epsilon_L\sigma_{L0}$. The effect of this constraint (which greatly reduces the complexity of the equations) is that increasing σ_{T0} (increasing the external transverse force), reduces μ_T (ϵ_T, ϵ_L, σ_{L0}, and μ_L held fixed), which is what we might physically expect. Case 1 is contained in Case 2.

The maximum electric field that can be achieved on the RFQ vanes without sparking is determined by the Kilpatrick criterion.[10] This field scales approximately by $1/\sqrt{\lambda}$. This scaling by λ is included in the scaling curves by defining $E_0 = \xi/\sqrt{\lambda}$. The Kilpatrick field for $\lambda = 0.705$ m is $E_0 = 19.9$ MV/m giving $\xi = 16.7$ MV/m$^{1/2}$. With present vacuum and surface preparation techniques, we can generally design for electric fields that are twice the Kilpatrick criterion.

Scaling Law Curves

We define the following scaling curve functions:

$$C_1 = [QI(1-\mu_T)^{3/4}] \,/\, [M_0(\epsilon_T/\lambda)^{3/2}\mu_L]; \qquad (\text{no } \xi/R_\epsilon) \quad (1)$$

$$C_2 = (M_0^2R_\epsilon^2\epsilon_T) \,/\, (\lambda^2\xi^2Q^2\sqrt{1-\mu_T}); \qquad (\text{no I}) \quad (2)$$

$$C_3 = \sqrt{C_1^2C_2^3} = (IM_0^2R_\epsilon^3) \,/\, (\mu_L\xi^3\lambda^{3/2}Q^2); \qquad (\text{no } \epsilon_T) \quad (3)$$

$$C_4 = \sqrt{C_1^4C_2^3} = [I^2M_0R_\epsilon^3(1-\mu_T)^{3/4}]/[\epsilon_T^{3/2}Q\xi^3\mu_L^2]; \quad (\text{no } \lambda) \quad (4)$$

where R_β = (synchronous velocity/minimum allowable synchronous velocity) and R_ϵ = (minimum RFQ vane radius/maximum beam radius). (Note that ξ appears with R_ϵ as ξ/R_ϵ. Increasing R_ϵ decreases the focusing effectiveness of the electric field.) The minimum allowable synchronous particle velocity (β_{min}) and the energy gain (δ) per RFQ cell (length = $\beta_s\lambda/2$) are

$$\beta_{min} = [-2\pi/(\phi_s\sigma_{L0})]\,[\epsilon_T\sigma_{T0}/(\lambda\sqrt{1-\mu_T})]^{1/2}, \text{ and} \quad (5)$$

$$\delta = [-\pi R_\beta^2\epsilon_T\sigma_{T0}M_0 \cot\phi_s] \,/\, [\phi_s^2\lambda\sqrt{1-\mu_T}], \quad (6)$$

where ϕ_s is the synchronous phase (negative for phase stable acceleration).

Functions C_1 to C_4 are unique functions of σ_{T0} for fixed values of μ_L/μ_T, ϕ_s, and R_β/R_ϵ. We have a parametric relationship, depending on σ_{T0}, between the quantities C_1, C_2, and any arbitrary function of C_1 and C_2 (C_3 and C_4 are examples). Curve C_2 versus σ_{T0} (unlike C_1) depends on ϕ_s, R_β, and R_ϵ. We can show that R_β and R_ϵ only appear with ϕ_s in the form $[R_\beta/(\phi_s \cdot R_\epsilon)]$, and as long as $[\sigma_{T0}/(\phi_s^2 \sin\phi_s)] \ll 1$, C_2 is relatively independent of ϕ_s, R_β, and R_ϵ. This situation often occurs in practice, and parameters

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picked for a particular part of an RFQ will be valid for the entire structure.

The variables that appear in C_1 to C_4 are I, Q/M_0, ϵ_T, λ, ξ/R_ϵ, μ_L, and μ_T. Function C_1 has no ξ/R_ϵ dependence, C_2 has no I dependence, C_3 has no ϵ_T dependence, and C_4 has no λ dependence. We generally design an RFQ for a given Q/M_0 and space-charge parameters μ_L and μ_T. However, situations frequently occur where only three of the four remaining parameters (I, ξ/R_ϵ, ϵ_T, and λ) have required values, which is why we chose functions C_1 to C_4. Because we have a parametric relationship between C_1 and C_2, choosing values for three of the four remaining parameters uniquely determines the fourth parameter and σ_{T0}.

