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A COMPUTER PROGRAM FOR BEAM PHASE SPACE WITH ARBITRARY CONFIGURATION

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Summary

A computer program intended for the beam transport system design of an initial beam phase space with arbitrary configuration is developed and some practical examples are worked out to show the application of the program.

Introduction

Measurements show that no practical beam phase space is a real ellipse as assumed by conventional beam transport theory. Recently the geometry of accelerator beam phase space with an arbitrary configuration has been developed.[1,2] Based on that geometry, a computer code, TRAC(Transport for Arbitrary Configuration), was designed for the transport of the beam phase space with an arbitrary configuration. The program can step through the beam line calculating the properties of the beam and output the parameters and the layout of the beam line, the beam envelopes, as well as the beam phase spaces at whatever point requested in the beam line.

Basic Theory [1]

In the transport theory of charged partical beam,[3] for a beam of finite dimensions the representative points lie within a six-dimensional hypervolume in phase space. Consider its two dimensional projection[4] (x,x') and define the beam envelope of that cross section as[1]

$$E = \mathrm{Max}\,|x|$$

1. Transport of Centrosymmetric Polygonal Phase Space

A centrosymmetric convex 2n-gonal phase space, defined by its n consecutive side lines $x+b_i x'-a_i=0$, $(i=1,2,\dots,k,\dots,n)$ and transporting through a field defined by the transfer matrix $M(z)=\begin{bmatrix} m_{11}(z) & m_{12}(z) \\ m_{21}(z) & m_{22}(z) \end{bmatrix}$,

generates a beam envelope

$$E(z)=\left|\frac{b_{k+1}a_k-b_k a_{k+1}}{b_{k+1}-b_k}m_{11}(z)+\frac{a_{k+1}-a_k}{b_{k+1}-b_k}m_{12}(z)\right|$$

$$(\zeta_k \le z \le \zeta_{k+1})$$

where ζ_k and ζ_{k+1} are solved from the equations

$$-\frac{m_{12}(\zeta_k)}{m_{11}(\zeta_k)}=b_k \quad , \quad \frac{m_{12}(\zeta_{k+1})}{m_{11}(\zeta_{k+1})}=b_{k+1}.$$

2. Transport of Centrosymmetric Smooth Convex Phase Space

In the three-dimensional phase space $(x_1=x, x_2=x', x_3=\frac{\Delta p}{p})$, a centrosymmetric smooth convex phase space is defined by $f(x_1,x_2,x_3;a,b,\dots)=0$ with its parameters a, b,... and the transfer matrix M(z) is replaced by a 3×3 matrix. From the equations[2]

$$f(x_1,x_2,x_3;a,b,\dots)=0$$

$$\frac{m_{12}(z)}{m_{11}(z)}=\frac{\partial f}{\partial x_2}\Big/\frac{\partial f}{\partial x_1}$$

$$\frac{m_{13}(z)}{m_{11}(z)}=\frac{\partial f}{\partial x_3}\Big/\frac{\partial f}{\partial x_1} \quad ,$$

one can solve x_1, x_2, x_3 in terms of M(z), a, b,...

$$x_1=\zeta_1(M(z);a,b,\dots)$$

$$x_2=\zeta_2(M(z);a,b,\dots)$$

$$x_3=\zeta_3(M(z);a,b,\dots) \quad ,$$

giving the beam envelope

$$E(z)=\left|m_{11}(z)\zeta_1(M(z);a,b,\dots)+m_{12}(z)\zeta_2(M(z);a,b,\dots) + m_{13}(z)\zeta_3(M(z);a,b,\dots)\right| \quad .$$

3. Transport of Beam Phase Space with Arbitrary Configuration

The beam envelope generated by a beam phase space with arbitrary configuration is identical with that generated by the equivalent centrosymmetric convex beam phase space defined by the convex cover of the union of the given phase space and its symmetric immage with respect to the origin of the phase space coordinates. Figure 1 gives an example of the equivalent centrosymmetric convex beam phase space.

