TRANSPORT AND ACCELERATION OF LOW-EMITTANCE ELECTRON BEAMS

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Abstract: Linear accelerators for colliders and for free-electron lasers require beams with both high brightness and low emittance. Their transport and acceleration is limited by single-particle effects originating from injection jitter, from the unavoidable position jitter of components, and from chromaticity. Collective phenomena, essentially due to wake fields acting within the bunch, are most severe in the case of high-frequency structures, i.e. a small aperture. Whilst, in the past, the transverse wake-field effects were believed to be most serious, we know that they can even be beneficial when inducing a corresponding spread in betatron oscillation either by an energy spread along the bunch or by an RF focusing system acting on the bunch scale. This paper evaluates the different effects by simple analytical means after making use of the smooth focusing approximation and the two-particle model. Numerical simulation results are used for verification.

1. Introduction

A linear accelerator consists of an injector, a damping ring, and a main linac. Since the longitudinal bunching motion ceases with γ^{-3} and the transverse forces decrease proportionally to γ^{-2}, electron injectors are normally simple and short. On the other hand, the beams of high brilliance needed for linear colliders or free-electron laser (FEL) drivers require special care in order to minimize transport aberrations and RF dynamic effects and to counteract space-charge and beam-induced fields. Many papers treat these subjects, and today there is good hope that RF laser guns[1] and progress in damping rings[1] will make it possible to supply beams of the required quality.

In the main linac we will have then to accelerate the beam to its final energy while preserving its emittance. For both linear colliders and FEL drivers the typical range of beam parameters is similar: 10^9–10^{10} electrons per bunch, ~ 0.1 mm r.m.s. bunch length, ~ 10^{-6} m normalized emittance, < 0.5% energy spread.

Nevertheless, the requirements for emittance preservation will be very different for the two machines. Linear colliders are extremely long. They will have a high RF frequency, 10 to 30 GHz, and will probably be single-bunch machines; FEL drivers are much shorter, have a lower RF frequency, 0.3 to 3 GHz, but have many bunches. Therefore, single-particle effects which are governed by the law of large numbers are more severe in the case of colliders. The same is true for the single-bunch transverse beam break-up (SBBU). It is caused by transverse beam-induced fields (wake fields) which scale with the third power of the RF frequency and whose effects depend strongly on the machine length. On the other hand, FEL drivers are subject to the weaker but cumulative multibunch beam break-up (MBBU). It is a coherent effect where the transverse EM_{11} mode, excited by the leading bunches, deflects the following bunches. The effect is well known and will not be treated here. References can be found in a recent paper[2] which studies different solutions to reduce the blow-up.

In this paper we investigate the most important single-particle and collective effects and possible cures. Much of the basic particle dynamics, the external focusing requirements, and the transverse perturbing effects are taken from Helm and Miller.[3] The presentation of wake fields and the two-particle model follows closely the work of Bane[4] and for the single-particle effects that of Ruth.[5] All numerical results refer to a hypothetical 1+1 TeV collider, the CERN Linear Collider (CLIC), which is under study at CERN.[6] The parameters used here are given in Table I.

Table I
CLIC Parameters used in this Paper

Energy range	E_0 = 5 GeV to E_f = 1 TeV
Accelerating gradient	E_a = 80 MV/m
Total length	L = 12.5 km
RF frequency	f_{RF} = 29 GHz
RF structure length, aperture	ℓ_{RF} = 0.25 m, 2a = 4 mm
Number of particles per bunch	$N_e = 5 \times 10^9$
r.m.s. bunch length	σ_z = 0.2 mm
Transverse wake-field slope at bunch centre	$W'_\perp = 1.69 \times 10^{21}$ V/C·m^3
Lattice:	$\mu = 90°$, FODO
magnet and cell length	$\ell_q, \ell_c \simeq \gamma^{1/2}$
Normalized emittance	$\epsilon_{nx} = 10^{-6}$ rad·m
Betatron wavelength	$\lambda_{\beta 0}$ = 5 m to $\lambda_{\beta f}$ = 70.7 m
Beam size	σ_{x0} = 8.9 μm to σ_{xf} = 2.4 μm
Focal length	f_0 = 0.44 m to f_f = 6.25 m
Total phase advance	μ_f = 2220 rad
Number of quadrupoles	N_q = 2828

2. Single-Particle Effects

2.1 Equation of Motion

We assume uncoupled x- and y-motion and a betatron wavelength much smaller than the distance needed to double the energy. The acceleration is then adiabatic and the transverse equation of motion becomes

$$x''(s) + \frac{p_0}{p} K(s)x(s) = -\frac{p_0}{p} \varrho^{-1}(s) \ , \tag{2.1}$$

where x is the transverse displacement of a particle at position s along the accelerator, p and p_0 are the actual and the design momentum respectively, K(s) is the focusing function, and $\varrho(s)^{-1}$ the local curvature due to all other but focusing fields. In contrast to circular machines, the dispersion increases with s in a linac. It is therefore useful to split the solution of Eq. (2.1) into a reference trajectory x_r, with $x_r(0) = x_r'(0) = 0$, and the betatron oscillation x_β which is determined by the initial conditions x_0, x_0', and K(s), only. Furthermore, it is sufficient, in most cases, to smooth the focusing system while keeping the local field errors $\varrho(s)$ discrete. Writing