SI units are used in generating the scaling curves with E_0 (V/m), ϵ_T (m•rad), M_0 [V, (m_0c^2/e)], and I (A), etc. We show σ_{L0} and the betatron function β_{Twiss} versus σ_{T0} in Fig. 1. (The betatron function is related to the beam size by $R_{BEAM} = \sqrt{\epsilon_T \beta_{Twiss}}$. For a harmonic oscillator, $\beta_{Twiss} = 1/\sigma_{T0}$, which is accurate for small phase advances in the Mathieu equation.) Figures 2 and 3 show C_1 and C_4 versus σ_{T0} for $\mu_L = \mu_T$, $(R_\beta/R_\epsilon) = 1.0$ to 2.0 and the following conditions. Figure 2: $\phi_s = -90°$; Fig. 3: $\phi_s = -30°$. Figure 4 shows the effect for $\mu_L \neq \mu_T$ (case 2) when $\phi_s = -90°$ and $(R_\beta/R_\epsilon) = 1$. Almost the same considerations apply to cases where $(\mu_L \neq \mu_T)$ as to where $(\mu_L = \mu_T)$, except that we get a different but unique scaling curve for each ratio of (μ_L/μ_T). This ratio depends only on the external forces through σ_{T0} and σ_{L0}. When σ_{L0} and σ_{T0} lie on the Fig. 1 curve, $\mu_L = \mu_T$; otherwise, $\mu_L \neq \mu_T$ and Fig. 4 gives an indication of what happens.

Discussion

Curves of σ_{L0} and C_1 versus σ_{T0} are independent of ϕ_s, R_β, and R_ϵ. Curve C_1 shows that, for a fixed σ_{T0}, ϵ_T, and μ_T, the length (λ) must decrease for a beam current increase. This relationship is independent of the beam's velocity. However, the particle bunch must sit inside the longitudinal phase-stable bucket; therefore, as the rf frequency increases, so does the minimum injection energy (Eq. 5).

Curve C_4 shows that an increase in current (fixed μ_T, ϵ_T, ξ) requires a decrease in σ_{T0}. This can be understood by looking at the betatron function versus σ_{T0} (Fig 1). For small zero-current tunes, a small increase in the tune depression (decreased tune) leads to a relatively large increase in the betatron function (beam size), which translates into a large reduction in the space-charge force. Going to higher frequencies and a smaller zero-current tune allows the betatron function to grow rapidly with current, causing small changes in the tune depression. Note that the phase advance per unit length does not decrease as rapidly as the phase advance per period for higher frequencies because the period length gets shorter as the frequency increases. Curve C_4 determines the value for σ_{T0} once a value for I is chosen. The value for λ is then determined by using one of the other curves.

We can use curve C_3 to maximize the beam current without regard to emittance and can determine the RFQ parameter regime using $I = C_3\xi^3\lambda^{3/2}\mu_L Q^2/(M_0^2 R_\epsilon^3)$. We can increase the current by decreasing σ_{T0} (increase C_3),

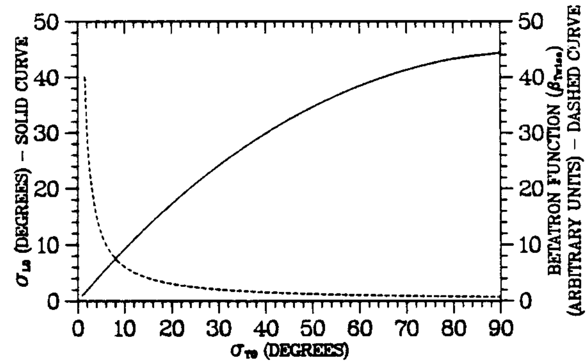

Fig. 1. σ_{L0} and β_{Twiss} versus σ_{T0} for $\mu_L = \mu_T$.

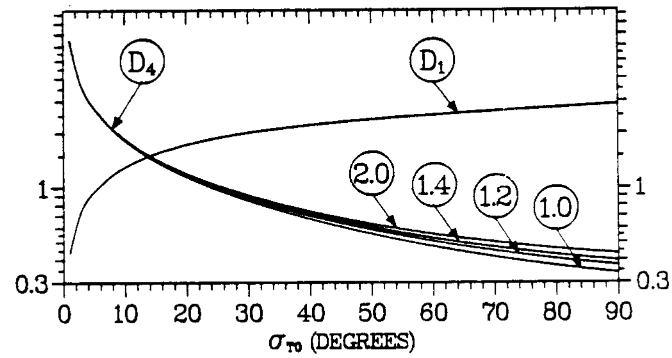

Fig. 2. Scaling law curves for $\mu_L = \mu_T$, $\phi_s = -90°$. Values for R_ϵ/R_β are given. $C_1 = D_1 \times 10^{-2}$, $C_4 = D_4 \times 10^{-6}$.

