LOW BETA LINACS AND ALTERNATING PHASE FOCUSSING*

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I. Introduction

The limitation in current in low β linacs comes primarily from the finite size of the separatrix in longitudinal phase space. And the size of the separatrix is determined by the non-linear character of the longitudinal motion. In this paper we try to estimate the maximum current which can be accelerated from the size of the non-linear term. Our intent is to try to obtain a general understanding of the current carrying capacity of an APF structure from such considerations, since a rigorous analytic treatment of a non-linear strong focussing force appears not to be feasible.

II. Analysis of Longitudinal Motion

We start with a conventional drift tube linac with a constant synchronous phase ϕ_s. Neglecting the effect of acceleration, the equation for the longitudinal motion is approximately

$$\frac{d^2\psi}{ds^2} + A\psi - \frac{B\psi^2}{2} = 0 \quad , \tag{1}$$

where s is the axial distance along the accelerator and we have included up to quadratic force terms in $\psi = \phi - \phi_s$. The parameters A and B are given, for small $|\phi_s|$, by

$$B \cong \frac{2\pi\, e\, E}{mc^2\, \lambda\beta^3} \quad , \quad A \cong B|\phi_s| \quad . \tag{2}$$

An "energy" integral gives the equation for the separatrix:

$$\left(\frac{d\psi}{ds}\right)^2 + A\psi^2 - \frac{B\psi^3}{3} = \frac{4A^3}{3B^2} \quad , \tag{3}$$

corresponding to the phase stable region

$$\frac{\psi_s - \psi_u}{2} < \psi - \psi_s < \psi_u - \psi_s \quad , \tag{4}$$

where ψ_s and ψ_u are the stable and unstable fixed points. For the conventional drift tube linac without space charge we have

$$\psi_s = 0 \quad , \quad \psi_u = 2A/B \quad . \tag{5}$$

The relation in Eq. (4) is a general result which depends only on the quadratic character of the force (or the cubic character of the potential in Eq. (3)).

We now introduce a linear space charge force by assuming that the beam bunch is a uniformly charged ellipsoid of radius R, length 2L, centered at a phase ψ_o which will be determined later. The differential equation for the phase motion then becomes

$$\frac{d^2\psi}{ds^2} + A\psi - \frac{B\psi^2}{2} = D(\psi - \psi_o) \quad , \tag{6}$$

where

$$D = \frac{f(L/R)}{R^2 L} \cdot \frac{3e\, Z_o\, \lambda I}{4\pi\, \beta^2\, Mc^2} \cong \frac{30\lambda\, e\, I}{RL^2\, \beta^2\, Mc^2} \quad . \tag{7}$$

Here $Z_o = 120\pi$ is the impedance of free space, I is the average beam current, and f(L/R) is a function of the relative ellipsoid axes which we approximate by $f(L/R) \cong R/3L$. The new stable and unstable fixed points can be shown to be

$$\psi_u = \frac{A}{B}\left[1 - 3\alpha + \left(1 - 3\alpha^2\right)^{1/2}\right] \quad , \tag{8}$$

$$\psi_\ell = \frac{A}{B}\left[1 - 2\left(1 - 3\alpha^2\right)^{1/2}\right] \quad , \tag{9}$$

where $\alpha = D/2A$, and ψ_ℓ is the opposite extreme of the separatrix from ψ_u. Here $3\psi_s = 2\psi_\ell + \psi_u$ as outlined in Eq. (4), and ψ_o, the ellipsoid center, is chosen to be $(\psi_\ell + \psi_u)/2$, corresponding to a completely filled bucket. The length of the beam bunch is then

$$2L = \frac{\beta\lambda}{2\pi}(\psi_u - \psi_\ell) = \frac{3A}{B}\left[\left(1 - 3\alpha^2\right)^{1/2} - \alpha\right] \quad , \tag{10}$$

in which case Eq. (7) can be written as

$$I = \frac{3}{40\pi}\beta ER\left[\alpha\left(\left(1 - 3\alpha^2\right)^{1/2} - \alpha\right)^2\right] \quad . \tag{11}$$

The maximum value of the bracket [] is .11, when $\alpha = D/2A = .23$, leading to

$$I = \frac{\beta RE}{380 \text{ ohms}}\left(\frac{A}{B}\right)^3 \quad . \tag{12}$$

The above analysis is quite crude, neglecting non-linear space charge forces and nonlinear focussing forces of higher order, as well as acceleration and non-uniform filling of the bucket. Nevertheless, Eq. (12) is reasonably consistent with present experience for both the conventional drift tube linac and the RFQ. We shall therefore use Eq. (12) as a guide for other linac configurations (APF) for which we will need to estimate the applicable values of A and B.