$$p = p_0(1 + \delta) \ , \qquad \delta \ll 1 \tag{2.2}$$

we obtain then

$$x_r'' + k^2(1 - \xi\delta)^{-1}x_r = (1 + \delta)^{-1}\varrho^{-1}(s) \tag{2.3}$$

$$x_\beta'' + k^2(1 - \xi\delta)^{-1}x_\beta = 0 \ . \tag{2.4}$$

The wave number $k = 2\pi/\lambda_\beta$ and the chromaticity $\xi = (\Delta k/k)/(\Delta E/E)$ are the same as for AG focusing. Having solved Eqs. (2.3) and (2.4) we take the acceleration into account by an adiabatic damping term and the AG focusing by reintroducing the betatron function

$$x(s) \rightarrow \left[\frac{\beta(s)}{\beta(s_i)} \frac{\gamma(s_i)}{\gamma(s)}\right]^{1/2} x(s) \ , \tag{2.5}$$

where s_i is the position of initial conditions. Nevertheless, the smooth focusing approximation oversimplifies the problem concerning several points[7]

- the machine ellipse stays constant and beams are always matched;
- all particles, independent of the phase advance, are stable;

- RF structures are continuous; therefore, accelerating-gradient and wake fields have to be scaled down (seems to be a small effect);
- transverse positioning errors depend on the average β-function and have a weaker influence (quite important, can give an order of magnitude difference in beam blow-up).

2.2 Injection Jitter

In principle, all static errors or slowly varying errors, with respect to the repetition rate, can be corrected either by realignment or feedback. Those errors which occur too fast for a feedback are assumed to be random and called jitter.

Let us consider a position jitter Δx at injection in an otherwise constant machine. Then, to keep the bunches in position at the end of the machine, the jitter must be small compared to the beam size at the beginning

$$\sigma_{\Delta x} \ll \sigma_{x0} , \tag{2.6}$$

or, in other words, the jitter in the injection slope must be small compared to the divergence of the beam

$$\sigma_{\Delta x'} \ll \sigma_{x0}/\beta_0 . \tag{2.7}$$

That means, for the CLIC example, that position and slope jitter at injection must be smaller than 9 μm and 11 μrad respectively.

2.3 Lateral-Displacement Jitter of Quadrupoles

A laterally displaced quadrupole acts like a well-aligned quadrupole plus a dipole magnet. The transverse kick of the dipole field can be simulated with a local curvature of

$$\varrho(s)^{-1} = (\Delta x_i/f_i)\delta(s-s_i) , \tag{2.8}$$

where f_i and Δx_i are the focal length and the displacement of the quadrupole at position $s = s_i$ and $\delta(s)$ is the Dirac δ-function. Then the reference trajectory, Eq. (2.3) and $\delta = 0$ together with Eqs. (2.8) and (2.5), becomes

$$x_i(s) = \frac{\Delta x_i}{f_i}\beta(s_i)\left[\frac{\beta(s)}{\beta(s_i)}\frac{\gamma(s_i)}{\gamma(s)}\right]^{1/2}\sin\mu(s,s_i)\,H(s-s_i) \tag{2.9}$$

with

$$\mu(s,s_i) = \int_{s_i}^{s} ds'/\beta(s') \qquad \text{phase advance between } s_i \text{ and } s$$

$H(s-s_i)$ Heaviside step function.

Let us now consider a lattice of constant phase advance per cell as for the CLIC study, Table I. In such a lattice, magnet and cell length, and therefore also f and β, scale with $\gamma^{1/2}$. Assuming all N_q quadrupoles in the machine to be displaced we simply add up the trajectories (2.9) and obtain at $s = L$

$$x(L) = \frac{\beta_0}{f_0}\sum_{i=1}^{N_q}\Delta x_i[\gamma(s_i)/\gamma_f]^{1/4}\sin\mu(L,s_i) . \tag{2.10}$$

The subscript 0 refers to the beginning of the machine. If the displacements Δx_i are random with a normal distribution we can apply the 'central limiting theorem' and obtain the r.m.s. value of the trajectory

$$\sigma_x = \frac{\beta_0}{f_0}\sigma_{\Delta x}\left[\sum_i [\gamma(s_i)/\gamma_f]^{1/2}\sin^2\mu(L,s_i)\right]^{1/2} ,$$

which, after replacing the sum by an integral and assuming a linear increase of $\gamma(s) = \gamma_0(1 + Gs)$, becomes

$$\sigma_x = \frac{\beta_0}{f_0}\sqrt{N_q/3}\,\sigma_{\Delta x} . \tag{2.11}$$

If we do not want emittance dilution, the reference trajectory must be small compared to the beam size at the end of the linac. This yields a limit to the tolerable magnet displacement jitter of

$$\sigma_{\Delta x} \ll \frac{f_0}{\beta_0}\sqrt{3/N_q}\,\sigma_{xf} . \tag{2.12}$$

Equation (2.12) requires the jitter to be smaller than 0.043 μm for CLIC.