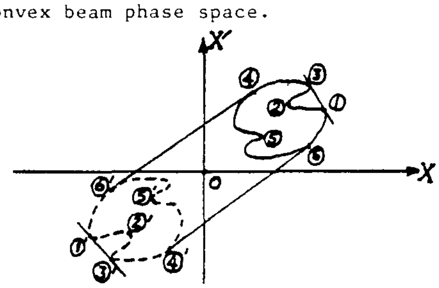

Fig.1. Phase space with arbitrary configuration ①②③④⑤⑥① and its equivalent centrosymmetric convex phase space ①③④⑥①'③'④'⑥'① .

4. Various Transfer Matrix

In a beam line, the transfer matrix M(z) can be regarded as an assembly matrix at Z in the N-th element of the system

$$M(Z)=R_N(Z-L)R_{N-1}(l_{N-1})\dots R_2(l_2)R_1(l_1)$$

where $L=l_1+l_2+\dots+l_{N-1}$ and l_i is the length of i-th element($i=1,2,\dots$).

For the drift length l_N, the transfer matrix of the element is

$$R_N(Z-L)=\begin{bmatrix}1 & Z-L\\ 0 & 1\end{bmatrix}, \qquad L\leq Z\leq L+l_N .$$

In the focusing plane of the quadrupole with length l_N the matrix is

$$R_{Nx}(Z-L)=\begin{bmatrix}\mathrm{COS}k(Z-L) & \frac{1}{k}\mathrm{SIN}k(Z-L)\\ -k\,\mathrm{SIN}k(Z-L) & \mathrm{COS}k(Z-L)\end{bmatrix}, \quad L\leq Z\leq L+l_N ,$$

and in the defocusing plane

$$R_{Ny}(Z-L)=\begin{bmatrix}\mathrm{CH}k(Z-L) & \frac{1}{k}\mathrm{SH}k(Z-L)\\ k\,\mathrm{SH}k(Z-L) & \mathrm{CH}k(Z-L)\end{bmatrix}, \quad L\leq Z\leq L+l_N .$$

In the bending magnet with uniform field, the matrices are

$$R_{Nx}(Z-L)=\begin{bmatrix}\mathrm{COS}(\frac{Z-L}{\rho}) & \mathrm{SIN}(\frac{Z-L}{\rho}) & \rho(1-\mathrm{COS}(\frac{Z-L}{\rho}))\\ -\frac{1}{\rho}\mathrm{SIN}(\frac{Z-L}{\rho}) & \mathrm{COS}(\frac{Z-L}{\rho}) & \mathrm{SIN}(\frac{Z-L}{\rho})\\ 0 & 0 & 1\end{bmatrix},$$

$$R_{Ny}(Z-L)=\begin{bmatrix}1 & Z-L\\ 0 & 1\end{bmatrix}, \qquad L\leq Z\leq L+\rho\alpha .$$

where α, ρ are the bending angle and the radius of curvature respectively.

Program Performance

1. Preprocessing

Since this program demands the initial phase space to be centrosymmetric convex, the beam phase space with arbitrary configuration must be replaced by its equivalent centrosymmetric convex phase space with the unit mm-mrad(see the preceding section on Basic Theory).

2. Input of the Initial Values

The initial information, including the preprocessed initial beam phase space and the layout of the beam line, is input by the user to the file called INDAT. Owing to centrosymmetry, only the half coordinates data of the initial beam phase space are needed. The beam line can be composed of drift space, quadrupole, bending magnet with uniform field and any other element whose transfer matrix can be expressed as discrete values step by step along the beam line.

3. Output

The beam envelopes and the beam phase spaces are printed by the output file OUDAT. If the program is executed on the graphical terminal one gets two figures in which the beam line, beam envelope and beam phase spaces are drawn in two planes (see Fig.2). In the figures the position and size of the elements in the beam line layout are specified by the input parameters with the letter 'F' for focusing quadrupole lens, 'D' for defocusing quadrupole lens, 'B' for bending magnet and 'N' for the element with discrete values. At the bottom of the figure there are the beam phase spaces at the point requested in the beam line with XP,YP representing X',Y' respectively.