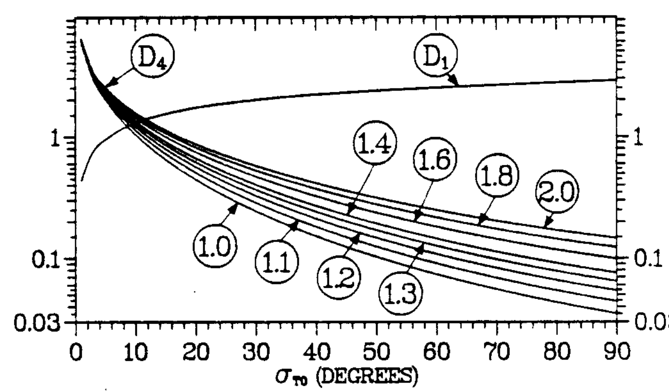

Fig. 3. Scaling law curves for $\mu_L = \mu_T$, $\phi_s = -30°$. Values for R_ϵ/R_β are given. $C_1 = D_1 \times 10^{-2}$, $C_4 = D_4 \times 10^{-6}$.

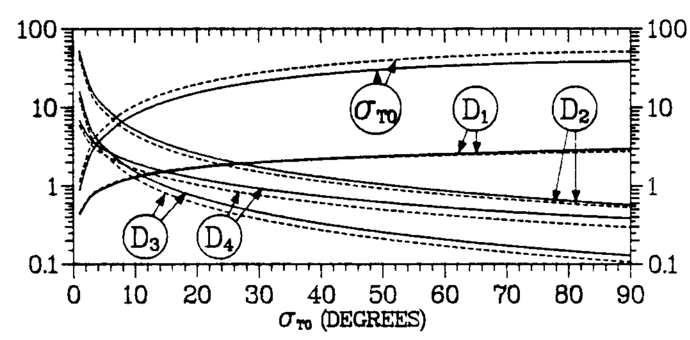

Fig. 4. Scaling law curves for $\mu_L \neq \mu_T$, $\phi_s = -30°$, $R_\beta/R_\epsilon = 1$, $\mu_L/\mu_T = 1.1$ (solid), $\mu_L/\mu_T = 0.9$ (dashed). $C_1 = D_1 \times 10^{-2}$, $C_2 = D_2 \times 10^{-2}$, $C_3 = D_3 \times 10^{-4}$, $C_4 = D_4 \times 10^{-6}$.

increasing the electric field (ξ), or increasing the rf wavelength (λ). What we do depends on the maximum acceptable emittance ($\epsilon_T \propto \lambda^2$ for fixed σ_{T0} using C_2), how much electric field breakdown we can tolerate (giving a maximum value for ξ), and what beam-energy-gain gradient is acceptable ($\delta \propto \sigma_{T0}$, Eq. 6). We use curve C_4 instead of C_3, when a maximum acceptable emittance and electric field are defined. Then we can study the tradeoffs between I, λ, and σ_{T0}.

Examples

We determine the RFQ parameters required to accelerate a He_4^{2} beam. We assume I = 0.1 A, ϵ_T = 1.2 x 10^{-6}m•rad and $\mu_L = \mu_T = 0.84$.[7,11] We require that R_ϵ = 2 and that the maximum electric field be twice the Kilpatrick field with a 1.4 field enhancement factor[12] (maximum electric field/E_0). We have M_0 = 3.73 x 10^9eV, Q = 2, ϕ_s = -90°, and ξ = 16.7 x 10^6 x 2 (Kilpatrick)/ 1.4 (enhancement factor) = 23.9 x 10^6 V/$m^{1/2}$. We calculate C_4 = 2.995 x 10^{-6} from Eq. (4) and find, using Fig. 2, that $\sigma_{T0} \simeq 4°$, and C_1 = 0.88 x 10^{-2}. Knowing C_1, we calculate λ = 0.80 m (375-MHz) from Eq. (1), and use Eq. (5) to calculate β_{min} = 2.93 x 10^{-2} (1.6-MeV kinetic energy). We have determined the minimum frequency and injection energy for this RFQ.