2.4 Tilt Jitter of RF Sections

An RF section with asymmetric power couplers, or with a symmetry error in construction, or which is tilted in the axial direction, gives a transverse kick to the beam. As an example we consider a section at position $s = s_i$ which is tilted by an angle α_i. The local curvature is

$$\varrho(s)^{-1} = \alpha_i[\Delta\gamma/\gamma(s_i)]\delta(s-s_i) , \tag{2.13}$$

with $\Delta\gamma$ being the energy gain in the section. Now, we can determine the reference trajectory in the same way as in the previous section and obtain at the end of the machine with N_{RF} sections:

$$x(L) = \Delta\gamma\beta_0(\gamma_0\gamma_f)^{-1/2}\sum_{i=1}^{N_{RF}}\alpha_i[\gamma_f/\gamma(s_i)]^{1/4}\sin\mu(L,s_i) , \tag{2.14}$$

with an r.m.s. value of

$$\sigma_x = \beta_0\Delta\gamma\sqrt{N_{RF}/(\gamma_0\gamma_f)}\,\sigma_\alpha . \tag{2.15}$$

Again we require the displacement to be small compared to the beam size so that the emittance is not diluted. The resulting tolerance on the jitter of the tilt angle is

$$\sigma_\alpha \ll \sigma_{xf}/(\beta_0\Delta\gamma)\sqrt{\gamma_0\gamma_f/N_{RF}} . \tag{2.16}$$

In the case of CLIC this means $\sigma_\alpha \ll 47$ μrad or, in terms of the displacement Δx of one end of the section with respect to the other, $\sigma_{\Delta x} \ll 12\mu$m.

2.5 Rotation Jitter of Quadrupoles

Damping rings naturally produce asymmetrical emittances, and one may want to preserve the asymmetry during acceleration. For instance, luminosity considerations and final focus design strongly favour flat beams in linear colliders. Of course, preservation of asymmetrical beams requires the control of the coupling between vertical and horizontal plane, as is introduced by rotated quadrupoles for instance.

Let us consider a beam which is much larger in the y- than in the x-direction. The y-motion is therefore relatively unperturbed, while the x-motion is strongly influenced.[5] A quadrupole of focal length f_i at position s_i, which is rotated by a small angle Θ_i, introduces a kick in the x-motion[3] which can be described by a local curvature of

$$\varrho(s)^{-1} = -2\frac{\Theta_i}{f_i}y(s_i)\delta(s-s_i) . \tag{2.17}$$

Comparing Eq. (2.17) with Eq. (2.8) we can proceed as in Section 2.3 and we find for the reference trajectory at the end of the machine:

$$x(L) = 2\frac{\beta_0}{f_0}y_0(\gamma_0/\gamma_f)^{1/4}\sum_{i=1}^{N_q}\Theta_i\cos\mu_y(L,s_i)\sin\mu_x(L,s_i) . \tag{2.18}$$

In deriving Eq. (2.18) we have included the adiabatic damping of the y-motion and we have assumed a lattice of constant phase advance as before. The r.m.s. value of x(L) follows from an ensemble average as

$$\sigma_x = \frac{\beta_0}{f_0}\sqrt{N_q}\,\sigma_{yf}\,\sigma_\Theta . \tag{2.19}$$

The trajectory displacement must be small compared to the beam size in the x-direction and the resulting tolerance of random rotations is given by

$$\sigma_\Theta \ll \frac{\sigma_{xf}}{\sigma_{yf}}\frac{f_0}{\beta_0}N_q^{-1/2} . \tag{2.20}$$

In CLIC the beam width in the x-direction is comparable with the one in the y-direction and both motions will strongly perturb each other. The above approximation is therefore not valid.

2.6 Chromatic Variation of the Reference Trajectory

From Eq. (2.3) we note that the reference trajectory has a chromatic dependence. In phase space this manifests itself by a changing position of the beam ellipse with s. The emittance becomes diluted. To estimate this effect we consider again the lateral displacement of quadrupoles as in Section 2.3. But this time we do not take into account the adiabatic damping and the β-function in order to simplify the calculation (it changes the result very little). We solve Eq. (2.3) with (2.8) and find for the chromatic part of the trajectory at the end of the linac:

$$x_{\delta_i} = x_i(\delta) - x_i(0) = \frac{\Delta x_i}{kf}\left[\frac{\sqrt{1-\xi\delta}}{1+\delta} \sin k \frac{L-s_i}{\sqrt{1-\xi\delta}} - \sin k(L-s_i)\right] . \quad (2.21)$$

In Eq. (2.21) we can distinguish two regimes:

i) Small chromatic variation $^1/_2\, \xi\delta kL \ll 1$.
Developing Eq. (2.21) up to first order in δ and $^1/_2\, \xi\delta kL$ yields

$$x_{\delta_i} \approx -\frac{1}{2}\xi\delta\frac{\Delta x_i}{f}(L-s_i)\cos k(L-s_i) . \quad (2.22)$$