III. Alternating Phase Focussing (APF)

We assume a symmetric APF, that is one in which the synchronous phase alternates between $-|\phi_s|$ and $+|\phi_s|$, each applying to a length ℓ of the linac. In this case the parameter A in Eq. (2) will alternate in sign, but B will not. We shall therefore obtain the equivalent value, A_{eq} by analyzing the linear problem

$$\frac{d^2\psi}{ds^2} \pm A\psi = 0 \left\{\begin{matrix} 0 < s < \ell \\ \ell < s < 2\ell \end{matrix}\right\} , \tag{13}$$

which leads to a phase advance per period, μ, given by

$$\cos\mu = \cos\Theta \cosh\Theta \quad , \quad \Theta = \ell\sqrt{A} \quad . \tag{14}$$

The equivalent smoothed focussing equation is then taken to be

$$\frac{d^2\psi}{ds^2} + \left(\frac{\mu}{2\ell}\right)^2 \psi = 0 \tag{15}$$

and A_{eq} is then

$$A_{eq} = \left(\frac{\mu}{2\ell}\right)^2 = \frac{\mu^2}{4\Theta^2} A \quad . \tag{16}$$

Since B is unchanged, we therefore expect, from Eq. (12), a reduction of the maximum current by the factor $(\mu/2\Theta)^6$. For $\Theta = \mu = \pi/2$ the reduction is substantial. For $2\Theta = 1.19\pi$, $\mu = \pi$ the reduction is a factor 2.7.

Our analysis needs to be taken with several grains of salt since the maximum current corresponds to longitudinal motion close to the border of stability. Nevertheless we believe that the region to be explored numerically is one for which Θ and μ are as large as possible, corresponding to a synchronous phase configuration such as

$$\phi_s = 75^\circ, 75^\circ, 75^\circ, -75^\circ, -75^\circ, -75^\circ, 75^\circ, \text{ etc.} \tag{17}$$

in order to make ℓ, Θ and μ sufficiently large. We are therefore hopeful that an APF structure can be designed so as to accelerate currents which are of the same order of magnitude as a conventional linac or an RFQ.

IV. Simulation of Longitudinal Motion

In order to test the above ideas, we have carried through a simulation involving an alternation of gap impulses and drift tube drifts for symmetric patterns like that in Eq. (17) for various values of the dimensionless parameters

$$k = \frac{2\pi\, eE\lambda}{\beta\, Mc^2} \quad , \quad \pm|\phi_s| \quad , \quad n \quad ,$$

where n is the periodicity of the APS configuration (n=6 for the pattern in Eq. (17)). We define the separatrix is that region of phase space for which the motion is not chaotic. Typical separatrices for the pattern in Eq. (17) are shown in Figs. 1 and 2 for k = .13, , n = 6, $|\phi_s| = 80^\circ$ and for k = .08, n = 8, $|\phi_s| = 80^\circ$. We find that the motion starts to be come chaotic before reaching the limit suggested by Eq. (16), particularly for large $|\phi_s|$. Nevertheless it does appear that one can find parameters for which the separatrix area is large, implying the ability to accelerate an appreciable current. We have also observed the validity of the relation in Eq. (4), suggesting that our assumption of small $|\phi_s|$ may be generously interpreted. In addition, we confirm that the transverse motion is also stable for the range of phases defined by Eq. (4).