In the second example, we determine some properties of an existing 425.0-MHz, 2.0-MeV proton RFQ linac[13,14] at the end of the "gentle buncher" section (the design "choke" point with beam energy = 1.01 MeV), which was designed to accelerate a proton beam with ϵ_T= 1.2 x 10^{-6} m•rad, M_0 = 938 x 10^6 eV, and I = 0.1 A. We determined the following parameters from a PARMTEQ computer run:[15] σ_{T0}= 20.1°, σ_{L0}= 19.8°, ϕ_s = -30.9°, and E_0 = (40 x 10^6 V/m)/(1.4 enhancement factor). Because σ_{T0} versus σ_{L0} lies close to the curve in Fig. 1, we assume $\mu = \mu_L = \mu_T$. From C_1 (Fig. 2), we obtain $[I(1-\mu)^{3/4}]/[M_0(\epsilon_T/\lambda)^{3/2}\mu]$ = 0.0175, and calculate μ = 0.805. There are several ways that this value for μ, which is close to the value where emittance growth could be a problem,[7,11] can be reduced for a fixed maximum electric field. Using curves for C_1 and Eq. (1), we see that μ can be reduced either by decreasing λ or increasing C_1 by increasing σ_{T0}. Curve C_4 shows that σ_{T0} can be increased by increasing E_0 (ξ), which can be done by decreasing the field enhancement factor.

As a third example, we determine the RFQ parameter regime required to accomplish the beam requirements of the second example with a significantly reduced μ. We require that ϵ_T = 1.2 x 10^{-6} m•rad, M_0 = 938 x 10^6 eV, Q = 1, ξ = 23.9 x 10^6 V/$m^{1/2}$ (see the first example), and I = 0.1 A . The initial RFQ synchronous phase will be -90° (bunching only), and the final phase -30° (bunching with acceleration). We require minimal emittance growth and choose $\mu_{T(L)}$ = 0.70. We consider the requirements for a 10% beam-current safety factor, a minimum RFQ vane radius that is 30% larger than the maximum beam radius (R_ϵ = 1.3), and $\beta_s = \beta_{min}$. We create Table I with the following prescription: calculate C_4 using Eq. (4); use C_4 and the appropriate figure to determine σ_{T0}, σ_{L0} and C_1; use Eq. (1) and the value for C_1 to calculate λ; calculate β_{min} from Eq. (5).

Table I. RFQ design data for Ex. 3.

I	R_ϵ	ϕ_s	C_4 x 10^{-6}	σ_{T0}	σ_{L0}	C_1 x10^{-2}	λ	FREQ. x 10^6	β_{min}	ENERGY MEV
0.10	1.0	-90°	0.432	66.7°	40.4°	2.60	0.67	445.	0.0110	0.057
0.10	1.0	-30°	0.432	20.5°	17.7°	1.75	0.52	579.	0.0478	1.073
0.11	1.3	-90°	1.149	20.7°	17.9°	1.76	0.49	617.	0.0163	0.125
0.11	1.3	-30°	1.149	11.4°	10.5°	1.37	0.41	726.	0.0673	2.131

The final RFQ design is a compromise between construction and beam-dynamics requirements. A design that includes the safety factors should perform as desired. An RFQ designed without the safety factors will likely experience beam scraping even with a "well-matched" beam. RFQ operation is most severely constrained at ϕ_s = -30°; and therefore we choose λ, where 0.52 > λ > 0.41 (579 to 726 MHz). The injection energy (for ϕ_s = -90°) will lie between 120 and 235 keV [use Eq. (5) with the appropriate σ_{T0}, σ_{L0}, and λ].

Conclusion

For a given synchronous phase, there is a unique family of scaling curves (μ_L/μ_T and R_ϵ/R_β) that relates the beam current, emittance, particle mass, and charge with the RFQ frequency, maximum vane-tip electric field, and space-charge tune depression if we assume equipartitioning and equal longitudinal and transverse tune depressions. We presented these scaling relationships, given by C_1 to C_4, to show the various tradeoffs involved in choosing RFQ designs and have provided curves to help choose starting points in parameter space for optimizing an RFQ for a particular requirement. Finally, we presented several examples for designing RFQs using our procedure.

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