For a sequence of N_q displaced quadrupoles we perform an ensemble average as in Section 2.3 and obtain the r.m.s. value of the trajectory as

$$\sigma_{x\delta} = \frac{1}{\sqrt{6}}\xi \sin\frac{\mu}{2}\sqrt{N_q}^{\,3}\,\delta\,\sigma_{\Delta x} , \quad (2.23)$$

where we have used $L = N_q\ell_c$ and $\ell_c/f = 2\sin\mu/2$.

ii) Large chromatic variation $^1/_2\, \xi\delta kL \gtrsim 1$.
This is the typical regime for linear colliders. Keeping only the zero-order term in δ of expression (2.21) gives

$$x_{\delta_i} \approx -2\frac{\Delta x_i}{kf}\sin\left[\frac{1}{4}\xi\delta k(L-s_i)\right]\cos\left[k\left(1-\frac{1}{4}\xi\delta\right)(L-s_i)\right] , \quad (2.24)$$

and after averaging over all N_q kicks we obtain the r.m.s. value of approximately

$$\sigma_{x\delta} = 2\frac{\sin\mu/2}{\mu/2}\sqrt{N_q}\,\sigma_{\Delta x} . \quad (2.25)$$

As before, we require the chromatic variation of the reference trajectory, Eqs. (2.23) and (2.25), to be small compared to the beam size. The resulting limit on the tolerable displacement jitter is then

$$\sigma_{\Delta x} \ll \frac{\sigma_{xf}}{\sqrt{N_q}\sin\mu/2}\times\begin{cases}\sqrt{6}/(\xi\delta N_q) & \text{for } ^1/_2\,\xi\delta kL \ll 1\\ \mu/4 & \text{for } ^1/_2\,\xi\delta kL \gtrsim 1\end{cases} \quad (2.26)$$

In CLIC the total phase advance kL is 2220 rad and an energy spread of 1% clearly results in a strong chromatic variation. The tolerable displacement jitter must therefore be smaller than 0.025 μm.

2.7 Chromatic Variation of the Corrected Trajectory

In the foregoing section we have seen, Eq. (2.24), that the chromatic deviation of the trajectory, due to a displaced quadrupole, grows like $\sin[^1/_4\,\xi\delta k(s-s_i)]$ along the linac. In principle, if the displacement is static, we can correct the trajectory and keep it within some limits. To estimate the chromatic variation of such a corrected machine let us, at first, consider a single displaced quadrupole in an otherwise ideal machine. For simplicity we again neglect acceleration. The kick at $s = s_i$ can be detected at $s = s_i + \ell_c$ and corrected by two kicks of

$-(2\Delta x_i/f)\cos k\ell_c$ at $s = s_i + \ell_c$,

and

$\Delta x_i/f$ at $s = s_i + 2\ell_c$.

For the rest of the linac a particle with reference momentum will have a zero trajectory. However, an off-momentum particle has a trajectory given to first order in δ by

$$x_{\delta_i} = x_i(\delta) - x_i(0) = -2\xi\sin^2\frac{\mu}{2}\delta\Delta x_i\sin\left[k\left(1+\frac{1}{2}\xi\delta\right)(L-s_i-\ell_c)\right] \quad (2.27)$$

at the end of the linac. Averaging over all N_q magnets we obtain the r.m.s. value of the trajectory as

$$\sigma_{x\delta} = \sqrt{2}|\xi|\sin^2\frac{\mu}{2}\delta\sqrt{N_q}\,\sigma_{\Delta x} . \quad (2.28)$$

The limit of the tolerable quadrupole displacement in a corrected machine is then

$$\sigma_{\Delta x} \ll \sigma_{xf}/\left(\sqrt{2}|\xi|\sin^2\frac{\mu}{2}\delta\sqrt{N_q}\right) . \quad (2.29)$$

For CLIC, it follows from Eq. (2.29) that $\sigma_{\Delta x} \ll 5.0\ \mu$m if the energy spread is 1%.

3. Wake fields and SBBU

In the case of relativistic bunches one wants to know the beam-induced fields, averaged over a certain length, which act on a co-travelling particle. These fields are called wake fields. Their longitudinal component causes the beam to lose energy and induces an energy spread along the bunch. Their transverse components, excited by the head of the bunch, deflect the tail from the axis and cause a blow-up or even a BBU.

As an example, Fig. 1 shows the wake fields of a Gaussian bunch in a CLIC accelerating structure. The longitudinal component is the one belonging to monopole (axis-symmetric) fields and is experienced by a probing particle at position z in the bunch independent of its transverse position. The transverse components belong to dipole fields. They are given per unit offset r of the bunch. A probing particle at position z and with an azimuthal angle ϕ with respect to the bunch experiences the fields

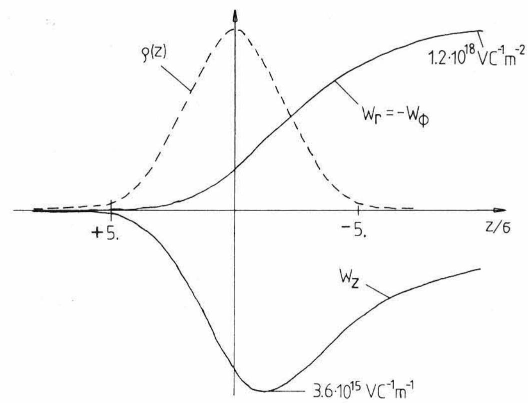

Fig. 1 Wake fields of the CLIC RF structure.