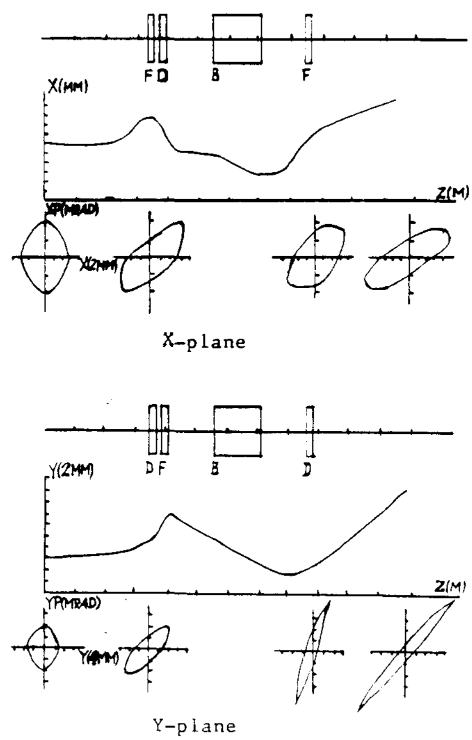

Fig.2. Beam envelopes of example 1.

Examples of Calculation

Examlpe 1: The initial beam phase space are the ellipse-paraboloids

$$\frac{X^2}{3^2}+(\frac{\Delta P}{P})^2/(0.025)^2=2(2-|X'|)$$

$$\frac{Y^2}{3^2}+(\frac{\Delta P}{P})^2/(0.025)^2=2(2-|Y'|)$$

Fig.2 shows their projections on X-X' plane and Y-Y' plane and the layout of beam line with the quadrupoles of strength $k=2.3\ m^{-1}$ and the bending magnet of bending angle $\alpha=90°$ and radius of curvature $\rho=1$ m.

Example 2: This example is intended to compare the difference between the two beam envelopes generated by the parallelogram phase space, defined by its two successive vertexes (2.55,-2.45), (3.35,1.48) and its inscribed ellipse phase space defined by its σ-matrix $\begin{bmatrix} 8.87 & -0.64 \\ -0\ 64 & 4.11 \end{bmatrix}$ through the same beam line. In Fig.3 the solid line represents the beam envelope generated by the parallelogram phase space, while the dashed line the beam envelope generated by its inscribed ellipse. It is interesting to note from Fig.3 that there is an obvious difference between these two beam envelopes. The maxmum relative difference is as high as 25% and 30% in Y plane and X plane respectively, and it shows that in X plane the waists of these two beam envelopes are not coincident in position, with the separation of 0.3m.

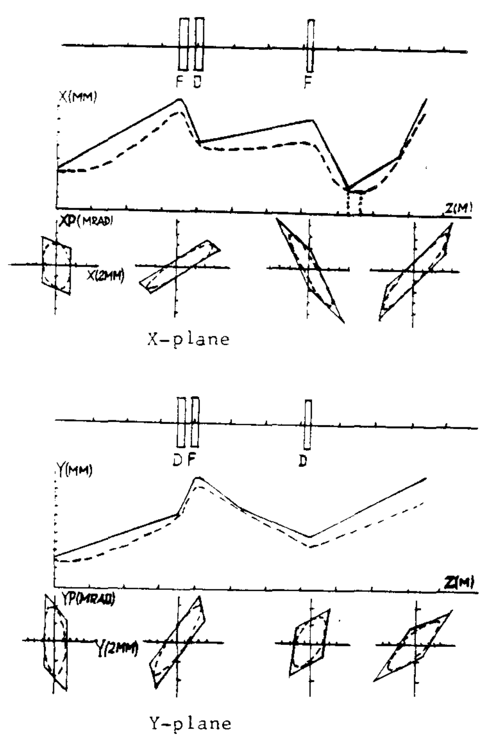

Fig.3. Comparison between the two beam envelopes of example 2.

References

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