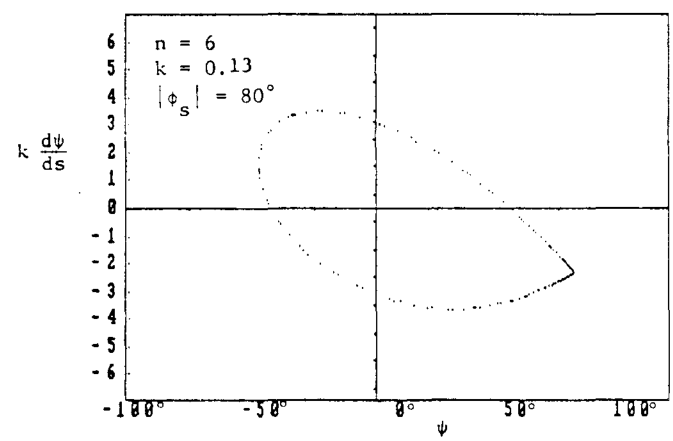

Figure 1

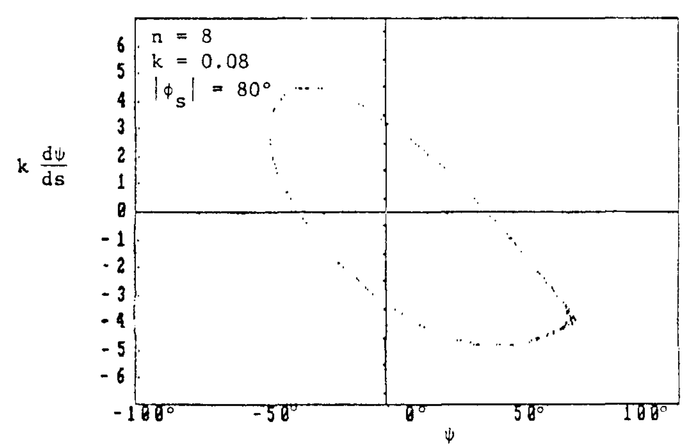

Figure 2

In Figs. 3 and 4, we show plots of ψ_u and $-\psi_\ell$ against $|\phi_s|$ for the parameters of Figs. 1 and 2 respectively. The small phase stable region for all but the largest values of $|\phi_s|$ is apparent. In addition, it appears that $-\psi_\ell/\psi_u \cong .5$ over wide range of $|\phi_s|$, suggesting that the quadratic force picture is a satisfactory approximation.

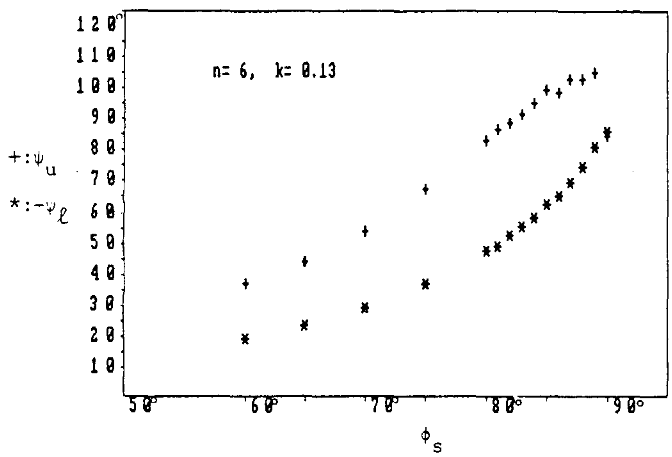

Figure 3

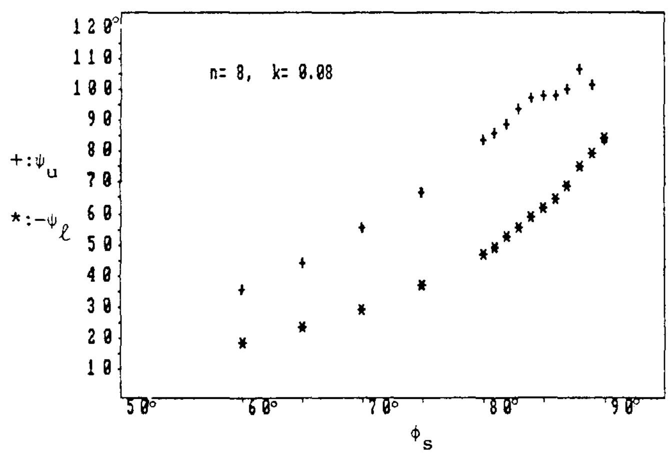

Figure 4

V. Comparison with Other Work

Detailed analytic work on APF has been carried out primarily in the Soviet Union and is summarized in Kapchinsky's book[1] as well as in the report by Wells.[2] Kapchinsky suggests that an asymmetric pattern will accelerate greater current, but reaches his conclusion by including the static transverse focussing which occurs for either an accelerated or a decelerated beam. Wells reports on estimates of currents as high as 300 m.a., although it is not clear whether currents of that order of magnitude have been achieved experimentally. It appears that the final determination of the current carrying capability of APF will require two parameter (r,z) orbit tracking including space charge.

VI. References

* Work supported by the Department of Energy.

1. I.M. Kapchinsky, Theory of Resonance Linear Accelerators, Harwood Academic Publishers, New York, 1985 (translated from a Russian book dated 1980); see also D.A. Swenson, Particle Accelerators 7, 61 (1976) for a discussion of related work at Los Alamos.

2. Nikita Wells, Radio Frequency Quadrupole and Alternating Phase Focussing Methods used in Proton Linear Accelerator Technology in the USSR, prepared for DARPA, published by the Rand Corporation, January 1985.