$r\cos\phi\,W_r(z)$ and $-r\sin\phi\,W_\phi(z)$

independent of its radial distance. All other wake fields, with higher azimuthal dependence, have normally a very small effect on low-emittance beams.

If all dimensions are scaled with the RF wavelength, we find that the wake fields scale as

$$W_\parallel \propto \lambda^{-2} , \quad W_\perp \propto \lambda^{-3} . \quad (3.1)$$

That is the reason why wake-field effects are particularly cumbersome in linear colliders where the RF frequency has to be high in order to keep the power consumption within reasonable limits. To reduce wake-field effects one can counteract them, as will be discussed in the next section, or one can reduce the wake fields themselves. Since they scale with the aperture 2a as

$$W_\parallel \propto a^{-1} , \quad W_\perp \propto a^{-2.8} , \quad (3.2)$$

and with the bunch length as

$$W_\parallel \propto \sigma_z^{-1} , \quad W_\perp \propto \sigma_z \quad \text{for } \sigma_z \lesssim 0.01\lambda_{RF} , \quad (3.3)$$

one may increase the aperture and decrease the bunch length. But, with increasing aperture the machine length also increases proportionally to $a^{-0.8}$ for equal power consumption. Short bunches have so-called higher-mode losses proportional to σ_z^{-1} and are not easy to make.

Now, let us consider a bunch travelling in a periodic focusing channel. The head of the bunch will excite transverse wake fields which act periodically on the tail in a way that the defocusing periods are longer than the periods of focusing. The net result is a blow-up of the tail. This effect has been analysed with a perturbation method.[8] Starting from the equation of motion for a particle

$$\frac{d}{ds}\left[\gamma(z,s)\frac{d}{ds}x(z,s)\right] + K(z,s)\gamma(z,s)x(z,s) = \frac{e}{mc^2}\int_z^\infty dz'\varrho(z')W_\perp(z'-z)x(z',s)\,, \tag{3.4}$$

where z is the longitudinal position in the bunch and s the coordinate along the linac, an asymptotic formula for the displacement x was derived:

$$x(z,L)/x_0 \approx \sqrt{E_0/(6\pi E_f)}\,\eta^{-1/6}\exp(3\sqrt{3}\eta^{1/3}/4)\times \cos(k_0L - 3\eta^{1/3}/4 + \pi/12)\,, \tag{3.5}$$

where η is a strength parameter given by

$$\eta = \left(\frac{1}{2}-\frac{z}{\ell}\right)^2\frac{eQW_0}{k_0E_0G}\ln\frac{E_f}{E_0} \gg 1\,. \tag{3.6}$$

Equation (3.5) was derived under the assumption of smooth focusing, $K(z,s) = k_0^2$, linear acceleration, $E(s) = E_0(1+Gs)$, and a bunch of constant charge distribution with length ℓ and total charge Q. The wake field depends linearly on z with a slope W_0/ℓ.

It is exactly the exponential factor in Eq. (3.5) that characterizes the blow-up. Note also that the phase depends on η, with the tail lagging behind the head. This phase lag is essential for the instability to develop. It also indicates how to cure it by introducing a spread in the wave number, which cancels the phase lag coming from the wake field.

In the case of CLIC the strength parameter is $\eta = 492$ at the bunch end, $z = -\ell/2$ ($\ell = 2\sqrt{3}\,\sigma_z$), and the tail is blown up by a factor of 165.

4. 'Landau Damping'

The stabilization of the transverse blow-up is most easily studied with the two-particle model. The bunch is modelled by two macro-particles, each of charge Q/2 and separated longitudinally by a constant distance $\ell = 2\sigma_z$. We neglect acceleration and assume smooth focusing. The trailing particle has a slightly different energy $E + \Delta E$. Then, the equations of motion are

$$x_1'' + k^2x_1 = 0$$
$$x_2'' + (k+\Delta k)^2x_2 = \frac{eQW_\perp(\ell)}{2E}x_1\,. \tag{4.1}$$

The first particle does not feel a transverse wake field and undergoes free betatron oscillation. The second particle experiences the wake force of particle 1 and has a wave number different by $\Delta k/k = \xi\Delta E/E$. We can easily solve Eqs. (4.1), with initial conditions $x_1(0) = x_2(0) = x_0$, and obtain for the difference between the two trajectories

$$x_2(s) - x_1(s) = -x_0\left[2 - \frac{eQW_\perp(\ell)}{2Ek\Delta k}\right]\sin\left(\frac{1}{2}\Delta ks\right)\sin\left(k+\frac{1}{2}\Delta k\right)s\,. \tag{4.2}$$

In Eq. (4.2) we distinguish three different cases:

i) No energy spread, $\Delta E = \Delta k = 0$.
The difference grows linearly with s, i.e.

$$x_2(s) - x_1(s) = x_0\frac{eQW_\perp(\ell)}{4Ek}s\sin ks\,. \tag{4.3}$$

Note that the growth is even much faster for a distributed charge, as shown in the previous section.

ii) Very small but finite spread, $\Delta k \neq 0 <<< k$.
The difference of the trajectories becomes

$$x_2(s) - x_1(s) \approx x_0\frac{eQW_\perp(\ell)}{2Ek\Delta k}\sin\left(\frac{1}{2}\Delta ks\right)\sin\left(k+\frac{1}{2}\Delta k\right)s\,. \tag{4.4}$$

It is beating with zeros at $\Delta ks = 2n\pi$ and maxima at $\Delta ks = (2N+1)\pi$.

iii) 'Landau damping',

$$\Delta k = \frac{eQW_\perp(\ell)}{4Ek}. \tag{4.5}$$

Both trajectories are equal. The extra phase advance of particle 2 owing to its lower energy, cancels the wake-field kick from particle 1.

Case (iii) is referred to as 'Landau damping' although it is somewhat different. The lack of growth is simply due to a cancellation of forces. More correctly it is called BNS damping, referring to the authors[9] who first studied the effect. For a real distributed bunch the criterion, Eq. (4.5), looks a little bit different

$$2k(z)\frac{d}{dz}k(z) = \frac{eQ}{2E}\frac{d}{dz}W_\perp(z)\,. \tag{4.6}$$

with the coordinate z going from head to tail.

By inspection of Fig. 1 we note that the transverse wake field of the CLIC accelerating structure is well approximated by a linear function in the core of the bunch. The stability criterion, Eq. (4.6), then requires a linear spread in the wave number with a slope $dk/dz = 215\ \mathrm{m}^{-2}$. This translates into an r.m.s. spread of $\Delta k/k = 3.4\%$ or an r.m.s. energy spread of $\Delta E/E = -2.7\%$.

5. RF Focusing

Instead of creating the spread in betatron wave number, necessary for Landau damping, by inducing an energy spread, one may design a system which focuses every part of the bunch differently. An RF quadrupole system does this. For highly relativistic particles it consists simply of a normal disk-loaded wave guide with slot-shaped irises. A charge passing off centre, in the direction of the slot, experiences a transverse magnetic field only and is focused. Perpendicular to the slot it feels the focusing magnetic force plus a defocusing electric force which is twice as large. The focusing gradient is[10]

$$B' = \pi\hat{E}_a\sin\phi/(c\lambda_{RF})\,, \tag{5.1}$$

with $\hat{E}_a$ being the peak accelerating field and ϕ the RF phase angle measured backwards from the crest. For CLIC the peak gradient is of the order of 100 T/m.

Every longitudinal position z within the bunch corresponds to a different phase angle ϕ and therefore a different focusing force. The wave-number spread is

$$\Delta k/k \approx \frac{4\pi z}{\lambda_{RF}\mu_0}\tan\frac{\mu_0}{2}\cot\phi_0 \quad \text{if} \quad \sigma_z \ll \lambda_{RF}\,. \tag{5.2}$$

The subscript 0 refers to the bunch centre.

In the case of CLIC, with a focusing system completely made of RF quadrupoles, the r.m.s. wave-number spread is $\Delta k/k = 23.8\%$ at $\phi_0 = 20°$. For Landau damping we only need $\Delta k/k = 3.4\%$, see Section 4, and we can therefore have a normal external focusing with some fraction of RF focusing. Also, the focusing strength can be reduced, thus allowing for relieved jitter tolerances.

6. Numerical Simulation

The only way to solve the complete equation of motion (3.4) is by numerical simulation. A computer code[11] has been written which takes into account the discrete nature of a real machine. It calculates transfer matrices for such elements as drift spaces, quadrupoles, and accelerating structures with wake fields. The bunch is divided longitudinally into a number of slices and each slice affects the motion of all slices behind it. All particles in one slice are assumed to have the same energy and velocity of light. The code transports each centroid (x, x′, y, y′) of a slice through the lattice to first order by multiplication of the transfer matrices. Later the code was extended[12] and made available under the name LTRACK. It has been used extensively to model the SLAC Linear Collider (SLC).[4]

Instead of tracking the slice centroids from element to element it is sufficient in many applications to smooth the focusing and to integrate Eq. (3.4). This is conveniently done by writing first-order differential equations for every centroid.[13] We have written a code LINBUNCH[14] which uses this method. The code includes δ-function wake fields, lateral random offsets of quadrupoles and RF sections, and also fractional RF focusing. In the following we give results obtained with LINBUNCH. They all refer to the CLIC parameters,

Table I. In contrast to Ref. 14 we use here the chromaticity of a 90° FODO lattice and not $\xi = -0.5$ as for smooth focusing.

Figure 2 shows the position of the slice centroids at the end of the linac in the case of zero energy spread. As calculated [Eq. (3.5)], the bunch tail is heavily blown up and lags behind in phase. The blow-up is not quite exponential and is therefore weaker—only a factor of 130 instead of 165 as predicted, since the transverse wake field grows less than linearly.

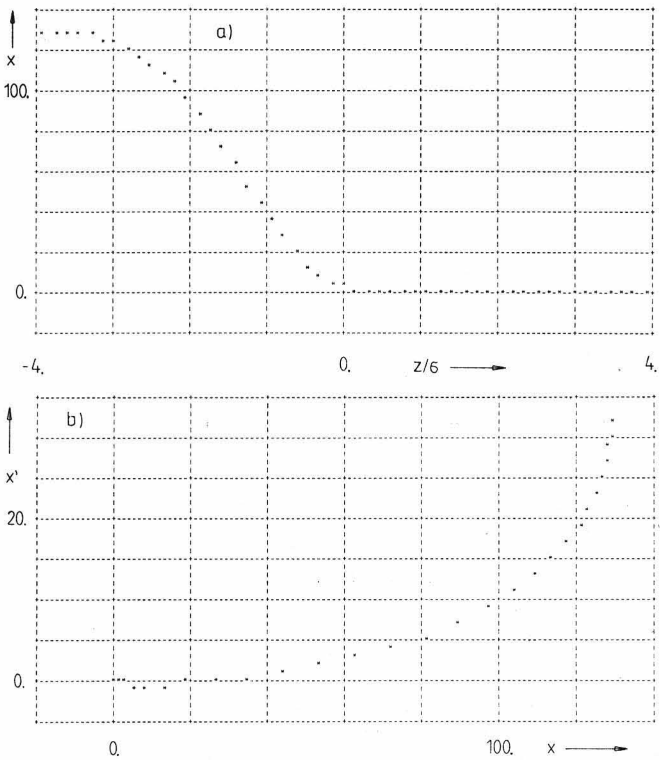

Fig. 2 a) Bunch shape and b) phase plot in the case of zero energy spread and unit value injection offset (at the end of the CLIC linac).

In principle, one can suppress any blow-up by shaping the wave number following Eq. (4.6). All slices would then oscillate coherently with an amplitude given by adiabatic damping. In the case of a real energy spread, resulting from the superposition of the RF field and the longitudinal wake field, the fit of the spread in wave number is poor. In order to make it fit, at least in the bunch centre, the bunch has to be let slip behind the crest of the RF field by a certain angle ϕ_{RF}. This means loss in accelerating gradient and an energy spread of the order of 3%. To reduce the induced spread, the first part of the linac can be driven with a negative phase required for damping, and the second part with a positive phase allowing for some blow-up. The position and the value of the phase jump are found by a trade-off between damping and energy spread. Table II lists a typical set of parameters for CLIC. Good damping was found to occur for an RF phase angle

Table II

A typical set of operational parameters and jitter requirements for CLIC; σ_x, σ_C, σ_Q are the r.m.s. values of the injection jitter and the displacement jitter of RF sections and quadrupoles, leading to 25% emittance increase

External focusing only:			
L_1 = 6.5 km:	$\phi_{RF1} = -7°$,	$\Delta E_{a1} = -3.6\%$,	$\Delta\gamma_1 = 2.7\%$
L_2 = 6.0 km:	$\phi_{RF2} = +10°$		
L_f = 12.5 km:	$\Delta E_f = -4.0\%$,	$\Delta\gamma_f = 1.2\%$	
	$\sigma_x = 9\ \mu m$,	$\sigma_C = 10\ \mu m$,	$\sigma_Q = 0.07\ \mu m$
Added on RF quadrupoles:			
	60° FODO lattice, $B'_{quad} = \hat{B}'_{RF}$,		$\phi_0 = 0°$, $\lambda_{\beta 0} = 25$ m
	$\phi_{RF} = +5°$,	$\Delta E_f = -3.3°$,	$\Delta\gamma_f = 0.9\%$
	$\sigma_x = 44\ \mu m$,	$\sigma_C = 11\ \mu m$,	$\sigma_Q = 1.4\ \mu m$

ϕ_{RF} between $-5°$ and $-9°$, corresponding to an energy spread $\Delta\gamma$ between -2.3% and -3.1%, in agreement with Eq. (4.6). This energy spread could be reduced to 1.2% at the end of the linac by choosing $\phi_{RF} = +10°$ in the second part of the machine.

Figures 3, 4, and 5 show the position of the slice centroids at the end of the linac for the operational parameters given in Table II. The case with an initial offset of the bunch (Fig. 3) demonstrates that the slices are well centred around the axis. The head particles oscillate freely with an amplitude given by adiabatic damping. They are out of phase, so their wake fields do not add up but interfere positively and damp the core of the bunch. In the next example (Fig. 4) the bunch is injected on-axis, but the RF structures are assumed to be laterally displaced in a random way. The head particles, again, do not experience any wake fields and stay aligned. The tail is blown up slightly. Finally, Fig. 5 shows the case of random quadrupole

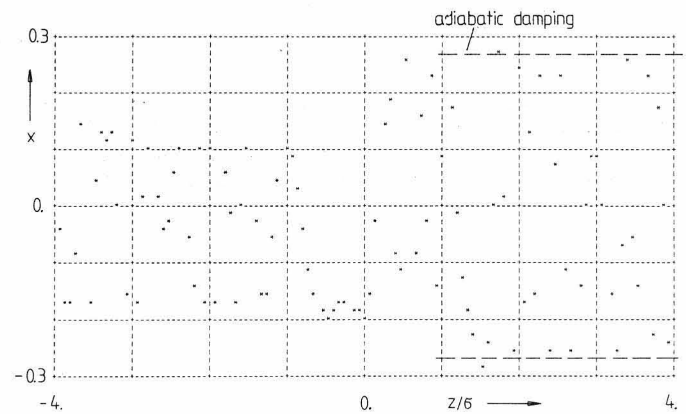

Fig. 3 Bunch shape in the case of real energy spread, a perfectly aligned machine, and unit value injection offset (at the end of the CLIC linac).

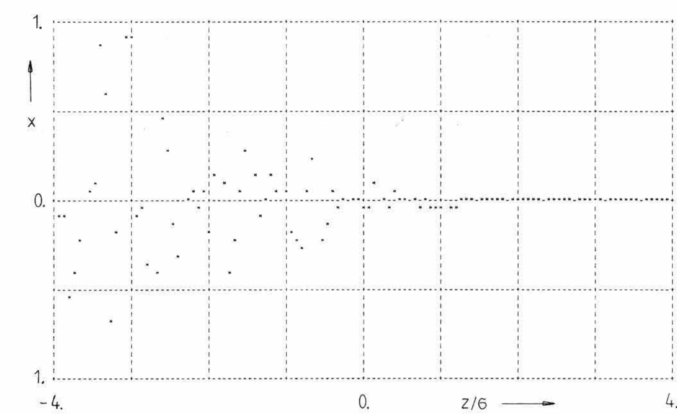

Fig. 4 Bunch shape in the case of random RF structure displacement, unit r.m.s. value, and on-axis injection (at the end of the CLIC linac).

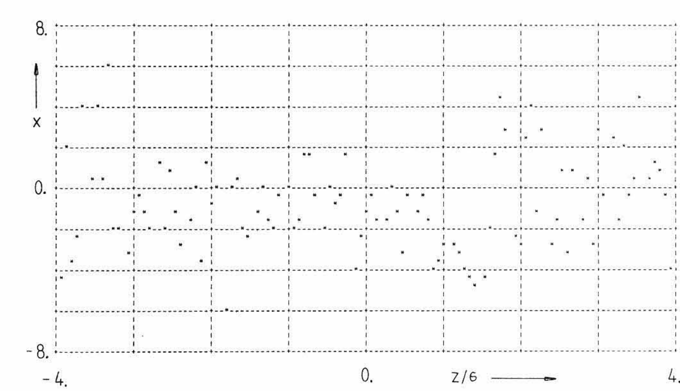

Fig. 5 Bunch shape in the case of random quadrupole displacement, unit r.m.s. value, and on-axis injection (at the end of the CLIC linac).

displacements. Head and tail are blown up equally. The core is slightly damped by wake fields. The resulting emittance blow-up allows us to fix upper limits for the r.m.s. jitter values in these three examples. As expected, the case of displaced quadrupoles is by far the worst. A tolerable r.m.s. displacement of 0.07 μm seems hard to achieve, although it is already larger than for a single particle [Eq. (2.12)].

Table II also shows the typical values for a focusing system consisting of external magnets plus RF quadrupoles. The RFQ system is thus designed so that the bunch centre is located at zero gradient, $\phi_0 = 0$, and the peak RF gradient is equal to the external gradient. This means that the RF system only provides a spread in betatron wave number and no focusing. The phase angle with respect to the normal RF can now be chosen so as to minimize the energy spread, yielding $\phi_{RF} = 5°$ and $\Delta\gamma = 0.9\%$. The slope of the wave number at the bunch centre and at injection is

$$dk/dz = \frac{-8\pi^2 \tan \mu_0/2}{(\mu_0 \lambda_{RF} \lambda_{\beta 0})} = 4353/\lambda_{\beta 0} \tag{6.1}$$

for $\mu_0 = 60°$. Since for Landau damping we require $dk/dz = 215\ m^{-1}$ (Section 4), we can reduce the focusing strength so that $\lambda_{\beta 0} = 20$ m. In fact, the simulations show that maximum damping happens at $\lambda_{\beta 0} = 25$ m. Figure 6 gives the bunch shape for these parameters in the

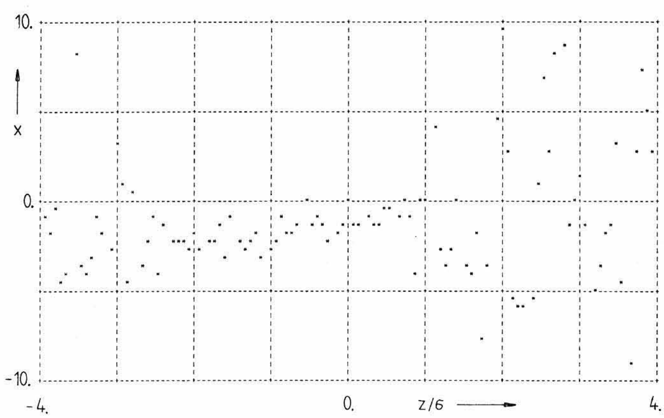

Fig. 6 As Fig. 5, but with an added on RF-quadrupole system (parameters in Table II).

case of random displacement of the external quadrupoles. The resulting limits for the tolerable jitter motion are much less serious than for external focusing only, but the limit for the quadrupole motion is still a big problem.

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